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Travel In Deep Dark Space with Giant LED Sail

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Abstract
This article discusses the scientific feasibility of sending a man-made device to a neighboring star beyond our solar system with limited fuel capacity and limited travel time in deep dark space using a giant LED sail.

1. The Challenge

The nearest neighboring star Proxima Centauri is 4.02×10^{16} m away, about 4 lightyears of distance. Currently Voyager 1 is traveling with speed of 1.7×10^{4} m/s (About 5.7×10^{-5} c, where c is the speed of light) from the distance of 2.35×10^{13} m away from the Sun. After it overcomes the gravitational potential of the Solar system from the current position, it will have the speed of 1.67×10^{4} m/s (5.58×10^{5} c). Assuming Voyager 1 is traveling straight away from the Sun with this roughly constant speed from its current position, it will need around 7.6×10^{4} years to reach Proxima Centauri. (The lower boundary estimation will be 7.5×10^{4} years, using the current speed of 1.7×10^{4} m/s. We ignore gravitational force from the Proxima Centauri star.) Thus, if we want a spaceship to reach this star within one hundred years, we need to provide the spaceship with many years of continuous thrust from limited on board fuel mass so that it can reach a very high speed, close to a tenth of light speed.

Since a spaceship has limited capacity to carry fuel and propellant, we need to find a high efficiency method of fuel usage to obtain propulsion for long, deep dark space travel. Based on Einstein's massenergy equation, photons can transform all of their energy into momentum. Using photons to obtain thrust might be a plausible way to fulfill this dream. Unlike solar sails, which use outside photons passively from a star we propose building a giant photon sail with an onboard LED photon source that uses energy produced by an onboard energy source, such as a macro nuclear fission generator [1,2]. We will discuss what power density of LED sail and size are required and what kind of energy transformation process should be considered for this kind of deep space travel.

Basically, there are five ways to obtain thrust from working energy for a rocket, namely chemical reaction, fission, fusion, star photons, and annihilation (matter and antimatter). Storage of antimatter in a real world is very difficult with today's storage technology for antimatter. It would be interesting if we could store positronium, and strip off the outside electrons for annihilation when using it. Since e and e+ have same mass, the electron actually does not form an outside electron cloud, so it is very hard to store this fuel. To compare the efficiency difference among those five processes of burning energy to obtain momentum, such as nuclear electric propulsion (NEP), nuclear thermal propulsion (NTP) [3] and chemical combustion, we can define an ideal up- limit pure number index of energy efficiency as below for comparison:

$$k_E = \frac{\Delta E}{mc^2}$$ *Equation 1*

Where $\Delta E=\Delta mc^2=(m-m_0)c^2$ is the energy generated by certain method such as chemical combustion, atomic fusion and fission, and m is the mass involved in such processes. Here m is the total special relativity mass used in the process. If the highest ideal momentum p is what we can obtain from such process, then another pure number index k_p is a good theoretical up-limit value to compare among those processes. We define ideal an up-limit momentum efficiency index k_p as

$$k_p = \frac{pc}{mc^2}$$ *Equation 2*

The index in Equation 2 will reflect the unit mass efficiency of such process that the total theoretical momentum a process could obtain. Consider the Einstein mass-energy equation

$$m^2c^4 = p^2c^2 + {m_0}^2c^4$$ Equation 3

we could rewrite Equation 2 as

$$k_p = \sqrt{k_E(k_E + 2\frac{m_0}{m})}$$ Equation 4

For a process only involves photon, since we have $mc_2=pc$, we get $k_p=1$ and $k_E=1$. This would be the highest theoretical efficiency index to obtain momentum from an energy transformation process. For none-photon process, only when $m_0\to0$ or $m\to\infty$, which means $u=v/c\to1$, that the propellant mass ejected from nozzle must have the speed of light or it is the photons that we could get $k_E=1$ and $k_p=1$. That is the reason we want to use photon for the thrust.

Let us look at LH_2 and LO_2 combustion. One mole H_2 can produce 241.8 kJ. The chemical reaction is

$$2H_2 + O_2 \to 2H_2O$$ Equation 5

One mole of H_2 and half mole of O_2 produces 241.8 kJ. The maximum theoretical efficiency index is $k_E=\Delta E/mc^2=4.85\times10^{-14}$. So, the highest theoretical $k_p\sim\sqrt{2k_E}=3.11\times10^{-7}$. For fission of U_{235}, it releases 83.14 TJ/kg. So $k_E=9.26\times10^{-4}$ and $k_p=4.30\times10^{-2}$. For deuterium-tritium fusion, we estimate that $k_E=9.41\times10^{-3}$ and $k_p=1.37\times10^{-1}$.

Process Name	k_E	k_p
$2H_2 + O_2 \to 2H_2O$	4.85×10^{-14}	3.11×10^{-7}
U_{235} Fission	9.26×10^{-4}	4.30×10^{-2}
deuterium-tritium fusion	9.41×10^{-3}	1.37×10^{-1}
Annihilation	1	1
Photon	1	1

Table 1 Ideal up limit pure number index of energy and momentum of different processes

The total efficiency index k_E with multiple processes such as NTP or NEP involving burning fuel, producing ions, and accelerating propellant etc. will be much smaller than the theoretical one and a final index would be in the form as $k = k_1k_2\ldots k_n$.

For nuclear fission rocket engine, the energy will be used to heat up propellant mass to be ejected or the energy will be converted into electric-magnetic energy to accelerate ions ejected from nozzle to obtain momentum [4], [5], [6], [7], [8]. NTP heats the hydrogen atom and NEP accelerates ion to be ejected out of the nozzle to obtain the thrust. The reason using hydrogen is it has the smallest mass of one mole number. On the other hand, using LED to emit the lightest propellant, the photon, for thrust and it needs no propellant mass. The longer the journey is the better off the LED sail is. It is reasonable to assume that overall efficiency indexes of NTP or NEP nuclear rocket engines are smaller than that of lighting LED.

Although laser emits energy beam (photons) in one direction, unlike high efficiency commercial available LED, which no cooling or fan system is needed [9], [10], [11], a laser system needs cooling system, energy pumping apparatus etc. so that overall efficiency, as so-called Wall-Plug Efficiency (WPE) is less than that of LED.

2. Efficiency Index of LED Sail Design

To maximize LED efficient, we need to reflect the photons emitted from LED to one direction, namely z direction. Below are two mirror formations as shown in Figure 1 and Figure 2. In Figure 1 the LED and wires are integrated into base materials. Figure 2 has an LED mounted on a cylinder within a parabolic mirror. The second structure is easier to repair should the LED go bad during a long travel. But its structure could be heavier. A set of panels, forming a giant junk sail, mounted with a large array of such LED cells could provide the thrust. Other design could use polymer panels and long strips of LED light in the center of each panel to forming the giant junk sail. Coated the sail panel with reflective grate strip film with saw like grate will have better reflection effect. Or to control the sail panel to have a two dimensional parabolic curvature. Or simply to make the sail panels as giant LED light panels.

Let us try a simple structure in Figure 2 and calculate the theoretical reflection index. For the simplicity we assume the LED is a ball and the mounting rod is very thin so its geographic size could be ignored. So that the phones reflected at the very bottom point of the mirror cold be ignored.

