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Verification of Goldbach's Strong and Weak Conjectures at Infinity Using Basic and Accessible Mathematics

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Abstract

This article uses two basic arguments: an even number denoted E' is either $E' = 2^n$ x E (E being even) or it is the sum of two or more even numbers. In the first case, $E' = 2^n$ x E, this paper shows that primes p < E/2 and if E = p + q (q being prime and > E/2) and if E' = p + qx (qx being prime) then $qx = (2^n - 1)p + 2^n q$ with n idicating the rank of the multiple such that n = 0 for E, n = 1 for 2 x E, n = 2 for 2^2 x E and so on. Therefore, 2^n x E = p + $(2^n - 1)p + 2^n q$. E' and E must share one common prime p < E/2 for this equation to be verified. The primes qx follow an exponential curve with correlation coefficient $R^2 \approx 1$. This proves that the prime p close to 0 has an infinity of possible equidistant primes qx and therefore supports verification of Goldbach's conjecture to infinity. In the second case, where E' is the sum of two or more even numbers, the paper shows in a simple way that an odd number and a prime number can be the sum of a minimum of three primes. This indicates that weak conjecture of Goldbach is deduced from the strong one. Finally, the paper discusses the significance of these results for proving the truth of both Goldbach's conjectures to infinity.

Keywords: Goldbach, Conjectures, Primes, Infinity, Exponential, Sum

1. Introduction

Goldbach's strong conjecture remains unsolved in mathematics. Until now, it has been tackled by pure and applied mathematics. In applied mathematics, the use of computers and informatics has made it possible to verify its authenticity to increasingly remote limits but an empirical verification is not a formal mathematical proof in the strict sense of the term, since mathematics is a hard science that works only with axioms or theorems [1,2]. In pure mathematics, some researchers have tried to tackle this by analyzing prime numbers or their equidistance from the natural integers or by analyzing the gaps between them ([3,4]. Others have used other approaches to prove Goldbach's strong conjecture [5-8].

But why does this conjecture resist demonstrability and defy all recognized mathematicians in the field? For two key reasons. The first is that we don't know what the most common and unvarying form of even numbers is: *are they all the sum of two primes to infinity, or of more primes, and are there any numbers that are exceptions to this rule or not? The other, even more difficult reason is that, if this common form exists, will it go from 0 to infinity?* This second reason is difficult to grasp, because we don't know whether prime numbers remain as dense at infinity. Indeed the prime number theorem being proved independently by Jacques Hadamard and Charles Jean de la Vallée Poussin in 1896 using ideas introduced by Bernhard Riemann (in particular, the Riemann zeta function) (Wikipedia), rather suggests that they become rarer at infinity whereas other mathematicians have demonstrated theorems that are well known today and which indicate that prime numbers go to infinity including Dirichlet theorem, Erd″os's proof, Euler's proof, and Bertrand's postulate; see also https://proofwiki.org/wiki/Number_of_Primes_is_Infinite) [9].

The real problem is that infinity is an abstract and very relative notion. The proof is that our infinity today is much further away than it was in Goldbach's day, and with the relentless progress of computing and computers, mathematics will reach limits never before known in the history. The other astonishing aspect of this conjecture is that it can be approached by basic elementary mathematics or with more complex formal concepts that are still subjects of research today. These conjectures still merit study and reflection on their significance in arithmetic at all levels. This article is therefore another attempt to shed light on this conjecture using basic, accessible mathematics.

In two previous articles (under review elsewhere), I emphasized that Goldbach's strong conjecture depends on the equidistance of the natural integers with respect to the primes, and that this conjecture obeys the equations 6x ± 1, while proposing new methods of converting an even number into the sum of two primes. The present article is a continuation of these two papers, this time focusing on the limits of equidistance to infinity and thus providing a new piece of information that closes this work and brings it to a definitive conclusion.

2. Results

Conversion of an even E > 4 and the evens 2^n x E in sum of two primes in line with Goldbach's strong conjecture. First case : E is an even 3n or multiple of 3.

1a. Lemma a : « if (2^{n-1} x E = p + q such that q > p) and (2^n x E = p + qx such that qx > p). Then qx = p + 2q and this is true to infinity ».

