Using Distributional Data Analysis Tools to Investigate the Glucose Density Distribution of the Mean Daily Glucose Values (eAG) for Three Type 2 Diabetes Patients Over an 18-Month Period Based on GH-Method: Math-Physical Medicine (No. 510)

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Abstract
The author read an article recently, “Glucodensities: a new representation of glucose profiles using distributional data analysis,” dated August 19, 2020, from arxiv.org (see Reference 1).

Incidentally, he has also made two further improvements on his glucose data analysis with his collected big data of sensor glucoses via a continuous glucose monitoring sensor device (CGM).

First, in addition to using the HbA1C, which is the mean value of the past 115 days of red blood cells carried glucoses, of a patient is the golden standard in evaluating diabetes conditions. He investigates the glucose fluctuation or GF (glucose excursion or glycemic variability) and then transforms the GF values from a wave’s time-space into an energy’s frequency-space via Fourier transform operations. Using this approach, he can then guesstimate the degree of damage on internal organs caused by the energies associated with glucose fluctuations. Although the GF research is one step deeper compared to the study of mean value of glucoses, such as HbA1C, it is still not deep enough to provide additional details and useful information hidden within the glucose waves.

Second, he realized that the average values or mean values of glucoses defined by the American Diabetes Association (ADA) such as the HbA1C or Time in Range (average glucose within a range) can only provide partial overviews of diabetes conditions. However, these basic biomarkers are still missing some hidden internal turmoil, i.e. glucose vibrations or severe stimulations, throughout certain selected timeframes due to all types of external and/or internal stimulators. Therefore, he has defined another term known as the glucose density (GD) in order to explore more and different information hidden within the glucose data and their waveforms. GD is defined as the occurrence frequency at a specific glucose value, for example 2.1% occurrence rate at 110 mg/dL glucose value over a selected time period of collected sensor glucoses. In this way, he can then calculate and examine each glucose value’s occurrence rate within a glucose range that is suitable to a specific patient.

If this glucose examination method is accepted by the medical community, it would be an extremely beneficial tool for doctors to be able to quickly study the conditions of their diabetes patients. Furthermore, the author has also programmed his algorithm into an iPhone APP software. Through the combination of his published papers and medical books along with a widely distributed APP for patient’s use in the future, he believes that worldwide type 2 diabetes (T2D) patients can benefit from his research work.

As a part of his follow-on research tasks, he plans to further examine his glucose density % resulted from certain food/diet nutritional types and exercise intensity levels. Hopefully, his research papers would not be limited within the scope of a “descriptive style using 26 alphabets” but instead as a “quantitative style using 10 digits”. Numbers do not lie as long as we don’t use fake, unorganized, and/or uncleaned data. Statistics is a tricky tool to use because it has the obvious characteristics of garbage in and garbage out (GIGO). It is also important to know that by using statistics with different selected time-windows for certain studies, the results would pro-
vide varying conclusions.

This part of the introduction assists the author to organize and summarize his thoughts and forces him to express the abstract ideas and theoretical concepts by writing them on paper, which has helped him before. Actually, there is nothing fancy about the above-mentioned analysis approaches; however, he would like to re-iterate what he has learned in the past and apply the mathematical tools to interpret certain interesting biophysical phenomena or solving some biomedical challenges.

In summary, the author has chosen to perform research work using the GD concept but with his own developed proprietary research software algorithm and APP software along with the collected CGM sensor glucose data of 3 T2D patients over the same period of 1.5 years from 3/8/2020 through 9/8/2021. Furthermore, he has also selected a consistent glucose range covering 39 mg/dL to 249 mg/dL with a total of 211 glucose points on the x-axis of GD diagram and the glucose density percentages (GD %) amplitude on the y-axis.

For most clinical practices, medical doctors use HbA1C as the golden standard to evaluate the conditions of their T2D patients. The HbA1C value represents the average glucose value of all glucoses over the past 90 to 120 days or perhaps 115 days based on the red blood cell’s lifespan; however, the HbA1C alone cannot tell doctors additional information other than the mean value. In fact, many other useful glucose information are still hidden within the body. Biomarkers such as the glucose variability (GV) or the glucose fluctuation (GF) can provide more information regarding the damage of a patient’s internal organs via glucose excursion which causes many diabetic complications, including both micro-vascular and macro-vascular diseases. Furthermore, the ADA has issued certain guidelines on time in range (TIR), time above range (TAR), and time below range (TBR) that can offer a general idea of how glucoses are distributed in three different ranges; TIR for normal conditions, TAR for hyperglycemic conditions, and TBR for hypoglycemic conditions. However, they are still the mean values within each Range, e.g. <70, 70-180, >180.

Therefore, these three biomarkers, HbA1C, GF, and Time in/above/below Range (TxR), are still missing the needed ability to provide more detailed glucose variations. Even the GV or GF are still dealing with another kind of mean values. Based on these observed shortcomings, glucose density or GD can fill in certain gaps of “missing information” from these three biomarkers, HbA1C, GF, and TxR.

