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Short Article

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Uchida's Step Function

Uchida Keitaroh*

Applied Mathematics Department

***Corresponding Author**
Uchida Keitaroh, Applied Mathematics Department, Japan.

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As I created a step function by using division by zero calculus, I will show how to create it.

$$y(x) = |x|$$
$$y'(x) = \frac{|x|}{x}$$
$$y'(0) = \frac{|0|}{0} = 0$$

This is well known as division by zero calculus.
The step function is created by using above.

$$x, b \in R$$
$$i, n \in N$$
$$1 \leq i \leq n$$
$$0 \leq x \leq b\ ,\ a_i = \frac{i}{n} \cdot b\ , a_n = b\ , 0 < b$$
$$f_i(x) = \frac{|x - a_i|}{x - a_i}$$

Regarding $f_i(x)$,

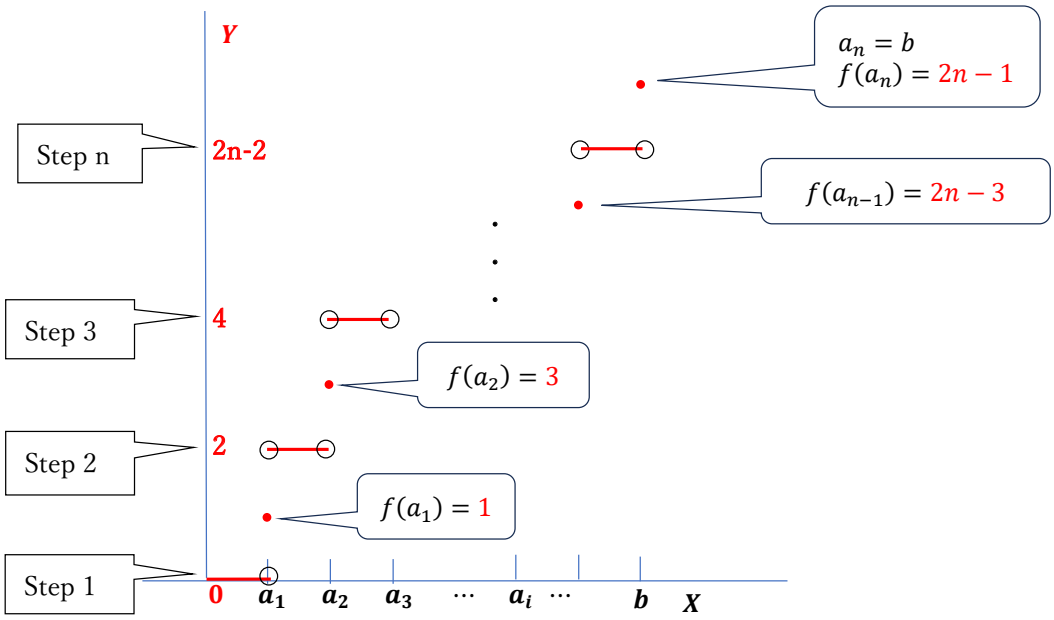
$$\text{for}\ \ 0 \leq x < a_i\ ,\ f_i(x) = -1.$$
$$\text{for}\ \ x = a_i\ ,\ f_i(a_i) = \frac{|a_i - a_i|}{a_i - a_i} = \frac{|0|}{0} = 0.$$
$$\text{for}\ \ a_i < x \leq b\ ,\ f_i(x) = 1\ \ except\ f_n(x)\ .$$
$$f_n(b) = 0.$$

Setting

$$f(x) = n + \sum_{i=1}^{n} f_i(x)\ ,$$

This is Uchida's Step Function, which has n steps.

Setting $y = f(x)$.



Therefore,

$$\frac{df(x)}{dx} = \sum_{i=1}^{n} \frac{\frac{|x-a_i|}{x-a_i}(x-a_i) - |x-a_i|}{(x-a_i)^2} = \sum_{i=1}^{n} \frac{|x-a_i| - |x-a_i|}{(x-a_i)^2} = 0$$

Reference

1. Okumura, H., Saitoh, S., Uchida, K. (2021). On the Elementary Function y = |x| and Division by Zero Calculus.