- For Goldbach's strong conjecture to be true, a natural number n > 2 must be surrounded before and after by two equidistant primes p and q such that q > p, the sum of which is 2n. In other words, if we pose any even number as E = 2n, p and q must be equidistant from E/2 = n such that E/2 - p = q - E/2. Let t = E/2 - p = q - E/2.
- p + t = E/2 and p + 2t = q ↔ p + q = (E/2 – t) + (E/2 + t) = 2 x E/2 = E.
- In this article, we look at the link between prime numbers q > E/2 of integers equal to 2 x E; 2^2 x E,...2^n x E.
- Be E any even > 4. Let E be 3n (multiple of 3). Let us first determine π(E/2) or all primes < E/2. Then primes < E/2 are denoted p and those > E/2 are denoted qx (or q, q', q'', q''' ,..).
- « Be E x 2 = Ex such that E = p + q and Ex = p + qx then qx = p +2q ». Indeed, Ex = 2E = 2(p + q) = 2p + 2q = p + (p + 2q) = p + qx ↔ qx = p +2q. Table 1 shows the examples of E = 30, E = 60 and E = 120. We have the primes p of π(E/2) including π(15), π(30), and π(60) and calculate E/ 2 – p = t. Then q = p + 2t for each number. But since 60 = 2 x 30 and 120 = 2 x 60 then if 30 = p + q ; 60 = p + q' and 120 = p + q'' then q' = p + 2q and q'' = p + 2q'.
- The three E/2 numbers (15 ; 30 and 60) are at the center of two equidistant primes whose sum will give 2 x 15; 2 x 30 and 2 x 60. Let p and q be the equidistant primes at 15, p and q' those at 30, and p and q'' those at 60. Hence q' = p + 2q, and q'' = 2 + pq'. Here are some examples ; 30 = 7 + 23 ; 60 = 7 + (7 + 2 x 23) = 7 + 53 ; and 120 = 7 + (7 + 2 x 53) = 7 + 113. This is true for all other primes < E/2 or π(E/2). Another Example 30 = 13 + 17 ; 60 = 13 + (13 + 2 x 17) = 13 + 47 ; and 120 = 13 + (13 + 2 x 47) = 13 + 107.
- Note p + 2q is not always prime, it might also be composite. We only focus on primes.
- Note that qx = p + 2q only when the numbers 2^n x E share p as an addition term with E.

E	**= 30 and E/2**	**= 15**	**2**	**E = 60 and E/2 = 30**		**4E = 120 and E/2 = 60**		
p	*t = E/2 - p*	*q*	*p*	*t = E/2 - p*	*q' = p + 2q*	*p*	*t = E/2 - p*	*q'' = p + 2q'*
3	12	27	3	27	57	3	57	117
5	10	25	5	25	55	5	55	115
7	8	23	7	23	53	7	53	113
11	4	19	11	19	49	11	49	109
13	2	17	13	17	47	13	47	107
			17	13	43	17	43	103
			19	11	41	19	41	101
			23	7	37	23	37	97
			29	1	31	29	31	91
						31	29	89
						37	23	83
						43	17	77
						47	13	73
						53	7	67

Table 1: The table shows primes denoted p of π(*E/2*) including E/2 = 15 ; E/2 = 30 ; and E/2 = 60. Let t = E/2 – p. Let q be any prime > E/2 such that p + q = 2 x E/2 = E. Note E = 30 ; 2E = 60 ; and 4E = 120 in the table.

Equidistant primes p and q ; or p and q' ; or p and q'' such that p + q = 30 ; p + q' = 60 and p + q'' = 120 are highlighted. The table shows that q' = p + 2q and q'' = p + 2q'. Note E = p + q ; 2E = p + q' and 4E = p + q'' according to Goldbach's strong conjecture.

- Table 2 extends Table 1 and shows 240 = 2 x 120 so E = 240 and E/2 = 120. Let p be any prime of π(120) < 120. If 120 = p + q'' and 240 = p + q''', we see that q''' = p +2q''. As an example 120 = 17 + 103 and 240 = 17 + (17 + 2 x 103) = 17 + 223. Or 120 = 47 + 73 and thus 240 = 47 + (47 + 2 x 73) = 47 + 193. This continues to infinity, *showing that p +2q generates an infinite number of primes that are equidistant from p.*
- Table 2 shows an important fact: equidistant primes q''' > E/2 (E/2 = 120) follow the gaps between p within π(E/2). In other words, to convert an even E into the sum of two primes, we must first determine π(E/2) and calculate E/2 - p = t and q = p

+2t. We see that the primes q > E/2 follow the gaps between the primes p < E/2. For example 240 = 7 + 233 and 240 = 13 + 227 we have 13 – 7 = 233 – 227 = 6. Or 240 = 7 + 233 and 240 = 47 + 193 then 47 – 7 = 233 – 193 = 40. This can also be seen in Table 1 This method can be pratical to convert any even E in sum of two primes.

