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Short Communication

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The Repulsion Force Between Ice Bodies in Dense Rings of Saturn, J.C. Maxwell Proposed in 1856

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Abstract

J.C. Maxwell concluded: disk of Saturn's dense rings can be stable only if it consists of swarm of unrelated meteorites. At that time, it was not possible to determine the nature of repulsion force between them. There is no yet model to describe this fact. The Cassini probe discovered dense rings consist of chunks of diamagnetic ice. Here we show how Saturn's magnetism could determine the repulsion force and how it allows the calculation of the equilibrium distance between ice bodies in dense rings.

1. Introduction

After Galileo observed the dense rings of Saturn in 1610, their features remain difficult to understand [1-3]. J.C. Maxwell in 1856 came to conclusion: the disk of dense rings of Saturn can be stable only if it consists of a swarm of meteorites unrelated to each other [4]. At that time, it was not possible to determine the nature of the repulsion force between them. There is no yet model to describe this fact. The Cassini probe (2004-2017) discovered that dense rings consist mainly of chunks of diamagnetic ice [5-8]. This leads to an idea that Saturn's magnetism induces the repulsion force, which determines the equilibrium distance between chunks of ice in dense rings.

2. Repulsion Force between chunks of Ice in Dense Rings

Chunks of diamagnetic ice that are nearby in an orbit are separated by magnetic repulsion and attracted to each other by gravity. From the balance of the forces of gravitational attraction and magnetic repulsion, we can obtain the equilibrium distance between the ice bodies in dense rings.

The force acting on a point magnetic dipole with a moment $\mathbf{m}_2$ in the magnetic field of another point magnetic dipole $\mathbf{m}_1$ at a distance r_0 is [9]

$$\mathbf{F}_m = \frac{3\mu_0}{4\pi r_0^5}\left[\mathbf{m_2}\left(\mathbf{m}_1\cdot\mathbf{r_0}\right)+\mathbf{m}_1\left(\mathbf{m}_2\cdot\mathbf{r_0}\right)+\mathbf{r_0}\left(\mathbf{m}_1\cdot\mathbf{m_2}\right)-5\mathbf{r_0}\left(\mathbf{m}_1\cdot\mathbf{r_0}\right)\left(\mathbf{m}_2\cdot\mathbf{r_0}\right)\right], \tag{9}$$

where $\mathbf{r}_0$ is the vector pointing from dipole 1 to dipole 2.
Under an assumption that the dipole moments are parallel and equal in magnitude,

$$\mathbf{F}_m = \frac{3\mu_0 m^2}{4\pi r_0^5}\mathbf{r_0} \tag{10}$$

The induced magnetic dipole moment $\boldsymbol{m}$ [5] is

$$\mathbf{m} = \frac{4\pi R^3\mathbf{B}_0}{\mu_0}\frac{\mu-\mu_0}{\mu+2\mu_0} \tag{11}$$

Then

$$\mathbf{F}_m = \frac{12\pi R^6 B_0^2}{\mu_0 r_0^5}\left(\frac{\mu-\mu_0}{\mu+2\mu_0}\right)^2 \mathbf{r_0}\,, \tag{12}$$

where B_0 is the magnitude of Saturn's magnetic field that magnetizes the chunk of ice.

Magnetic field of Saturn's magnetic dipole in the magnetic equator plane in an infinitesimal approximation is [9]:

$$\mathbf{B}_0\left(\mathbf{r_{Sp}} \perp \mathbf{m_S}\right) = -\frac{\mu_0}{4\pi r_{Sp}^3}\mathbf{m_S} \tag{13}$$

where $\mathbf{r}_{\mathbf{sp}}$ is the radius vector from Saturn's magnetic dipole.

Thus, the force acting on the ice body 2 (eq. 12) in the equatorial plane is

$$\mathbf{F}_m\left(\mathbf{r_{Sp}} \perp \mathbf{m_S}\right) = \frac{3\mu_0 R^6 m_S^2}{4\pi r_0^5 r_{Sp}^6}\left(\frac{\mu-\mu_0}{\mu+2\mu_0}\right)^2 \mathbf{r_0}\,. \tag{14}$$

The attractive gravitational force acting on the ice body 2 with the mass M_{p2} induced by the ice body 1 with the mass M_{p1} is

$$\mathbf{F}_g = -\frac{GM_{p1}M_{p2}}{r_0^3}\mathbf{r_0} \tag{15}$$

The relationships for repulsion force $\mathbf{F}_{\mathrm{m}}$ and attraction force $\mathbf{F}_{\mathrm{g}}$ as a function of distance between the ice bodies are schematically shown in Figure 1.

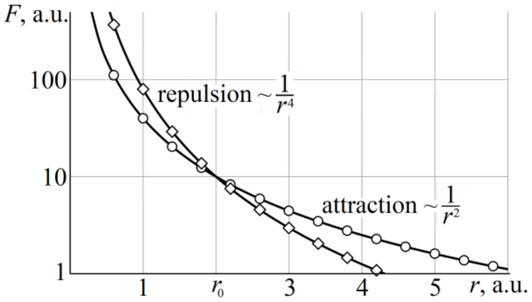

Figure 1: Dependence of the Repulsion and Attraction Forces on Distance between Ice Bodies.

At the equilibrium distance r_0, we observe a stable position of the ice chunks inside dense rings.

If the masses of ice bodies are equal ($M_{p1} = M_{p2} = M_p$), the balance of the gravitational and magnetic forces occurs at $\mathbf{F}_g + \mathbf{F}_m = \mathbf{0}$. Then

$$r_0 = \left(\frac{R}{r_{Sp}}\right)^3 \frac{m_S}{M_p}\left|\frac{\mu-\mu_0}{\mu+2\mu_0}\right|\sqrt{\frac{3\mu_0}{4\pi G}} \tag{16}$$

Considering that $M_p = 4\pi R^3 \rho_p / 3$, we get:

$$r_0 = \left(r_{Sp}\sqrt{\frac{4\pi}{3}}\right)^{-3} \frac{m_S}{\rho_p}\left|\frac{\mu-\mu_0}{\mu+2\mu_0}\right|\sqrt{\frac{\mu_0}{G}} \tag{17}$$

Thus, the equilibrium distance between ice chunks varies inversely with their density ρ_p and decreases as the third power of distance from the planet. Accurate calculation of the equilibrium distance between ice bodies requires knowledge of the characteristics of dense rings, such as their composition, density of space ice, magnetic permeability, etc. The close chunks of ice in the rings also affected by centrifugal force, but its contribution to the balance of forces is insignificant due to the small distance between the ice chunks in compared to the distance to Saturn.

3. Conclusion

Saturn's magnetism could determine the repulsion force between ice bodies in dense rings of Saturn due to diamagnetism of ice. The equilibrium distance between chunks of ice in dense rings emerges from the balance of gravitational attraction and magnetic repulsion between them. This model is supported by observations from the Cassini probe.

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