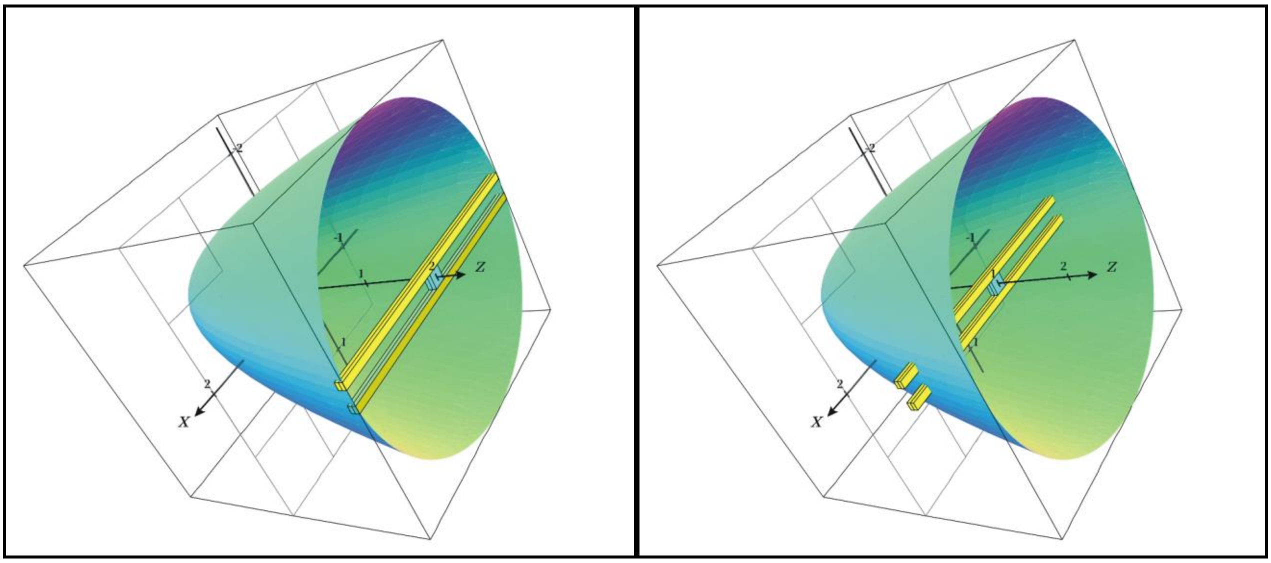

Figure 1. A parabolic reflect mirror cell with LED light source, similar to the hand light. The focal point O is the LED bulb inside the cell.

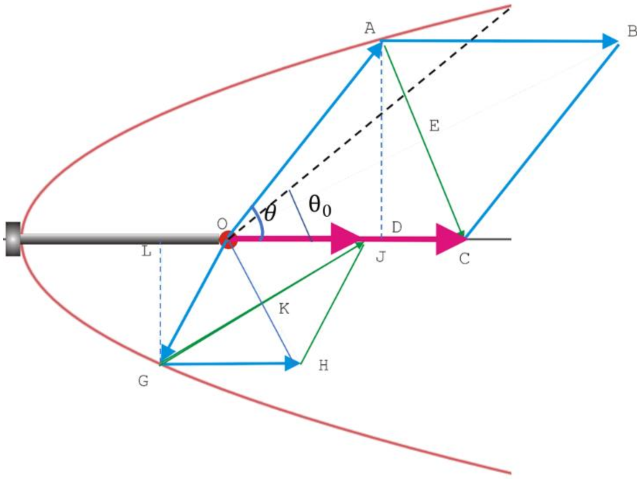

Figure 2. Parabolic mirror with ane LED bulb at the focal point.

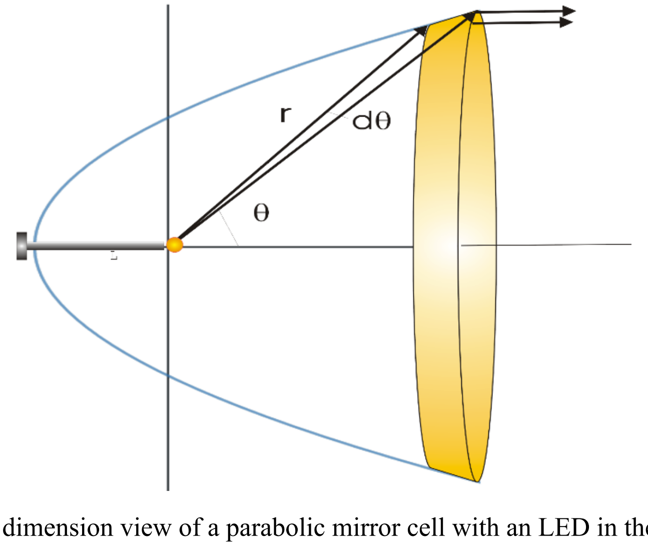

Figure 3. Three dimension view of a parabolic mirror cell with an LED in the focal point

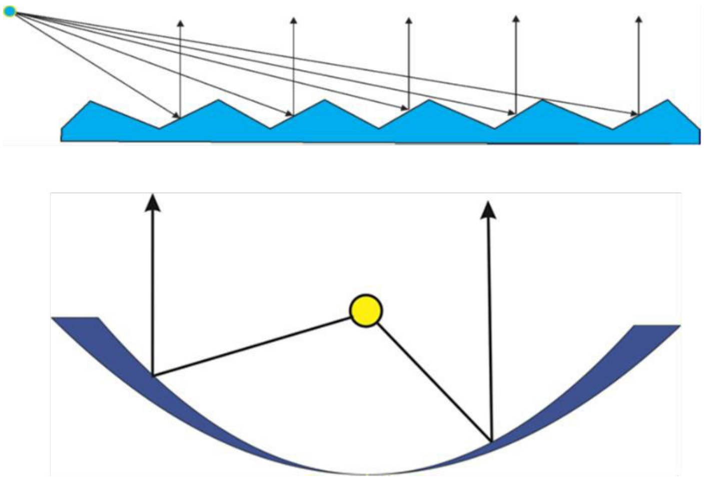

Figure 4. Long reflection film with parabolic shape controlled by memorized end-edge wire, or coated with grate strip to reflect more photon in z direction

For simplicity we denote light speed c=1 to simplify discussion. O is the light source, which can emit energy E per second uniformly. Angle $\angle AOC=\theta$. Angle 0 is the angle as shown in Figure 2 connected from the mirror edge to the focal point. For those photons with angle less than 0 , the total momentum the light bulb could obtain at Z direction, is

$$P_a = \frac{E}{4} sin^2(\theta_0)$$

Equation 6

Now we will calculate those photons contribution with angle larger than θ_0. Let us assuming the mirror is perfect, and the reflection factor is 1. There are two parts forces, one acts on the LED ball and other one acts on the mirror and reflected at Z direction. The total effective momentum the cell obtains at Z direction after integration for all the beam for all angle θ larger than θ_0 we have

$$P_b = \frac{E}{2}(1 + cos\theta_0)$$

Equation 7

Combine Equation 6 and Equation 7, assume effective mirror reflection index k as a constant for the simplicity, we have

$$P/E = \frac{1}{2}(k(1 + \ cos\theta_0) + \frac{1}{2}\ sin^2\theta_0) \qquad \text{Equation 8}$$