- Note that for an even E 3n, E/2 – p = t is either prime or composite numbers that are not 3n.
- Another important fact is that calculating the second prime q (or q', q'', q''',...) as in Tables 1 and 2 shows that we have all the conversions to the sum of two primes of the numbers E/2. This can be seen in the first two columns orresponding to p and (E/2 – p) of E/2 = 30; E/2 = 60 in Table 1 and the case of E/2 = 120 in Table 2. This means that the conversion of a number 2^n x E always depends on that of the number 2^{n-1} x E. For example, in Table 2 we have all the conversions into the sum of two primes of the number E/2 = 120 (the first two columns corresponding to p and E/2 - p) from which we deduce that of E/2 x 2 = 120 x 2 = 240. This is undoubtedly a very efficient method for converting any even number > 4 into the sum of two primes in line with Goldbach's strong conjecture. However, note that t = E/2 – p of (2^{n-1} x E) number is not always prime but can also be composite, but in all cases, determine the conversion of the (2^n x E) number.

p	t = E/2 - p	q''' = p +2q''
3	117	237
5	115	235
7	113	233
11	109	229
13	107	227
17	103	223
19	101	221
23	97	217
29	91	211
31	89	209
37	83	203
41	79	199
43	77	197
47	73	193
53	67	187
59	61	181
61	59	179
67	53	173
73	47	167
79	41	161
83	37	157
89	31	151
97	23	143
101	19	139
103	17	137
107	13	133
109	11	131
113	7	127

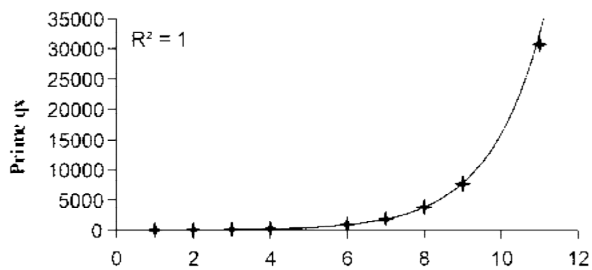
Table 2: The table shows primes denoted p of $\pi(E/2)$ of E = 240 and E/2 = 120 (note 240 = 2^3 x 30). Let t = E/2 – p. Let q be any prime > E/2 such that p + q = 2 x E/2 = E. Equidistant primes p and q''' are highlighted. The table shows that q''' = p + 2q'' (q'' of E = 120 and E/2 = 60 of Table 1). Then E = 240 = p + q'''.

1b. Lemma b : « Primes p + 2q of of successive 2^n x E numbers line up on an exponential curve with a correlation coefficient $R^2 \approx 1$ »

Be E = p + q and let us consider the numbers 2^n x E = p + qx. The table 3 starts with E = 30 = 7 + 23. Then 2^n x E = 7 + qx are determined and qx calculated kowing that if 2^{n-1} x E = p + q then 2^n x E = p +qx such that qx = p + 2q. The graphic 1 shows that the prime numbers qx equidistant at p (= 7) from the numbers 2^n x E are strongly correlated with a correlation coefficient $R^2 \approx 1$ and line up on an exponential curve. The graph shows the qx primes according to their rank or order of appearance. Whether you increase the number of qx primes by calculating other numbers 2^n x E, the qx primes always line up on an exponential curve. If we set E = p + q, this shows that prime number p < E/2 has an infinite number of equidistant primes qx > E/2 of form p + 2q resulting from numbers of type 2^n x E.

2^n x 30	E	E = 7 + qx
n = 0	30	23
n = 1	60	53
n = 2	120	113
n = 3	240	233
n = 5	960	953
n = 6	1920	1913
n = 7	3840	3833
n = 8	7680	7673
n = 10	30720	30713

Table 3: Be 2^{n-1} x E = p + q and 2^n x E = p + qx. The table gives primes qx = p + 2q for E = 30 = 7 + 23 and p = 7 of successive 2^{n-1} x E and 2^n x E numbers listed from top to bottom in the second column. Note that not all E = 7 + qx are primes but can also be odd composite (like the cases of n = 4 or n = 9).



Graphic 1: Primes qx of successive 2^{n-1} x E and 2^n x E numbers line up on an exponential curve with a correlation coefficient $R^2 \approx 1$ according to their rank or order of appearance (n value). The graphic is from table 3 with p = 7 and E = 30 = 7 + 23. The graphic shows primes qx such that 2^n x E = p + qx and qx = p + 2q with q from the preceding 2^{n-1} x E.

1C. The equation of the exponential progression of the qx factors of the numbers 2^n x E.

