ISSN: 2639-0108

Short Article

Advances in Theoretical & Computational Physics

The Relation between Hyper-Exponential Functions and Napier Number

Uchida Keitaroh*

Department of Applied Mathematics

*Corresponding Author
Uchida Keitaroh, Department of Applied Mathematics, Japan.

Submitted: 2024, Jun 26; **Accepted:** 2024, Aug 01; **Published:** 2024, Aug 05

Citation: Keitaroh, U. (2024). The Relation between Hyper-Exponential Functions and Napier Number. *Adv Theo Comp Phy, 7*(3), 01-02.

Special functions that satisfy the following differential equation are defined as hyper-exponential functions of n-order.

$$\frac{d^n y(x)}{dx^n} = f(x)y(x)$$

The hyper-exponential function can be said to be a family of functions of such special functions.

I created the symbol **exph** to represent the hyper-exponential function.
Exph stands for hyper-exponential function, which means infinite series as follows.

$$x \in R\,, f(x) \in R, i \in N_0, j \in N_0, n \in N$$

$$f(x):$$

A differentiable function defined by some interval containing zero, which is an integrable and bounded function.

$$seed(x;0) = 1$$
$$seed(x;j) = \frac{x^j}{j!} \quad (j = 1,2,\cdots,n-1)$$

$$k_0(x) = seed(x;j) \quad (j = 0,1,2,\cdots,n-1)$$
$$k_{i+1}(x) = \int_0^x \cdots \int_0^x dx^n\, f(x)\, k_i(x)$$
$$exph_j^n\{x; f(x)\} = \sum_{i=0}^{\infty} k_i(x)$$

And for,

$$y(x) = exph_j^n\{x; f(x)\}$$

The following differential equation is obtained.

$$\frac{d^n y(x)}{dx^n} = f(x)y(x) \ .$$

Thus, **exph** is a form of representation of the hyper-exponential function.

Napier number:

$$\boldsymbol{e} = 1 + \sum_{i=1}^{\infty} \frac{1}{i!} = 2.718281828\cdots \ .$$

Exponential function:

$$e^x = exp(x) = exph_0^1(x;1) = 1 + \sum_{i=1}^{\infty} \frac{x^i}{i!} \quad .$$

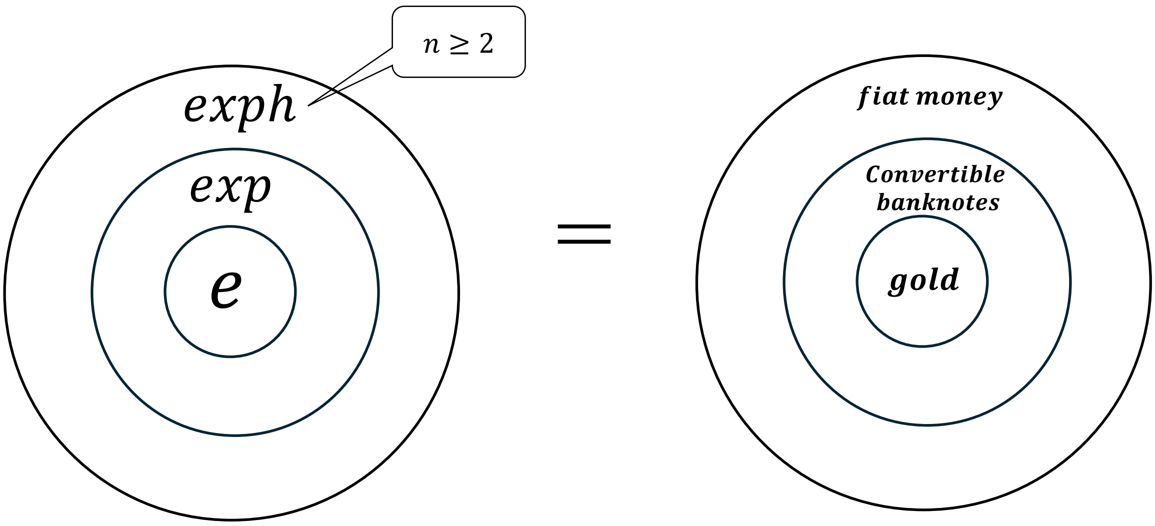
The following feature of hyper-exponential functions of first-order is important.

$$exph_0^1\{x; f(x)\} = e^{\int_0^x f(x)dx}.$$

Thus, hyper-exponential functions of first-order are tied to the Napier number **e**. For this reason, if the Napier number is compared to gold, not only the exponential function but also hyper-exponential functions of first-order can be compared to convertible banknotes.

On the other hand, hyper-exponential functions of second-order or higher-order cannot be described by using the Napier number, so they can be compared to fiat money.

What I am arguing is that, just as humanity has come to use fiat money as currency, the time has come for researchers to use hyper-exponential functions of second-order or higher-order, which are not tied to the Napier number, in various fields such as mathematics, physics, and science and technologies.



As mentioned above, hyper-exponential functions contain the exponential function. And hyper-exponential functions of second-order or higher-order are not tied to the Napier number.

I eagerly recommend that all of humanity, including researchers, understand this and make active use of **exph.**