By using the CGM sensor glucoses and the author developed APP software program on the iPhone, he can generate three sets of GD data and GD curves for 3 different T2D patients individually and then combine them into one diagram with the same scales of both x-axis and y-axis. Through a closer examination of the combined diagrams of GD % for the 3 T2D patients, the author can describe his 4 key observations:

1. The waveforms of the three T2D patients are extremely similar to each other with their GD % peak levels and associated glucose ranges within the same proximity to each other. This observation means that all 3 patients have similar diabetes conditions over the past 18 months. Actually, both the male and female patients, who are 70+ years old, have long-term and severe T2D history for over 20 years. However, through a stringent lifestyle management program, they have been able to put their diabetic conditions under control. According to the ADA 2021 consensus report, both male and female patients belong to a “remission stage” at present time (see papers No. 505 and No. 506). The young patient is 48-year-old male with a 6-year history of T2D whose glucose conditions are not as severe as the 70+ year old diabetes patients.

2. The author has selected a rather strict definition of “normal glucose range” within 70-120 mg/dL. Using this definition, the majority of their glucose data are located within this normal glucose range: male patient has 52%, female patient has 52%, and young patient has 59% of their CGM glucoses within this 70-120 mg/dL range. Furthermore, their average GD % value within this “normal glucose range” are also low, 1.7% to 1.9%. This indicates that their average GD % peak values are low as well. The finding signifies that all of them have higher percentages of glucoses points which are lower glucose.

3. From examining the possibility of having hyperglycemia (>180 mg/dL) or hypoglycemia (<70 mg/dL), all three clinical cases have shown low possibilities (within the range of 1.2% to 2.3%). This finding reveals that they have lower risks of having either hyperglycemia or hypoglycemia.

4. The 3 GD waveforms are extremely similar in shape with the GD waveforms from the paper in Reference 1, except for their peak GD percentages are within 1% to 2.5% (with only three T2D patients over an 18-month timeframe) while the peak GD percentages in Reference 1 are within 2% to 6% (with many more severe T2D patients over relatively shorter timeframes).

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Methods

MPM Background

To learn more about his developed GH-Method: math-physical medicine (MPM) methodology, readers can read the following three papers selected from his ~500 published medical papers.

The first paper, No. 386 describes his MPM methodology in a general conceptual format. The second paper, No. 387 outlines the history of his personalized diabetes research, various application tools, and the differences between biochemical medicine (BCM) approach versus the MPM approach. The third paper, No. 397 depicts a general flow diagram containing ~10 key MPM research methods and different tools.

In particular, his paper No. 453 illustrates his GH-Method: math-physical medicine in great details, “Using Topology concept of mathematics and Finite Element method of engineering to develop a mathematical model of Metabolism in medicine in order to control various chronic diseases and their complications via overall health conditions improvement”.

Glucose Density (GD%)

The author took the following photo directly from the beginning part of Section 3 in the Glucodensities’ paper (Reference 1), because he is not quite familiar with how to write English articles with LATEX math symbols using Page application on iPad.

\[ \text{3 Definition and Estimation of the Glucodensity} \]

For patient \( i \), denote the gathered glucose monitoring data by pairs \((t_j, X_j)\), \( j = 1, \ldots, m_i \), where the \( t_j \) represent recording times that are typically equally spaced across the observation interval, and \( X_j \) is the glucose level at time \( t_j \). Note that the number of remark \( m_i \), the spacing between them, and the overall observation length \( T \) can vary by patient. One can think of these data as discrete observations of a continuous latent process \( Y(t) \), with \( X_j = Y(t_j) \). The glucodensity for this patient is defined in terms of this latent process as:

\[ \tilde{f}(\sigma) = \frac{1}{T} \int_{\sigma} \tilde{f}(\sigma) d\sigma \]

is the proportion of the observation interval in which the glucose levels remain below \( \sigma \). Since \( \tilde{f}(\sigma) \) are increasing from 0 to 1, the data to be modeled are a set of probability density functions \( \tilde{f}_i \), \( i = 1, \ldots, n \).

Of course, neither \( \tilde{f}_i \) nor the glucodensity \( \tilde{f} \) is observed in practice, but one can construct an approximation through a density estimate \( \hat{f}(\sigma) \) obtained from the observed sample. In this case of COM data, the glucodensity may have different support and shape. Therefore, we suggest using a non-parametric approach to estimate each density function. For example, using a kernel-type estimator, we have:

\[ \hat{f}(\sigma) = \frac{1}{2 \pi} \sum_{j=1}^{m_i} f_j(\sigma - X_j) \]

where \( \hat{f} \) is the smoothing parameter and \( f_j(\sigma) = \frac{1}{2} K(\frac{\sigma}{h}) \). The choice of \( K \) does not have a big impact on the efficiency of the estimator, but the value of \( h \) is crucial.

For the case of any individual patient \( i = 1 \), the author can then ignore the index \( i \) and only use \( j = 1, \ldots, T \), where \( T \) is the overall observation length of glucooses, or in his case, the total \( T \) is 211 (from 39 mg/dL to 249 mg/dL).