- Let E = p + q (p and q are prime numbers) such that q > p and q - E/2 = p - E/2. Let qx be any prime of 2^n x E. We have 2 x E = p + (p +2q) as seen above and so qx = p + 2q. For 2^2 x E = 2p + 2(p + 2q) = 4p + 2^2q = p + (3p + 2^2q). For 2^3 x E = 2p + 2(3p + 2^2q) = p + (7p + 8q). For 2^4 x E = 2p + 2(7p + 8q)) = p + (15p + 2^4q) and so on. Therefore 2n x E = p + ((2^{n-1}) p + 2^nq)) → qx = (2^{n-1})p + 2^nq with n ≥ 0. The exponent n is the variable which represents the order of the multiple, so n = 0 for E ; n = 1 for 2 x E ; n = 2 for for 2^2 x E ; and n = 3 for 2^3 x E and so on.
- Using standard mathematical equation notation, we have *y* = *qx* = (2^x - 1)a + 2^xb so that x is the variable which corresponds to the order of the multiple, so x ≥ 0. We have x = 0 for 2^0 x E ; x = 1 for 2^1 x E and increases by one unit from one multiple to the next. We have the constants a and b such that a = p and b = q and 2^0E = E = a + b = p + q.
- Note that this equation doesn't always produce a prime, but it possibly produce an infinity of primes at positions we cannot predict unless we perform a primality test. However, this equation proves that p has an infinite number of possible equidistant (2^x - 1)a + 2^xb primes, which are essential if Goldbach's strong conjecture is to be true to infinity.
- Note that *every* prime p < E/2 common to successive 2^{n-1} x E and 2^n x E numbers has infinitely many possible equidistant primes qx. The larger the number 2^n x E, or the more it tends to infinity, the more an infinity of numbers p < E/2 has an infinity of equidistant primes qx, which follow an exponential progression. We can extend this further and say that the numbers p of π(2^nE/2) or π(2^{n-1}E) (note p < E/2) have an infinite number of primes qx (note qx > E/2) to the exponential, which infinitely increases the chances that the primes p have equidistant primes qx and thus proves Goldbach's conjecture to infinity.
- *An even number that is a multiple of 3 results from the equations 6x ± 1. In fact, all prime numbers and their multiples except those of 2 and 3 are 6x ± 1 and so, to have an even number*

3n, two prime numbers 6x + 1 and 6x - 1 must add up. E = (6x + 1) + (6x - 1) = 6x.

- E = (6x + 1) + (6x - 1) = 6x. And so, to obtain a prime number from E/2 of an even number 3n, you need E - (6x - 1) = 6x + 1 or E - (6x - 1) = 6x + 1. But since 6x ± 1 is either prime or a multiple of primes (except those of 2 ad 3), we have either a prime number or multiple of primes that are 6x ± 1 (Table 1 and 2). This applies to even E/2 of even numbers 3n (see Table 1 for E = 60, E/2 = 30; and E = 120, E/2 = 60 and Table 2 for E = 240, E/2 = 120). For example 60 = 13 + 47 and we have 13 is 6x + 1 and 47 is 6x – 1. And therefore to get primes numbers from 60, we must calculate 60 – 7 ; 60 – 11 ; 60 – 13 and so on. This will not give primes at all times but all primes that add up to make 60.
- But if E/2 of an even number 3n is odd, then 2n that are not multiples of 3 must be subtracted or added (see table 1, the number E = 30 and E/2 = 15). For example 15 – 2 = 13 ; 15 – 4 = 11 and 15 – 8 = 7. All these rules apply to both addition and subtraction.

3. Conversion of an Even E > 4 and the Evens 2^n x E in sum of two Primes in line with Goldbach's Strong Conjecture. Second Case : E is not 3n

We follow the same method as for E = 3n. An even number E is either 3n or not, and in both cases gives either an even E/2 number or an odd E/2 number. We have seen the case of E = 3n with E/2 odd or even. Here we see the example of an even number E = 170 (E/2 = 85) which is not 3n and whose E/2 is odd. Then come the cases of even E/2.

Since E/2 is odd, it makes sense to add even numbers t to it to obtain primes qx = E/2 + t. The primes qx are equidistant from p. *Table 4A show that t = E/2 - p is always a multiple of 3 when qx is prime (except with 3).* We also see that in all cases q = p + 2t which is also logical because E = p + q with q > p and p = E/2 - t and q = E/2 + t and therefore q = p + 2t. For example, in Table 4A, we have q = 167 = 3 + (82 x 2) or q = 163 = 7 + (78 x 2). Note that the equation q = p + 2t applies to any even E > 4 sum of two primes p and q, but here *the remarkable fact is that for an even number that is not 3n and whose E/2 is odd, the values of t are all 3n when q is prime (except with 3)*. This is true for any prime number p of π(*E/2*) which has an equidistant prime q.

