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Review Article

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The MIMO Data Transfer Line with Seven-Frequency Octonion Carrier

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Abstract

Currently, with the development of public relations and production systems, there is a need to increase the capacity of communication systems and information transmission. It has been shown theoretically that it is possible to increase throughput by using multidimensional signals in space instead of real signals on a plane. It is now accepted that a multidimensional space, Multiple-Input Multiple-Output (MIMO), can be formed using multiple antennas to transmit and receive in physical space. However, as physicists point out, such space is three-dimensional, and with the addition of time it is four-dimensional.

It is clear that in such a physical space, when using more than 2 antennas for transmission and 2 for reception, it is impossible to obtain a gain in throughput of more than 4 times, since according to the laws of cybernetics, the diversity at the channel input will not be transmitted to the exit. It follows that it is necessary to reconsider existing views on the dimension of physical space. Previously, in the work the MIMO data transfer line with three-frequency quaternion carrier, it was shown that it is possible to use a hypercomplex quaternion number as a model of physical space. In this case, the dimension of space will be equal to 4 with 3 imaginary (spatial) axes and one scalar axis. In addition, combinations of three quaternion angular frequencies on the imaginary axes formed 4 single-frequency channels. Accordingly, the gain in throughput compared to real signals reached 4 in orthogonal axes and 4 in frequencies.

In this work, an octonion with 7 imaginary (spatial) axes and one scalar is used as a mathematical model of physical space. It is shown that the dimension of physical space will be 8 with 64 single-frequency channels in the form of combinations of 7 angular frequencies. Hence, the gain in throughput will be 8 in orthogonal axes and 64 in frequencies.

Keywords: MIMO, Hypercomplex, Quaternion, Octonion, Throughput, 5G, 6G

1. Introduction

With the development of the Internet, the problem of increasing the throughput of communication channels has become acute. It has been shown theoretically that throughput can be increased if multidimensional Gaussian processes are used as signals [1]. Multidimensional Gaussian or other functions can only be described in spaces of the same dimension. The multidimensionality of a functional space can be determined by the number of coordinate axes in the space. When the coordinate axes are orthogonal, the space will have a maximum volume.

Like Euclidean space, a point M in n-dimensional space is determined by the value of its coordinates M (x_1, x_2 ,..., x_n) Therefore, we must use a Many-Input – Many-Output (MIMO) scheme with the ability to separate the input and output points with minimal energy loss. In this case, maximization of throughput is achieved when the number of inputs is equal to the number of outputs of the multidimensional space [1].

However, in this case a question arises. If, according to the theory of relativity, physical space is four-dimensional, taking into account the time coordinate, then it is impossible to obtain in this space using spatially separated antennas a gain in throughput of more than four. In accordance with the laws of cybernetics, the throughput of a communication channel must be equal to the diversity of transmitted information at the channel input [2]. Otherwise, to transmit the incoming diversity, it will be necessary to increase the transmission time. For example, it took billions of years to obtain the diversity that exists on Earth. The result is a human whose brain contains 86 billion neurons. If people exchanged information with each other through 3- dimensional space, it would take them centuries to education. In other words, it is obvious that complex organisms exchange information using spaces (channels) with a dimension corresponding to their complexity.

Research carried out in showed that hypercomplex spaces can be considered as a mathematical model of multidimensional physical space [3-10]. Thus, in works [3-5], quaternion Fourier transforms of 4-dimensional pulse vectors were presented, with the help of which 4 pulse spectra were calculated. The work [6] presents similar calculations of the Laplace transform for 4- dimensional vectors [7-8]. Works show the possibility of increasing the throughput of MIMO communication lines by 2 times using complex signals and 4 times using single-frequency quaternions. The works show that the hypercomplex model of physical space based on the quaternion is not only 4-dimensional in coordinates with one scalar axis and 4 imaginary ones, but also 4-dimensional in spatial frequencies [9, 10]. From 3 reference frequencies 4 positive combination frequencies are obtained.