For the cell as shown in the left one of Figure 1 with $\theta_0=\pi/2$, for an ideal reflection index as k=1 for this mirror cell, it will be 75%. As a whole system efficiency rate, the total rate will combine with electricity to light efficiency of the LED bulb. People often use overall wall plug efficiency (WPE). Current WPE of a cool light LED so far is 18%. So, in real world the up limit overall efficiency index of LED cell today would be 0.75×0.18=0.135 for $\theta_0=\pi/2$. The area for improvement would be to improve WPE and the reflection index of the mirror surface

Thus, a photon sail thrust of parabolic mirror cell will be calculated by

$$Thrust = \frac{(S \times D)}{C} \times W \times F \qquad \text{Equation 9}$$

Where S is the sail panel size, D is disc compact factor of cell array, C is one cell size, W is one LED bulb power, and F is the index calculated by Equation 8. For a sail size as 400 m^2, cell size as *r*=1 *cm*, LED power as 1 *watt*, disc compact factor [12] as 80%. And F=0.7 with $\theta_0=0.5\pi$ and *k*=0.94. We can get that there will be total of 1.02×10^6 cells, consuming 1.02 Megawatts. The sail power is 0.717 Megawatts. Roughly it is 962 horsepower. If WPE is 20%, it needs a 5.09 Megawatts generator. Another simple design is to coat whole sail panel with LED material and make the sail as a giant LED light panel and at the edge with some reflection mirror curtain.

Commercially available 18 watts LED has a 7x7 mm^2 base. Thus, a 100 m^2 sail will become a 36.7 megawatts giant LED. With a factor of 25% effective photon emitted in z direction, it provides 9.0 megawatts thrust or nearly 12 k HP. Total power needed is

184 megawatts if WPE is 20%. INL website [13] stated that it has a capacity of 1 to 50 megawatts, and the EU website [14] reported that it has a capacity of 1 to 20 megawatts. Their size is close to a shipping container. So, this LED sail power level is achievable with today's technology. Although further discussion of fuel mass to spaceship mass ratio would require much more power of the LED sail and higher fuel momentum index. It is worth for further detailed technology analysis.

Solar photon force near earth orbit is about $1.0\times10^{-5} N/m^2$ using average wavelength as 550 nm and average sun photon flux as $3.77\times10^{21}\ s^{-1}m^{-2}$. For a commercial LED of only 18 watts on a 7 (mm)2 base, and with 25% of z reflection factor, it is equivalent to $3.05\times10^{-4} N/m^2$, about 30 times of that of solar photon pressure near the earth orbit. The power of LED and the size of sail can be increased to meet the needs of a reasonable mass spaceship.

For sail with reflecting mirror, a challenging is to find a mirror material which can resist the heat of high-power photon beam in a high vacuum environment. It needs to disperse a huge amount of heat to vacuum space should a sail mirror absorb single digit percentage of photon energy. For a giant LED sail without mirror, the heat produced by LED base still is a challenging problem. To disperse the heat from LED base into a vacuum environment will be an engineering challenging task.

3. Travel Time Estimation

Einstein special relativity theory between force ***f***, mass *m* and acceleration ***a*** is a well-known equation [15] when we only consider one-dimensional movement. A simplified approach is that we can ignore the fission mass loss comparing to the spaceship mass, and we can treat the spaceship mass as a constant. Although this may not be valid if huge energy is needed for a long distance travel [26-30].

$$a = a_0\gamma^{-3}$$

Equation 10

Where we denote c=1, $u=v/c$,and $\gamma^{-1}=\sqrt{1-u^2}$. When only consider the fourth-dimension variant ***t***:

$$f = ma\gamma^3$$

Equation 11

Thus, we can do integral of this deferential equation:

$$\frac{du}{dt} = \frac{f}{m}(1-u^2)^{3/2}$$

and we have

$$\frac{u}{\sqrt{1-u^2}} = \frac{fT}{m} + \frac{u_0}{\sqrt{1-u_0^2}}$$

Equation 13

$$v = \frac{fT/mc + v_0/\sqrt{c^2-{v_0}^2}}{\sqrt{1+(fT/mc)^2}}c$$

Equation 14

Here T is the total acceleration time. When v/c or $fT/(mc)$ is a very small value, above equation become Newtonian one as $v=aT+v_0=fT/m+v_0$. If an LED sail can provide a constant 1.0×10^3 Newton force, we can calculate total travel time to Proxima Centauri star for a 104 kg mass spaceship in Table 2. Due to deceleration at the destination, there is an optimal overall travel time depends on total acceleration time. For 12 years of acceleration, the spaceship will take 46 years to reach Proxima Centauri. Again, for a very long trip assuming spaceship mass is a constant and ignoring the fuel mass change will not be valid since it needs a large amount of fuel mass. We will discuss a constant power case next.

For high-speed travel, we should assume the power w provided by sail is a constant instead of the constant force. Special relativity would be

$$\frac{du}{dt} = \frac{w}{mc^2u}(1-u^2)^{3/2}$$

Equation 15

We got an energy conservative equation at the condition of spaceship mass as a constant.

$$wT = \frac{mc^2}{\sqrt{1-u^2}} - \frac{mc^2}{\sqrt{1-{u_0}^2}}$$

Equation 16

$$u = \sqrt{1 - \frac{m^2c^4\left(1-{u_0}^2\right)}{\left(wT\sqrt{1-{u_0}^2}+mc^2\right)^2}}$$

Equation 17

Assuming the spaceship needs deceleration when approach the destination, from the chart of Figure 6, if the spaceship undergoes around 20 years of acceleration, it will take about 375 years to reach Proxima Centauri.

acceleration time vs speed and travel time: constant force				
acc time (y)	speed (m/s)	travel time(y)	fT/(mc)	u
1	3.16E+06	404.8	0.0105	0.0105
2	6.31E+06	203.9	0.0210	0.0210
3	9.46E+06	137.6	0.0316	0.0315
4	1.26E+07	105.0	0.0421	0.0420
5	1.58E+07	85.9	0.0526	0.0525
6	1.89E+07	73.4	0.0631	0.0630
7	2.20E+07	64.8	0.0736	0.0734
8	2.52E+07	58.6	0.0842	0.0839
9	2.83E+07	54.1	0.0947	0.0943
10	3.14E+07	50.6	0.1052	0.1046
11	3.45E+07	47.9	0.1157	0.1149
12	3.76E+07	45.9	0.1262	0.1252
13	4.06E+07	44.3	0.1367	0.1355
14	4.37E+07	43.1	0.1473	0.1457
15	4.68E+07	42.2	0.1578	0.1559
16	4.98E+07	41.6	0.1683	0.1660
17	5.28E+07	41.1	0.1788	0.1760
18	5.58E+07	40.8	0.1893	0.1860
19	5.88E+07	40.7	0.1999	0.1960
20	6.18E+07	40.6	0.2104	0.2059
61	1.62E+08	68.9	0.6417	0.5401
68	1.75E+08	75.3	0.7153	0.5818
69	1.76E+08	76.2	0.7258	0.5874
70	1.78E+08	77.2	0.7363	0.5929
71	1.80E+08	78.1	0.7469	0.5984
72	1.81E+08	79.0	0.7574	0.6038