	E = 170 and E/2 = 85	
p	t = E/2 - p	q = p + 2t
3	82	167
5	80	165
7	78*	163
11	74	159
13	72*	157
17	68	153
19	66*	151
23	62	147
29	56	141
31	54*	139
37	48	133
43	42*	127
47	38	123
53	32	117
59	26	111
61	24*	109
67	18*	103
71	14	99
73	12*	97
79	6	91
83	2	87

Table 4A: Conversion of an even number E to sums of two primes p and q. E is not 3n and whose E/2 is odd. Here the number E = 170 and E/2 = 85 is shown as a example. Note that prime q = p + 2t knowing that t = E/2 – p. Also note that t is always 3n when q = p + 2t is prime *except with p = 3*. Values of t that are 3n (and that give primes q) are marked by an asterix. E = p + q.

- The table 4Bwhich is a continuation of the previous one (Table 4A) shows that if 2^{n-1} x E = p + q then 2^n x E = p + qx so that qx = p + 2q. The same rule applies to any even number E > 4. In table 4B ; E = 340 = 2 x 170 (170 being shown in table 4A). For instance 170 = 3 + 167 and 340 = 3 + 337 with 337 = 3 + (167 x 2). In this example, 170 and 340 have no primes p in common except 3 and qx = p + 2t. Note that 340 is not 3n and E/2 is even this time.
- The analysis has been extended to 680 (Table 4C), and it can be seen that equidistant primes always occur with t-values that are multiples of 3. Note here that 170 and 340 have only 3 as a common prime that participates in their sum with 170 = 3 + 167 and 340 = 3 + 337. Similarly, 340 and 680 have only 3 primes in common (Tables 4B and 4C). But 170 and 680 have more common primes including 7, 19, 61, 67 and 73. If we follow the values in Tables 4A, 4B and 4C, we can see that either qx is prime or a multiple of 3 or composite, and the reverse is true for t. Again all t values are 3n but this time they are odd beause E/2 is even

	E = 340 E/2 = 170	
p	t = E/2 - p	q = p + 2t
3	**167**	**337**
5	165	335
7	163	333
11	159	329
13	157	327
17	153	323
19	151	321
23	**147***	**317**
29	**141***	**311**
31	139	309
37	133	303
41	129	299
43	127	297
47	**123***	**293**
53	117	287
59	**111***	**281**
61	109	279
67	103	273
71	**99***	**269**
73	97	267
79	91	261
83	87	257
89	**81***	**251**
97	73	243
101	**69***	**239**
103	67	237
107	**63***	**233**
109	61	231
113	**57***	**227**
127	43	213
131	39	209
137	33	203
139	31	201
149	**21***	**191**
151	19	189
157	13	183

163	7	177
167	**3***	**173**

Table 4B: Conversion of an even number E to sums of two primes p and q. E is not 3n and whose E/2 is even. Here the number E = 340 and E/2 = 170 is shown as a example. Note that primer q = p + 2t knowing that t = E/2 – p. Also note that t is always 3n when q = p + 2t is prime *except with p = 3.* Values of t that are 3n (and that give primes q) are marked by an asterix. E = p + q.

E = 680 E/2 = 340		
p	t = E/2 - p	q = p + 2t
3	337	677
5	335	675
7	**333***	**673**
11	329	669
13	327	667
17	323	663
19	**321***	**661**
23	317	657
29	311	651
31	309	649
37	**303***	**643**
41	299	639
43	297	637
47	293	633
53	287	627
59	281	621
61	**279***	**619**
67	**273***	**613**
71	269	609
73	**267***	**607**
79	**261***	**601**
83	257	597
89	251	591
97	243	583
101	239	579
103	**237***	**577**
107	233	573
109	**231***	**571**
113	227	567
127	213	553
131	209	549
137	203	543
139	**201***	**541**
149	191	531
151	189	529
157	**183***	**523**
163	177	517
167	173	513
173	167	507

179	161	501
181	**159***	**499**
191	149	489
193	**147***	**487**
197	143	483
199	141	481
211	129	469
223	**117***	**457**
227	113	453
229	111	451
233	107	447
239	101	441
241	**99***	**439**
251	89	429
257	83	423
263	77	417
269	71	411
271	**69***	**409**
277	63	403
281	59	399
283	**57***	**397**
293	47	387
307	**33***	**373**
311	29	369
313	**27***	**367**
317	23	363
331	**9***	**359**
• 337	**• 3***	**• 353**

Table 4C: Conversion of an even number E to sums of two primes p and q. E is not 3n and whose E/2 is even. Here the number E = 680 and E/2 = 340 is shown as a example. Note that prime q = p + 2t knowing that t = E/2 – p. Also note that t is always 3n when q = p + 2t is prime *except with p = 3.* Values of t that are 3n (and that give primes q) are marked by an asterix. E = p + q.