The quaternion-based mathematical model of physical space does a good job of simulating the color patterns of space and the behavior of photons. Without going into details, color models have 3 primary colors, for example R, G, B, forming 8 colors of the rainbow, which can be defined as positive and negative [11]. Light travels in 3-dimensional space in the form of electromagnetic waves. Electromagnetic waves are described in 3-dimensional space by Maxwell's equations using the Hamilton operator for 3 imaginary units *i, j, k*. The photon has no mass, but it manifests itself as a "particle" (quaternion scalar) and causes the photoelectric effect. Photon energy is transmitted by quanta using a scalar part that changes in amplitude in a wave-like manner. That is, the photon manifests itself both as a particle, or rather a scalar without mass, and as a wave.

If we consider time as a cyclic rotation with a certain period, then for a three-frequency quaternion the time will depend on the position point in space, since the rotation frequency at this point is determined by the sum of 3 frequencies on the coordinate axes. This fact is a confirmation of the theory of relativity and the connection between space and time. It is also known that light, as an electromagnetic wave, has polarization. Polarization of light confirms the presence of an orthogonal coordinate system in physical space.

The purpose of this work is to study the octonion as a model of a physical multidimensional space with 7 imaginary orthogonal axes and one scalar, as well as 64 frequency channels formed by 7 frequencies on imaginary orthogonal axes, to increase throughput.

2. Materials and Methods for Solving the Problem

We write the mathematical model of a MIMO channel with the same number of inputs and outputs in the form [1, 9, 10]:

$$\mathbf{s}(t) = \mathbf{H}(t)\mathbf{x}(0) + \mathbf{n}(t), \quad (1)$$

where $\mathbf{H}(t)$ – square channel matrix of dimension *MxM*, $\mathbf{x}(0)$ - *M* – dimensional vector of information symbols, $\mathbf{n}(t)$ – zero-mean noise vector with circular symmetry.

Octonion (Octave) is obtained by doubling the quaternion $q = s + ix + jy + kz$, where *s, x, y, z* are real numbers, *i, j*, k are imaginary units. We write the octonion in algebraic form as [12]:

$$o = se + xi + yj + zk + s_1e_1 + x_1i_1 + y_1j_1 + z_1k_1. \quad (2)$$

where $s, e, x, y, z, s_1, x_1, y_1, z_1$ – real numbers, $i, j, k, e_1, i_1, j_1, k_1$ – imaginary units.

Let's present operations with imaginary units in the form of a table.

Table 1: Operations of Multiplication of Imaginary Octonion Units in Representation (2).

$\times$	e	i	j	k	e_1	i_1	j_1	k_1
e	e	i	j	k	e_1	i_1	j_1	k_1
i	i	$-e$	k	$-j$	i_1	$-e_1$	$-k_1$	j_1
j	j	$-k$	$-e$	i	j_1	k_1	$-e_1$	$-i_1$
k	k	j	$-i$	$-e$	k_1	$-j_1$	i_1	$-e_1$.
e_1	e_1	$-i_1$	$-j_1$	$-k_1$	$-e$	i	j	k
i_1	i_1	e_1	$-k_1$	j_1	$-i$	$-e$	$-k$	j
j_1	j_1	k_1	e_1	$-i_1$	$-j$	k	$-e$	$-i$
k_1	k_1	$-j_1$	i_1	e_1	$-k$	$-j$	i	$-e$

We write the exponential function of the octonion (2) as

$$\mathrm{e}^{o}=\exp\left\{se+xi+yj+zk+s_1e_1+x_1i_1+y_1j_1+z_1k_1\right\}=\mathrm{e}^{se}\,\mathrm{e}^{xi}\,\mathrm{e}^{yj}\,\mathrm{e}^{zk}\,\mathrm{e}^{s_1e_1}\,\mathrm{e}^{x_1i_1}\,\mathrm{e}^{y_1j_1}\,\mathrm{e}^{z_1k_1}\,. \tag{3}$$

Let us denote the radius of rotation in 8D space as

$$\mathrm{e}^{s}=r=\sqrt{s^2+x^2+y^2+z^2+s_1^2+x_1^2+y_1^2+z_1^2}\,.$$

As we see, the radius of rotation is equal to the modulus of the octonion $|o|$.