Table 2: Total travel time with different acceleration time

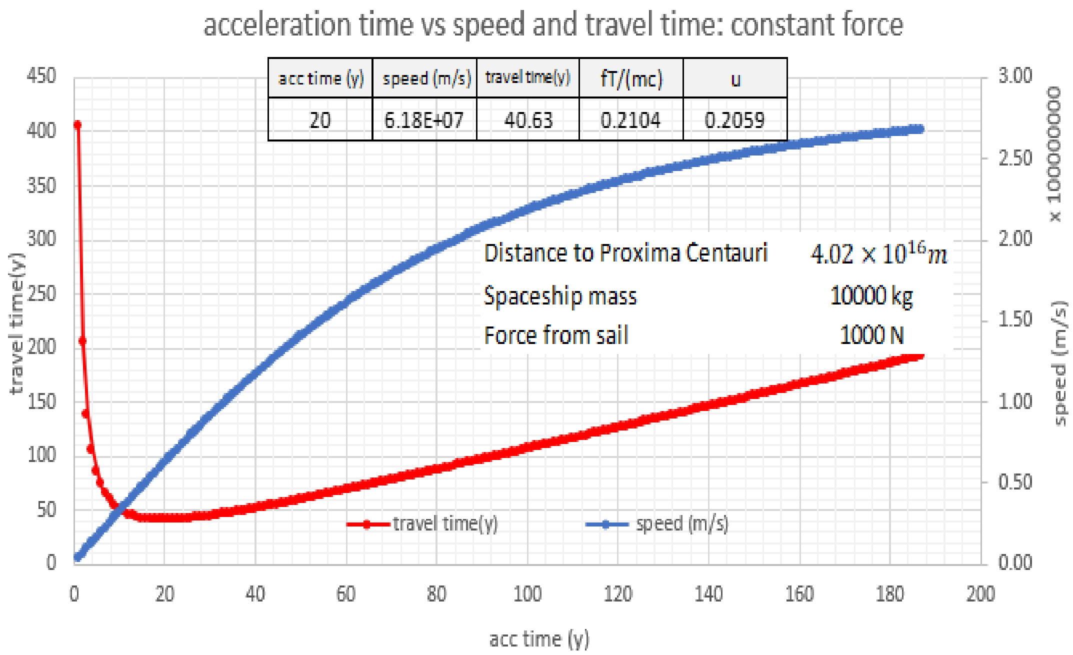

Figure 5: Total travel time to Proxima Centauries with thrust force of 1 $\times 10^3$ N in the phases of acceleration and deceleration.

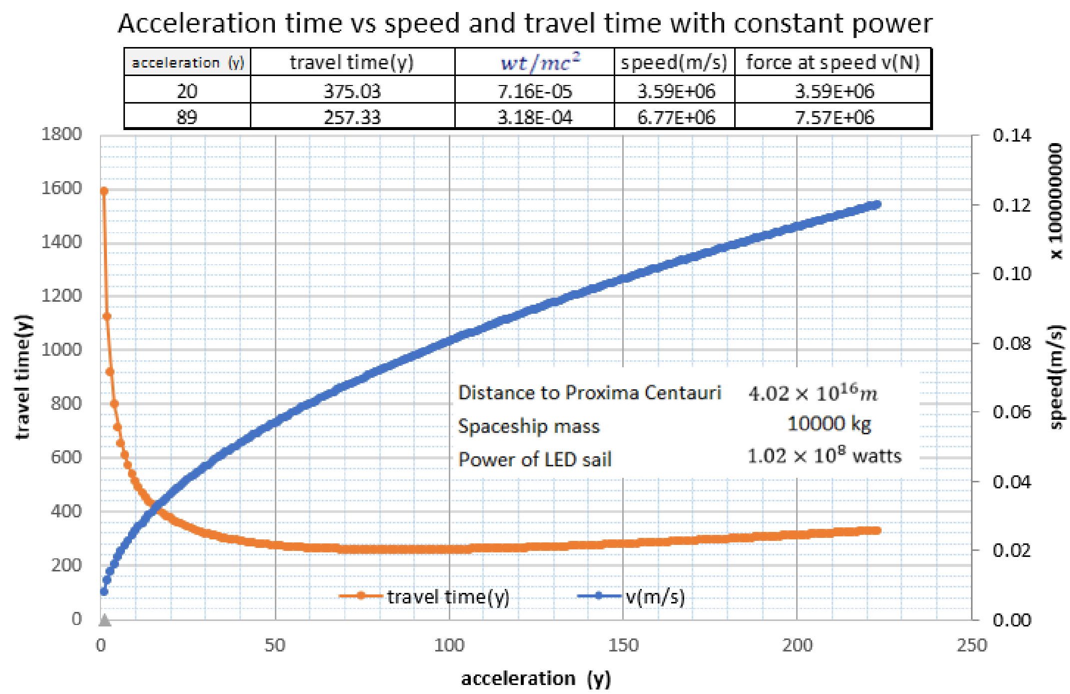

Figure 6: Total travel time to Proxima Centauries for a spaceship with constant sail LED power with the phases of acceleration and deceleration.

4. Fuel Mass Estimation

A reasonable simplification is to assume that it is a one-dimensional movement for a long-distance travel and the rate of burning mass is a constant during the acceleration and deceleration phases. From the earth observer the mass of spaceship is m_s The efficiency ratio is k to obtain the momentum from the energy by an engine to burning the mass so $dp = -kdm$. Initial speed is $u_0 = v_0/c$. At the end of acceleration phase the speed is $u_1 = v_1/c$, the acceleration time is t, and total used up fuel mass is m_a. Using special relativity momentum equation and taking derivative of it we got those equations below.

$$dp = \frac{mdu}{(1-u^2)^{3/2}} + \frac{udm}{(1-u^2)^{1/2}}$$

Equation 18

$$dp\sqrt{1-u^2} - udm = \frac{mdu}{(1-u^2)}$$

Equation 19

For acceleration $dp=-kdm$

$$\frac{dm}{m} = \frac{-du}{(1-u^2)(u+k\sqrt{1-u^2})}$$

Equation 20

thus, we have

$$ln\left(1+\frac{m_a}{m_s+m_d}\right) = \int_{u_0}^{u_1} \frac{du}{(1-u^2)\left(u+k\sqrt{1-u^2}\right)}$$

Equation 21

and

$$\frac{m_a}{m_s+m_d} = \frac{\frac{u_1}{\sqrt{1-u_1{}^2}} - \frac{u_0}{\sqrt{1-u_0{}^2}}}{k+\frac{u_0}{\sqrt{1-u_0{}^2}}}$$

Equation 22

Where m_d is the fuel mass for deceleration phase. For deceleration $dp=kdm$ the equation will be:

$$\left(k-\frac{u}{\sqrt{1-u^2}}\right)dm = \frac{mdu}{(1-u^2)^{3/2}}$$

Equation 23

Since $dm<0$ and $du<0$ so we have the constrain condition as $(k-u_1/\sqrt{1-u_1^2})>0$. This is understandable that from the earth observer, only when a mass traveling fast than u from the spaceship could it reduce the speed of the spaceship. Since $k<1$ so $(k-u_1/\sqrt{1-u_1^2})<1$. The relationship of k and u_1 is decided by acceleration phase. Thus, in this case u has an up limit due to the efficiency index of k. For example, if we do not dump used fuel for U_{235} fission energy, since $k=0.043$, we have $u_1< 0.04296$ of speed of light.