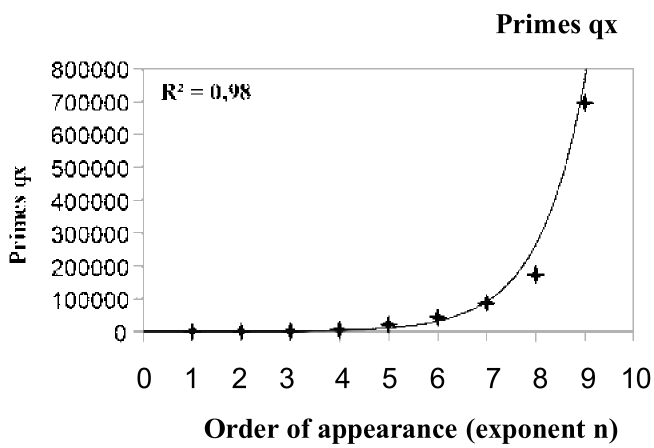
- An even number E that is not a multiple of 3, is such that E/2 = p + 3n with p prime. And so to obtain a prime of this type of number, we need to subtract 3n and so p = E/2 - 3n. If E/2 is even, 3n is odd, and if E/2 is odd, 3n is even. This is shown in tables 4A, B and C. Example in Table 4A, we have E = 170 and E/2 = 85. The table shows E/2 = 85 = p + t = p + 3n when q = p + 2t is prime. Example 85 = 7 + 78 with 78 = 3n and q = 7 + 2 x 78 = 163 that is prime → 170 = 7 + 163. Another example from Table 4B with E = 340 and E/2 = 170. We have 170 = 29 + 141 with 141 = 3n and q = 29 + 2 x 141 = 311 that is prime → 340 = 29 + 311. A final example from table 4C with E = 680 and E/2 = 340. We have 340 = 157 + 183 with 183 = 3n and q = 157 + 2 x 183 = 523 that prime → 680 = 157 + 523.

- There are three types of even numbers 6x, 6x + 2 and 6x + 4. The last two are not 3n. We have 6x + 2 = (6x + 1) + (6x + 1), so we need two 6x + 1 primes. Whereas 6x + 4 can also be considered as 6x - 2 and are therefore the sum of two prime numbers 6x - 1. But remember that 6x ± 1 are not always prime but can also be multiples of prime numbers except 2 and 3. Here are some examples of how to convert even numbers into the sum of two prime numbers we've just seen in Tables 4A, 4B and 4C by using here the equations 6x ± 1. For example 170 = 6 x 28 + 2 and so it is 6x + 2 and we have 170 = 13 + 157 with 13 = 2 x 6 + 1 and 157 = 6 x 26 + 1 thus both 6x + 1. While the number 340 = 6 x 56 + 4 so it is 6x + 4 or 6x – 2 and we have 340 = 83 + 257 with 83 = 13 x 6 + 5 and 257 = 6 x 42 + 5 being both 6x - 1. Indeed 6x – 1 is also 6x + 5. We also have 680 = 6 x 113 + 2 so it is 6x + 2. We have for example 680 = 283 + 397 with 283 = 6 x 47 + 1 and 397 = 6 x 66 + 1 thus being both 6x + 1. In fact Goldbach's strong conjecture obeys equations 6x ± 1

- Indeed if p and q are equidistant primes such that p + q = E then E/2 - p = q – E/2 = t. Therefore, the appearance of a prime number depends upon t that must be 3n for a non-3n E/2 number to get primes except with 3. If E/2 is even t = 3n is odd and if E/2 is odd t = 3n is even. Whereas for even numbers 3n (Table 1 and 2), t is odd and is either prime or composite but never a multiple of 3 when 3n E/2 is even. But if 3n E/2 is odd, then t = 2n but never 3n. For such numbers, p > 3. All these rules apply to both addition and subtraction. These rules are *essential conditions for Goldbach's strong conjecture to be true for Even numbers.*

- We then calculate the qx of successive numbers 2^{n-1} x E and 2^n x E umbers as above except this time E is not multiple of 3. Here we take the example of E = 170. If we examine the primes qx of 2^{n-1} x 170 and 2^n x 170 numbers, we see that they all follow an exponential curve as before, although this time the correlation coefficient R2 ≈ 0.98 (Graphic 2 and the inserted table). They obey to the same equation as above such that 2^x x E = a + $(2^x - 1)$a + 2^xb with qx = $(2^x - 1)$a + 2^xb.