His gathered CGM glucose data by pairs \((t_j, X_j)\), \( j = 1, \ldots, T \), where the \( X_j = Y(t_j) \) = CGM glucose and the \( t_j \) represents recording times (every 15 minutes for 96 times each day). Therefore, he can simplify the above equation in the photo further into a simplified equation for one patient only. The GD for himself can be defined in terms of a continuous format as follows:

\[ T \]

\[ GD(x) = \frac{Y(t)(dt)}{T} \]

\[ with \ x_1 < Y(t) < x_2 \]

where \( x_1 \) and \( x_2 \) are \( \frac{1}{2} \) boundaries of his selected glucose range.

The GD for himself can also be defined in terms of a discrete format as follows:
$T$

$GD(x) = \left( \sum_{j=1}^{Y(t)} \right) / T$

with $x_1 < Y(t) < x_2$

where $x_1$ and $x_2$ are boundaries of his selected glucose range.

He uses above equations to develop his APP software program on iPhone device to calculate three GD% values of these three T2D patients and then draw the three associated GD% curves.

Results

Figure 1 shows the 90-days moving average CGM sensor glucose (eAG) curves for the three T2D patients during a period of 1.5 years from 3/8/2020 to 9/8/2021. The related information of these T2D patients are listed below in a format of (age, years of having T2D, average eAG):

Male: (74, 26, 115 mg/dL)
Female: (73, 21, 108 mg/dL)
Young: (48, 6, 107 mg/dL)

Figure 1: Glucose Density (GD%) curves of FPG, PPG, eAG (5/8/2018 - 9/8/2019)

Figure 2: The author’s combined 3 GD% curves of FPG, PPG, eAG, and Reference 1’s GD% curve

The bottom diagram in Figure 2 illustrates the 3 GD% waveforms in comparison to the GD% waveforms from Reference 1 showing similar waveforms. Except, these 3 patients’ peak-lumps of GD% are located within 1.5% to 2.5% over 18 months, while the peak-lumps GD% in Reference 1 are within 2% to 6% (with many more severe T2D patients over relatively shorter timeframes, e.g. 1-2 weeks).

Figure 3 depicts two summarized observations of GD%. The top diagram demonstrates the summation of GD% and the average GD% of 3 patients within the “normal glucose range” of 70-120 mg/dL. The following table lists in the format of (summation GD%, averaged GD%):

<table>
<thead>
<tr>
<th></th>
<th>Summation GD%</th>
<th>Averaged GD%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>52%</td>
<td>1.7%</td>
</tr>
<tr>
<td>Female</td>
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<td>1.7%</td>
</tr>
<tr>
<td>Young</td>
<td>59%</td>
<td>1.9%</td>
</tr>
</tbody>
</table>

The top diagram of Figure 2 shows the combined GD% curves of these three patients. They have similar GD% “peak-lumps” located within the glucose range of 90 mg/dL to 120 mg/dL with GD% amplitudes within the range of 1.5% to 2.5%. However, most of the GD% values near the glucose data point $x_1$ (39-65 mg/dL) and the glucose data point $x_2$ (200-249 mg/dL) having 0% of GD% values. Therefore, the author has truncated most of the 0% GD values near point $x_1$ and point $x_2$, and then combined the 3 GD% curves into one diagram with identical glucose ranges from 39 mg/dL ($x_1$) to 249 mg/dL ($x_2$) which has a total glucose range value of 211, i.e. $T = 211$. 

The following table lists in the format of (summation GD%, averaged GD%):

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The above equations to develop his APP software program on iPhone device to calculate three GD% values of these three T2D patients and then draw the three associated GD% curves.
Both of the male and female patients have identical summation of GD% value (52%) and average of GD5 value (1.7%). The young patient has higher values for summation of GD% value (59%) and average of GD% value (1.9%) which means he has a better glucose condition than the elderly male and female patients.

The bottom diagram demonstrates the possibilities of having either hyperglycemia (>180 mg/dL) or hypoglycemia (<70 mg/dL). The following table lists in the format of (hyperglycemia %, hypoglycemia %):

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<td>(1.5%, 1.5%)</td>
<td></td>
</tr>
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<td>(1.6%, 2.3%)</td>
<td></td>
</tr>
<tr>
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<td>(1.2%, 1.5%)</td>
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All 3 patients have low risk in terms of having hyperglycemia or hypoglycemia. With a closer examination, we would discover that the female case is higher than the male case, while the male case is higher than young case.

Conclusions
In summary, the author has chosen to perform research work using the GD concept but with his own developed proprietary research software algorithm and APP software along with the collected CGM sensor glucose data of 3 T2D patients over the same period of 1.5 years from 3/8/2020 through 9/8/2021. Furthermore, he has also selected a consistent glucose range covering 39 mg/dL to 249 mg/dL with a total of 211 glucose points on the x-axis of GD diagram and the glucose density percentages (GD %) amplitude on the y-axis.

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References
For editing purposes, majority of the references in this paper, which are self-references, have been removed for this article. Only references from other authors' published sources remain. The bibliography of the author’s original self-references can be viewed at www.eclairemd.com.

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