$$\frac{dm}{m} = \frac{du}{(1-u^2)(k\sqrt{1-u^2}-u)}$$

Equation 24

$$\frac{m_d}{m_s} = \frac{\frac{u_1}{\sqrt{1-u_1{}^2}} - \frac{u_0}{\sqrt{1-u_0{}^2}}}{k-\frac{u_1}{\sqrt{1-u_1{}^2}}}$$

Equation 25

Thus, the total fuel mass is $m_f= m_a + m_d$.

$$\frac{m_f}{m_s} = \frac{2k\left(\frac{u_1}{\sqrt{1-u_1{}^2}} - \frac{u_0}{\sqrt{1-u_0{}^2}}\right)}{\left(k-\frac{u_1}{\sqrt{1-u_1{}^2}}\right)\left(k+\frac{u_0}{\sqrt{1-u_0{}^2}}\right)}$$

Equation 26

We need reconsider the Equation 19 to calculate the fuel mass for the case of dumping used fuel with speed of u' with the momentum p' (where dm <0). From the observer of earth coordinate, in this case assume the *dm*<0. At time *t*, the spaceship has mass m, speed u and momentum *p*. At t+dt, they are *m*+*dm*, *u*+*du, p*+*dp*, and dumped used fuel as dm with speed u' relative to earth observer. The energy used for reducing the momentum so $dp= -kdm$>0 for acceleration, and $dp= kdm$<0 for deceleration. Using special relativity momentum equation, we have

$$p(t+dt)-p(t)=\frac{mdu}{(1-u^2)^{3/2}}+\frac{udm}{(1-u^2)^{1/2}}+dp'$$

Equation 27

$$(dp-dp')\sqrt{1-u^2}-udm=\frac{mdu}{1-u^2}$$

Equation 28

Where *dm*<0 and *u*'>0, so *dp*'>0, assume its velocity in earth coordinate is u'

$$dp'=\frac{-u'dm}{\sqrt{1-u'^2}}$$

Equation 29

For acceleration phase and dumping used fuel with speed of u' , we have:

$$-\frac{dm}{m}=\frac{du}{(1-u^2)\left(\left(k-u'/\sqrt{1-u'^2}\right)\sqrt{1-u^2}+u\right)}$$

Equation 30

And integral the left side we have

$$ln\ (1+\frac{m_a}{m_s+m_d})=\int_{u_o}^{u_1}\frac{du}{(1-u^2)((k-u'/\sqrt{1-u'^2})\sqrt{1-u^2}+u)}$$

Equation 31

For deceleration phase with dump speed *u*' no less than *u*, relative to earth coordinate.

$$-\frac{dm}{m}=\frac{du}{(1-u^2)\left(\left(-k-u'/\sqrt{1-u'^2}\right)\sqrt{1-u^2}+u\right)}$$

Equation 32

and

$$ln\ (1+\frac{m_d}{m_s})=\int_{u_o}^{u_1}\frac{du}{(1-u^2)((k+u'/\sqrt{1-u'^2})\sqrt{1-u^2}-u)}$$

Equation 33

For the case with u'=u, That is equivalent to dump the used fuel with relative zero speed to spaceship. We have

$$ln\ \left(1+\frac{m_a}{m_s+m_d}\right)=ln\ \left(1+\frac{m_d}{m_s}\right)=\int_{u_o}^{u_1}\frac{du}{k(1-u^2)^{3/2}}=\frac{1}{k}(\frac{u_1}{\sqrt{1-{u_1}^2}}-\frac{u_0}{\sqrt{1-{u_0}^2}})$$

Equation 34

With u_0=0.00001 and u_1=0.04 , k =0.043, the ratio would be m_f/m_s=26.96. This is a huge fuel mass requirement. For k =0.4 and u_1=0.08 the mass ratio is about 0.5. Table 3 lists the ratio of fuel and spaceship mass with different u_1 and k. If we reuse and then dump the used fuel in the cases of nuclear fission energy, the mass ratio will be much smaller than that of no dumping. The best would be matter-antimatter energy generator since the momentum index k is much larger.

				no dumping used fuel			dumping used fuel		
k	u_0	u_1	$u_{1\,max}$	m_a/m_s	m_d/m_s	m_f/m_s	m_a/m_s	m_d/m_s	m_f/m_s
0.0430	0.00001	0.0400	0.0430	13.478	13.485	26.963	3.897	1.536	5.433
0.0430	0.00001	0.0408	0.0430	18.839	18.848	37.687	4.094	1.584	5.678
0.0430	0.00001	0.0416	0.0430	30.879	30.894	61.773	4.303	1.634	5.937
0.0430	0.00001	0.0424	0.0430	82.700	82.738	165.438	4.526	1.685	6.211
0.0800	0.00001	0.0600	0.0797	3.021	3.021	6.042	2.373	1.120	3.493
0.0800	0.00001	0.0612	0.0797	3.280	3.281	6.561	2.478	1.152	3.630
0.0800	0.00001	0.0624	0.0797	3.582	3.583	7.165	2.590	1.185	3.775
0.0800	0.00001	0.0637	0.0797	3.937	3.938	7.876	2.708	1.220	3.927
0.2000	0.00001	0.0800	0.1961	0.670	0.670	1.340	0.737	0.494	1.231
0.8000	0.00001	0.1000	0.6247	0.144	0.144	0.287	0.152	0.134	0.286

Table 3: Minimum fuel mass to spaceship mass ratio for different index *k*

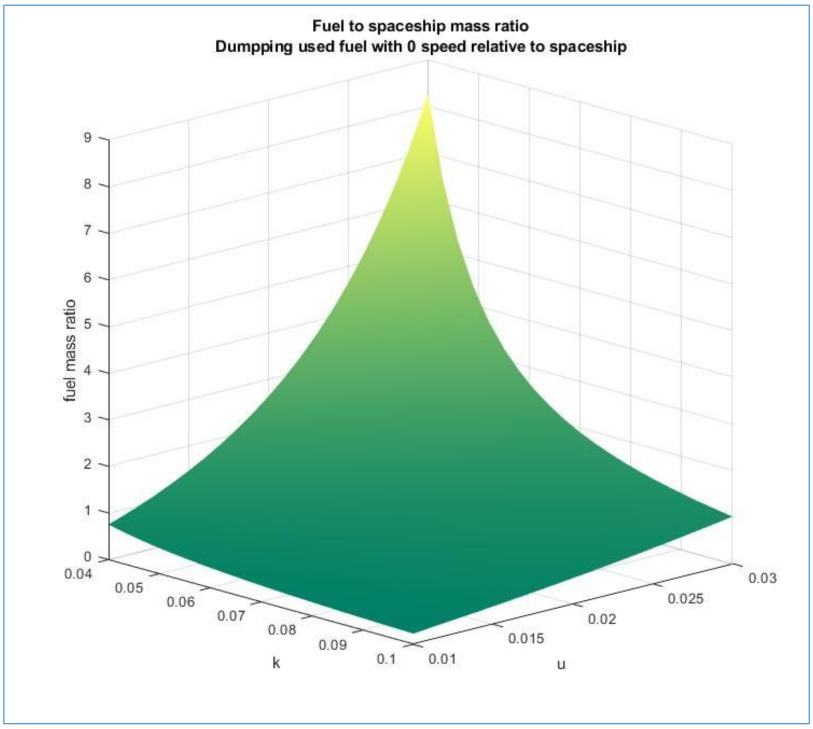

Figure 7: For dumping used fuel with relative speed 0 to spaceship, initial speed to lightspeed as u_0=0.0001. Fuel mass to spaceship mass ratio relationship with finial speed u and momentum index k. Here uses c = 1.