E x 2n	E x 2^n =	Primes qx
170 x 2^0	170	167
170 x 2^1	340	337
170 x 2^2	680	677
170 x 2^5	5440	5437
170 x 2^7	21760	21757
170 x 2^8	43520	43517
170 x 2^9	87040	87037
170 x 2^{10}	174080	174077
170 x 2^{12}	696320	696317



Graphic 2: Primes qx of 2^{n-1} x 170 and 2^n x 170 numbers line up on an exponential curve with a correlation coefficient $R^2 \approx 0.98$ according to their rank or order of appearance.

The graphic is from the inserted table with p = 7 and E = 170 = 7 + 163. The graphic shows primes qx of 2n x E. We have 2n x E = p + qx such that qx = p + 2q with q from the preceding 2n -1 x E. Note (2x - 1)a + 2xb is not always prime (case of n = 3, n = 4, n = 6, n = 11).

4. Conclusions

- E = p + q such that q > p and both q and p are primes then p < E/2 and q > E/2 such that p = E/2 – t and q = E/2 + t. The primes p and q are said to be equidistant at E/2. Note E/2 is any integer < 2 odd or even. *This is the first basic principle for the Goldbach's strong conjecture to be true.*
- The numbers 2 x E are also evens and can be converted to sums of p + qx. In this case, qx = p + 2q. E = p + q and 2 x E = p + qx → qx = p + 2q.
- Let us take E, 2E, 2^2E, 2^3E, 2^4E,...2^nE then the rule applies whatever the number 2^nE we start with. We can start, for example, with 2^{10}E, then calculate qx of 2^{11}E ; 2^{12}E ; 2^{13}E ;... 2^nE or even more. The larger the number 2nE, the more prime numbers π(2^nE/2) will contain. *All calculated primes qx line up on an exponential curve.*
- Note that this is true for any prime p of π(E/2). Indeed, the rule applies for each prime factor of π(2^nE/2). For example if we take the number 96 and 96/2 = 48 = 7 + 41 we therefore have p = 7 and qx = 7 + 2 x 41 = 89. Therefore 96 = 7 + 89. For 96 x 2 = 192, we have qx = 7 + (2 x 89) = 185 (not prime), then for 96 x 4 = 384, we have qx = 7 + (2 x 185) = 377 (not prime), and then 96 x 8 = 768 with qx = 7 + (2 x 377) = 761 which is prime and so 768 = 7 + 761. If we start with 48 = 17 + 31 then 96 = 17 + (2 x 31) = 17 + 79, then 96 x 2 = 192 has qx = 17 + (2 x 79) = 175 (not prime) ; and 96 x 4 = 384 has qx = 17 + (2 x 175) = 367 which is prime then 384 = 17 + 367 and so on. *Therefore, each p of π(E/2) has an infinity of possible equidistant primes qx resulting from 2n x E.*

- Calculating primes p of $\pi(E/2)$ is a very efficient method to find equidistant primes q knowing that q = p + 2t such that t = E/2 – p.
- In all cases of Evens E and if E = p + q → qx of 2^nE = (2^x - 1) a + 2^xb and 2^x x E = a + (2^x - 1)a + 2^xb with qx = (2^x - 1)a + 2^xb (x > 0). The exponent x represents the order or rank of the multiple 2^x x E with x = 1 for 2 x E if we start with E = p + q.
- Each number qx = (2^x - 1)a + 2^xb is either prime or not. But by continuing the operations, and if a number qx exists, it will be qx = (2^x - 1)a + 2^xb.
- There is an important distinction to note. The prime number qx of 2^n x E follows the *equation 2p + q (or y = qx = 2a + b)* with q = b belonging to the preceding number 2^{n-1} x E. But if we take all the numbers qx of the successive numbers 2^{n-1}E and 2^nE, then they follow an approximate exponential equation because this depends on the correlation coefficient, which is never = 1 with any accuracy.

5. The other Case Where an Even Number is the Sum of Smaller Even Numbers

Let's assume that E = E1 + E2 such that E2 > E1. If we follow Goldbach's strong conjecture, we have E1 = p1 + q1 and E2 = p2 + q2. Suppose p1 = p2 then E = p1 + q1 + p1 + q2 = p1 + (p1 + q1 + q2) and if E = p1 + qx (qx prime) then qx = p1 + q1 + q2. Because E = E1 + E2 then E1 < E/2 and E2 > E/2 which means that π(E2) contains all the primes of π(E1). In addition, π(E) contains both π(E1) and π(E2). Therefore, E, E1, and E2 could possibly have common primes p < E/2 to be put into the sum of two primes. Example 7 + 13 = 20 and 7 + 41 = 48. Given that 68 = 7 + 61 then 61 = 7 + 13 + 41. Example 11 + 19 = 30 and 11 + 43 = 54 and since 84 = 11 + 73 then 73 = 11 + 19 + 43. This means that if E = E1 + E2 with E, E1 and E2 sharing one prime p to be in sums of two primes like E = p1 + qx ; E1 = p1 + q1 and E2 = p2 + q2 → qx = p1 + q1 + q2.

Indeed if E = E1 + E2 and if we suppose E, E1 and E2 share one common prime number p1 < E/2, then we have E1 = p1 + q1 and E2 = p2 + q2. Thus we have either E = p1 + qx such that qx is prime or E = p1 + c such that c is composite (multiple of primes). In this case we have qx or c = p1 + q1 + q2. Because qx or c are odd then an odd is sum of three primes which is the Goldbach's weak conjecture. We see that the strong conjecture leads to the weak one. But the latter only indicates the minimum number of primes an odd number could consist of.

If E is the sum of n even numbers that share a prime number p1 < at their halves, then E = p + qx or E + c and qx or c will be the sum of several primes that tends to infinity. So three primes is the minimal form of the sum of an odd number. The latter can be the sum of as many primes as it is large. The infinity of prime numbers also results from the addition of pre-existing primes. The case of an even number being the sum of two or more odd numbers will not be discussed, as it follows the same argument and leads to the same result.

6. Discussion

If an even number E is equal to the sum of two primes p and q such that q > p, then all numbers 2n x E can be written as sums of the same prime p and possible but infinite primes of type p + 2q. The more E tends to infinity, the more $\pi(E/2)$ tends to infinity, the more the possible number of primes p +2q tends to infinity (as there are infinite number of primes p < E/2). An infinite even number 2^n x E therefore has an infinite possibility of having a prime number qx equidistant to a number p of $\pi(E/2)$. Goldbach's conjecture is therefore true to infinity. The primes qx of successive 2^{n-1} x E and 2^n x E (n > 1) all line up on an exponential curve following the equation qx =(2^x- 1)p + 2^xq with the initial E = p + q. We know that between 0 and E/2 there are more primes than between E/2 and E, but given the infinite number of p of $\pi(E/2)$ as E tends to infinity, this increases to infinity the probability that qx equidistant to every p of $\pi(E/2)$ will continue to exist.

There is a notable difference between factoring an integer into a product of prime factors and Goldbach's strong conjecture of converting an even number into the sum of two primes. To illustrate this, let's consider a biprime number Bn = s x t (s and t are primes). In this case s < square root of Bn and t > square root of Bn. Now let us suppose that the square root of Bn is > to any known prime number, however large, so t must be a new prime number that tends even further to infinity. We see that for a biprime number, the square root is the limit. Whereas for Goldbach's conjecture, for an even number E = p + q (p and q are prime and q > p), E/2 is the limit. If E/2 is > any known prime number, however large, then q must be a new prime number. Let's call this new prime q'. In fact, all primes p < E/2 that are infinite will produce all possible differences E/2 - p. And so there should be a prime p such that E/2 - p = q' - E/2. In fact, prime numbers which are 6x ± 1 advance by a distance of 6 (between 6x - 1 on the one hand and 6x + 1 on the other) and there are infinite distances 2n between 6x - 1 and 6x + 1. The prime q' will be in continuity with the primes that precede it, and it will always be possible to find a prime p < E/2 such that E/2 - p = q' - E/2. This is why Goldbach's strong conjecture depends on a critical density of primes before E/2.

Now suppose that an even number E' is at infinity and that E' = 2^n x E. Assume that E'/2 is > all known primes denoted p and q, and if E = p + q, then there must exist a prime q' = (2^x- 1)p + 2^xq such that p + q' = E'. The prime q' has infinitely many possibilities. Again, this depends on the critical density of primes < E'/2.

Consequently, all the results and arguments in this article plead and demonstrate the validity of Goldbach's strong conjecture at infinity. But Goldbach's two conjectures, strong and weak, teach us only the minimum number of primes of which an even or odd number can be the sum.

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