When the total mass changes due to large amount of fuel mass is used, Equation 16 and Equation 17 are no longer valid. For acceleration phase, we need to combine Equation 15, Equation 30 and Equation 32 that dumping the used fuel mass with relative speed 0 to spaceship (u'=u). Thus, for acceleration and deceleration we could get mass and speed relation below respectively:

$$m = m_0\, e^{\frac{-u}{k\sqrt{1-u^2}}+\frac{u_0}{k\sqrt{1-u_0{}^2}}}$$

Equation 35

$$m = m'_0\, e^{\frac{u}{k\sqrt{1-u^2}}+\frac{-u_1}{k\sqrt{1-u_1{}^2}}}$$

Equation 36

Where $m_0 = m_s + m_a + m_d$, and $m'_0 = m_s + m_d$. Where m_s is the mass of spaceship, m_a is the total fuel mass used for acceleration phase and the m_d is the total fuel mass used for future deceleration phase. And u_1 is the speed at the end of acceleration phase. Then the ship will travel in constant speed of u_1, then reduces the speed to u_0 when it reaches the destination star.

Combine Equation 15, Equation 35 and Equation 36. we will have three phases differential equations

$$\frac{du}{dt} = \frac{w}{m_0 c^2 u}(1-u^2)^{3/2}\, e^{\frac{1}{k}\left(\frac{u}{\sqrt{1-u^2}} + \frac{-u_0}{\sqrt{1-u_0{}^2}}\right)} \quad ; \quad acceleration\ phase\ (u: u_0 \rightarrow u_1)$$

$$\frac{du}{dt} = 0 \quad ; \quad constant\ speed\ travel\ phase$$

$$\frac{du}{dt} = \frac{w}{m'_0 c^2 u}\left(1-u^2\right)^{3/2} e^{\frac{1}{k}\left(\frac{-u}{\sqrt{1-u^2}} + \frac{u_1}{\sqrt{1-u_1{}^2}}\right)} \quad ; \quad deceleration\ phase\ (u: u_1 \rightarrow u_0)$$

Equation 37

This set of equation needs numerical solution to estimate the travel time. One way is using fuel requirement as first priority, using m_s,m_d,u_0,k, and Equation 36, we could get speed u_1. Then we could use u_1 to get m_a from Equation 35. Thus, from Equation 37 we could calculate travel time for the three phases for the target distance. Another way is using speed requirement as first priority, using m_s, u_0, u_1, k and Equation 35 and Equation 36 we could obtain needed fuel mass m_a, m_d, then from Equation 37 and the target distance we will get the travel time for the three phases. First method is based on fuel technology status and the second method is based the travel time needed. The code is listed at the end of the citation section.

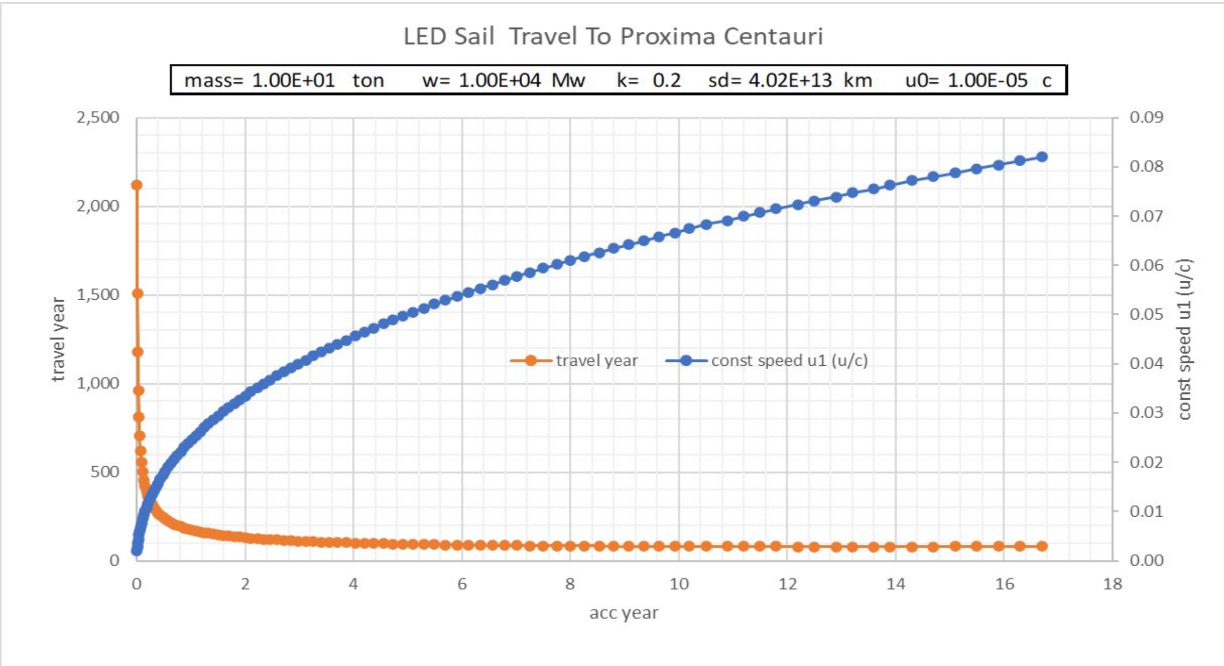

Figure 8: Travel time vs acceleration time

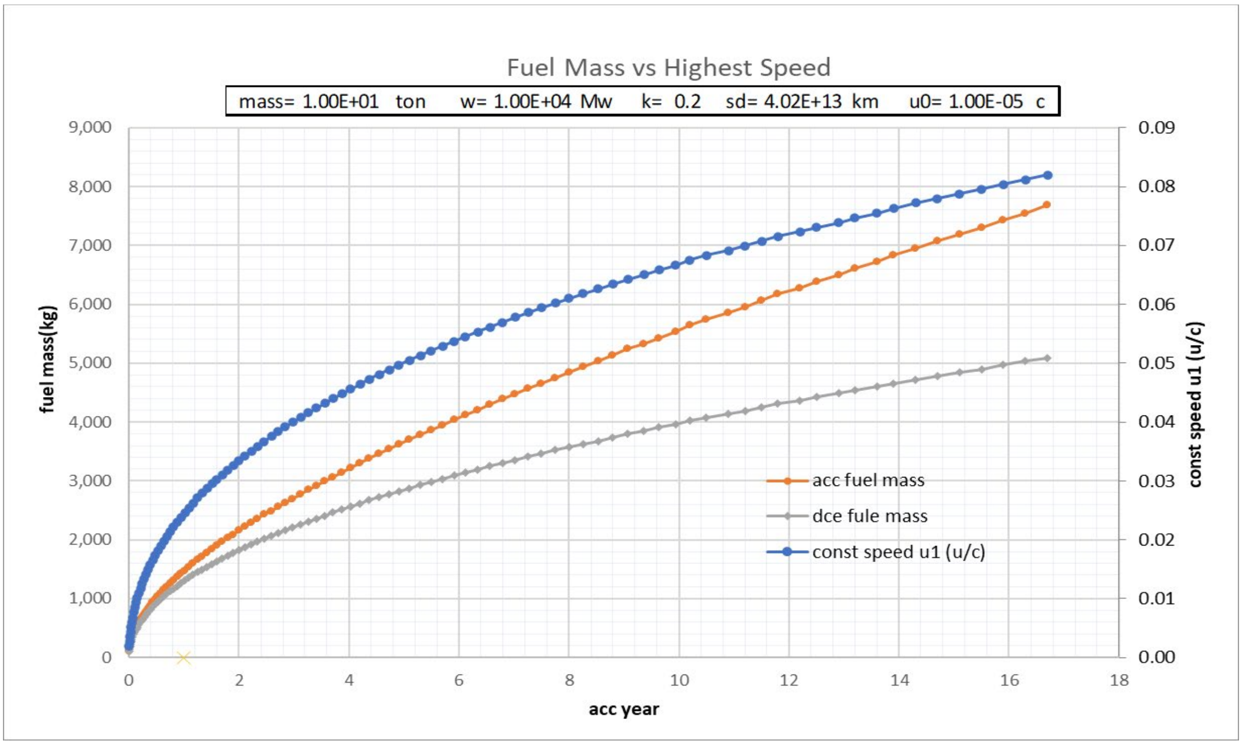

Figure 9: Fuel mass required vs acceleration time

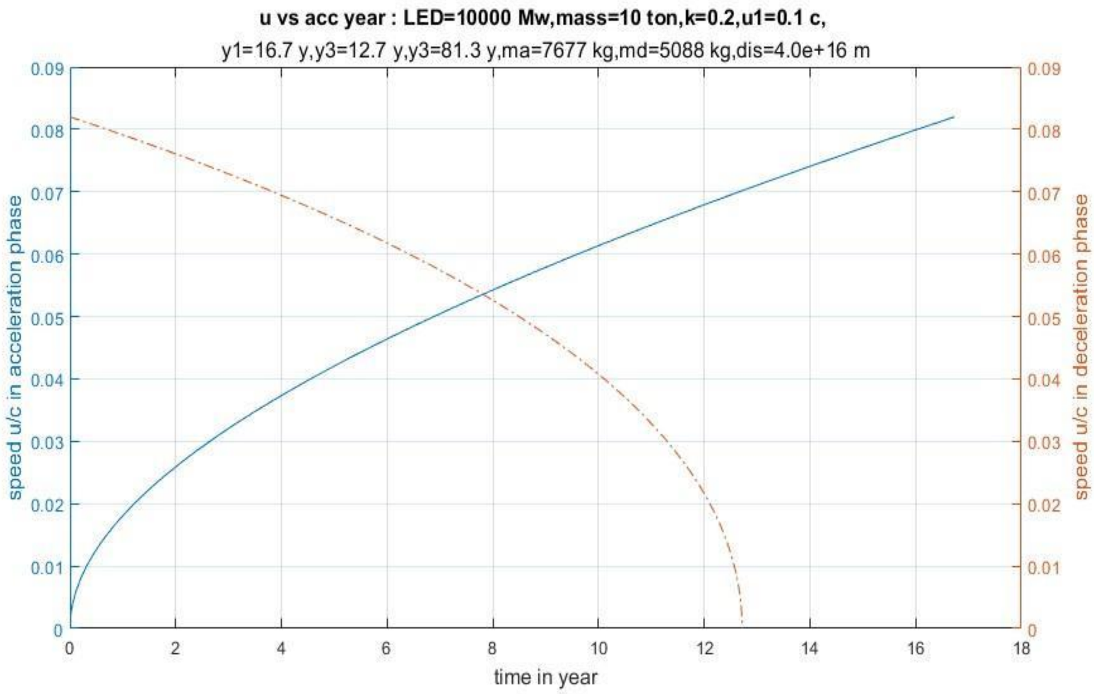

Figure 10: Acceleration phase and deceleration phase speed vs time for u_1=0.082 c

5. Other Considerations

Sending and receiving signals between home and a 4.25 lightyear distance source is very challenging. We could send signals back home by blinking LEDs as morse signals or send microwaves using the sail frame as a giant antenna. However, a critical problem exists in that the signal beam must be sufficiently focused. Table 4 shows that for a point source 4.25 lightyears away, a beam with an angular divergence of 0.1 degrees will have a beam radius of 7×1013 m at Earth. If the source has 2 MW power, with a 50 m radius microwave telescope as reflection surface, Earth would only receive 1×10-18 watts of that energy. Unlike radio telescopes that can accumulate the signal as time being, communication between two faraway parties will require each bit of signal be received within certain time interval [31-35]. We call it as the bandwidth requirement. This makes the task even more challenging. Of cause the minimum theoretical divergent angle is decided by diffraction formula as $1.22\lambda/D$ where D is the effective diameter of sail and λ will be the effective LED wavelength.

Source Energy (watts)	Beam divergence (degree)	Distance (m)	Spot radius near earth from a point source (m)	Antenna r (m)	Receiver Energy (watts)
2×10^{6}	0.1	4.021×10^{16}	7.02×10^{13}	50	1.02×10^{-18}

Table 4: The power the earth telescope could receive from a 4.23 lightyear away source

With long distance travel in the interstellar space, it must consider the possibility of sail being damaged by the space dust. In the phase of constant speed travel, the sail should retreat back to fuselage for protection. Replacement of damaged LED and sail will need an autonomous AI service robot. To make the sail panel multi-purpose is desired such as a microwave antenna as well as a photovoltaic panel.

6. Conclusion

Using photons to obtain thrust has high efficiency in converting energy to momentum compared with other methods using regular propellent whose rest mass is not zero. With the use of LED sails, conversion of fuel mass into thrust photons will have high fuel efficiency if macro nuclear generators are used as the energy source. There are still many unsolved technology issues that need to be developed to fulfill this dream. Developing new kinds of atomic fuel with high momentum index kp is critical for a reasonable fuel to spaceship mass ratio for atomic energy powered space travel [36-43]. Implementation of disposing of used fuel methods are also required. Looking forward, the ultimate goal is to develop a new kind of nuclear fuel that uses matter-antimatter in a future macro fusion nuclear generator should a new storage technology of anti-matter be invented. Dispersing base heat of LED sails in a vacuum environment will be a challenge, as well as developing a huge light-weight LED sail structure. Receiving a signal sent back from the spaceship 4 light years away is another challenge. Folding and unfolding a giant sail and replacing failed parts autonomously during travel also will be a challenge.

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```
%% calculate time needed to travel to a remote star
%% input parameters for the scripts  %%
% u0        %inital spaceship speed  in m/s
% sd:       %distance to remote star in m
% ms:       %spaceship mass in kg
% w :       %LED sail power in watts
% k :       %momentum index p = k*delta(m)
%% Return data
% year1:       %acceleration years in phase one
% year2:       %travel years with constant speed in phase two
% year3:       %deceleration years in phase three
% u1:          %constant speed in phase two
% syear:       %total travel years
% ma:          %fuel mass in kg for accerlation
% md:          %fuel mass in kg for decerlation
% dis1:        %distance in phase one
% dis2:        %distance in phase two
% dis3:        %distance in phase three
%% input values
w   = 1.0E10;    %LED sail power in watts
ms  = 10000;     %ship mass in kg
k   = 0.2;       %momentum index k = delta_p/delta_mass, p = k*dm
c   = 2.9979E8;  %speed of light in m/s
ys  = 3.1558E7;  %~365*24*60*60;  %seconds in one year 31557600
sd  = 4.0198E16; %target star distance in m
u0  = 0.00001;   %initial speed relative to lighspeed in m/s
N   = 100;       %total points to calculate
u10 = 0.002;     %first u1 speed
u11 = 0.082;     %last  u1 speed
%delta increse of u1
du1 = (u11-u10)/(N-1);
%% output  data to text file
fileID = fopen('led_shiptime.csv','w');
fprintf(fileID,'mass=;%.0e;ton;w=;%.0e;Mw;k=;%.2f;sd=;%.2e;km;u0=;%0.1e;c\n',
ms/1000,w/1000000,k,sd/1000,u0);
fprintf(fileID,'acc year;const year;dec year;const speed;travel year;acc
mass;dce mass;dis1 ;dis2 ;dis3 ;\n');
draw=0;
for u1=u10:du1:u11
    draw=0;
    if u1==u11
        draw=1;
    end
    [year1,year2,year3,syear,ma,md, dis1,dis2,dis3] = ...
                              shiptravel(u0, u1, w, k, ms, sd,c,ys, draw);
    fprintf(fileID,'%.2e;%.2e;%.2e;%.2e;%.2e;%.2e;%.2e;%.2e;%.2e;%.2e\n',...
                 year1,year2,year3,u1,syear,ma,md, dis1,dis2,dis3);
end
fclose(fileID);
fclose('all');
```

code 1 Main script

```
function [year1,year2,year3,syear,ma,md, dis1,dis2,dis3] = ...
                              shiptravel(u0, u1, w, k, ms, sd,c,ys, draw)
  % get ma and md of fuel requirement
  md = get_dc_mass(ms,k,u0, u1);
  ma = get_ac_mass(ms, md, k,u0, u1);
  % Acceleration phase, phase one, from u0 to u1
  uxspan=[u0,u1];
  [ux,t1] = ode45(@(ux,t1) odefcnac(ux,t1,w,ms,ma,md,k,c,u0), uxspan, u1);
  % integeral ux to obtain dis1 from u0 to u1
  dis1 = trapz(t1,ux);
  j = size(t1,1);
  tt1 = t1(j);  %get the time of phase one
  % Deceleration phase, phase three, from u1 to u0
  uyspan=[u1,u0];
  [uy,t3] = ode45(@(uy,t3) odefcnde(uy,t3,w,ms,   md,k,c,u1), uyspan, u0);
  t33 = t3(1)-t3;
  %integeral uy to obtain dis3 from u1 to u0
  dis3 = trapz(t33,uy);
  j = size(t33,1);   % time used for phase three
  tt3 = t33(j);
  %phase two distance
  dis2 = sd - dis1 -dis3;
  %calculate time for phase two, constant speed with u1
  tt2 = dis2/(u1*c);
  %total time
  syear = (tt1+tt2+tt3)/ys;
  year1 = tt1/ys;  year2 = tt2/ys;  year3 = tt3/ys;
  % draw curves
  if draw>0
      u1s =sprintf('u1=%.1f c, ',u1);
      y1s =sprintf('y1=%.1f y,',year1);
      y3s =sprintf('y3=%.1f y,',year3);
      syears =sprintf('y3=%.1f y,',syear);
      mds =sprintf('md=%.0f kg,',md);
      mas =sprintf('ma=%.0f kg,',ma);
      wm  =w/1000000;
      wms =sprintf('LED=%.0f Mw,',wm);
      msk =ms/1000;
      mks =sprintf('mass=%.0f ton,',msk);
      ks  =sprintf('k=%.1f,',k);
      sds =sprintf('dis=%.1e m',sd);
      yyaxis left
      ty1=t1./ys;
      plot(ty1,ux,'-');
      xlabel('time in year','FontSize',12);
      ylabel('speed u/c in acceleration phase','FontSize',12);
      titletext =['u vs acc year : ', wms, mks,ks,u1s];
      stitletext=[y1s,y3s,syears,mas,mds,sds];
      title(titletext,stitletext,'FontSize',12)
      grid on
      hold all
      yyaxis right
      ty3=t33./ys;
      plot(ty3,uy,'-.');
      ylabel('speed u/c in deceleration phase','FontSize',12);
  end
end
```

code 2 Function calculate for travel time

```
%% deceleration phase mass need from u0 and u1
function md = get_dc_mass(ms,k,u0, u1)
    % ms = m1*exp(1/k*(u0/sqer(1-u0^2) - u1/sqrt(1-u1^2)))
    % m1 = ms + md        %initial mass of deceleration phase
    ux  = exp(1.0/k*(u0/sqrt(1-u0^2) - u1/sqrt(1-u1^2)));
    m1  = ms/ux;
    md  = m1 - ms;
end
%% acceleration phase mass need from u0 and u1, md
function ma = get_ac_mass(ms, md, k,u0, u1)
    % ms = m0*exp(1/k*(u0/sqrt(1-u0^2) - u1/sqrt(1-u1^2)))
    % m0 = ms + md + ma %initial mass of deceleration phase
    m1  = ms + md;
    ux  = exp(1.0/k*(u0/sqrt(1-u0^2) - u1/sqrt(1-u1^2)));
    m0  = m1/ux;
    ma  = m0 - m1;
end
%% deceleration phase, get speed from md mass
function u = get_dc_speed(ms,md,k,u1)
    % m = m1*exp(1/k*(u/sqrt(1-u^2) - u1/sqrt(1-u1^2)))
    % m0 = ms + ma + md  %initial mass of acceleration phase
    % m1 = ms + md       %initial mass of deceleration phase
    m01=ms+md;
    lnmm1 = ln(ms/m01);
    u00=exp(u1/(k*sqrt(1-u1^2)));
    u01=lnmm1*u00;
    u02=ln(u01)*k;  %u02 must positive
    u03=u02*u02;
    % u/sqrt(1-u^2) = u02
    u=sqrt(u03/(1+u03));
end
%% acceleration phase, get speed from ma mass
function u = get_ac_speed(ms,md,ma,k,u0)
% m = m0*exp(1/k*(-u/sqrt(1-u^2) + u0/sqrt(1-u0^2)))
% at the end of accerlation phase mass is ms+md
m00=ms+md+ma;
m01=ms+md;
lnmm0 = ln(m01/m00);
u00=exp(u0/(k*sqrt(1-u0^2)));  % u00 must negtive
u01=lnmm0/u00;
u02=-ln(u01)*k;
u03= u02*u02;
% u/sqrt(1-u^2) = u02
u=sqrt(u03/(1+u03));
end
%% phase one:   time need to obtain u in acce phase (t using s as unit)
function dtduac = odefcnac(u,t,w,ms,ma,md,k,c,u0)
  m0 = ms + ma + md;
  dtduac =1.0/((w/(m0*c*c*u))*(1-u*u)^(3/2)*(exp((u/sqrt(1-u*u)-u0/sqrt(1-
u0*u0))/k)));
end
%% phase three: time need to reduce u in dec  phase (t using s as unit)
function dtdude = odefcnde(u,t,w,ms,md,k,c,u1)
  m1 = ms + md;
  dtdude =1.0/((w/(m1*c*c*u))*(1-u*u)^(3/2)*(exp((u1/sqrt(1-u1*u1)-u/sqrt(1-
u*u))/k))) ;
end
```

code 3 help functions