Using Euler's formula, we obtain expression (3) in polar representation:

$$\begin{aligned}\mathrm{e}^{o}&=\mathrm{e}^{se}\left(\cos x+i\sin x\right)\left(\cos y+j\sin y\right)\left(\cos z+k\sin z\right)\times\\&\times\left(\cos s_1+e_1\sin s_1\right)\left(\cos x_1+i_1\sin x_1\right)\left(\cos y_1+j_1\sin y_1\right)\left(\cos z_1+k_1\sin z_1\right).\end{aligned} \tag{4}$$

In radio engineering problems, coefficients for imaginary units in a complex or hypercomplex representation of signals have the physical meaning of angles written in radians. Typically, these angles change over time. Therefore, let us imagine them as functions of time and write them in the form of angular frequencies:

$$x(t)=\omega_i t\,,\ y(t)=\omega_j t\,,\ z(t)=\omega_k t\,,\ s_1(t)=\omega_{e_1}t\,,\ x_1(t)=\omega_{i_1}t\,,\ y_1(t)=\omega_{j_1}t\,,\ z_1(t)=\omega_{k_1}t\,,$$

where ω_i, ω_j, ω_k, ω_{e_1}, ω_{i_1}, ω_{j_1}, ω_{k_1} – angular frequencies on orthogonal imaginary coordinate axes j, k, e_1, i_1, j_1, k_1.

By multiplying complex numbers in the polar form of notation (4) and replacing the angles of sines and cosines with the corresponding functions of time, we obtain the expression for the octonion in trigonometric representation. The exponential function of the octonion will also be an octonion, therefore, we obtain function (4) in the form:

$$\begin{aligned}&f\left(o(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)\right)=\\&=p(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)+iu(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)+\\&+jv(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)+kw(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)+\\&+e_1p_1(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)+i_1u_1(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)+\\&+j_1v_1(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)+k_1w_1(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)\,.\end{aligned} \tag{5}$$

After grouping similar terms with imaginary units, the components in expression (5) will take the form:

$$\begin{aligned}&p\left(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t\right)=\\&\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)-\\&-\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)+\\&+\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)-\\&-\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)-\\&-\cos(\omega_i t)\cos(\omega_k t)\cos(\omega_{e_1}t)\cos(\omega_{j_1}t)\sin(\omega_j t)\sin(\omega_{i_1}t)\sin(\omega_{k_1}t)-\\&-\cos(\omega_i t)\cos(\omega_k t)\cos(\omega_{i_1}t)\cos(\omega_{k_1}t)\sin(\omega_j t)\sin(\omega_{e_1}t)\sin(\omega_{j_1}t)+\\&+\cos(\omega_j t)\cos(\omega_{e_1}t)\cos(\omega_{j_1}t)\sin(\omega_i t)\sin(\omega_k t)\sin(\omega_{i_1}t)\sin(\omega_{k_1}t)+\\&+\cos(\omega_j t)\cos(\omega_{i_1}t)\cos(\omega_{k_1}t)\sin(\omega_i t)\sin(\omega_k t)\sin(\omega_{e_1}t)\sin(\omega_{j_1}t)+\\&+\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\sin(\omega_i t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)-\\&-\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_i t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)+\end{aligned} \tag{6}$$

$$+\cos(\omega_i t)\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_{e_1} t)\cos(\omega_{k_1} t)\sin(\omega_k t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)+$$
$$+\cos(\omega_k t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_{e_1} t)\sin(\omega_{k_1} t);$$

$$u\left(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t\right)=$$
$$-\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_i t)-$$
$$-\cos(\omega_i t)\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_j t)\sin(\omega_k t)+$$
$$+\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_{e_1} t)\cos(\omega_{j_1} t)\sin(\omega_k t)\sin(\omega_{i_1} t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_{i_1} t)\cos(\omega_{k_1} t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{j_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{k_1} t)\sin(\omega_j t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_k t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\sin(\omega_j t)\sin(\omega_{e_1} t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{j_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_{i_1} t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_k t)\cos(\omega_{i_1} t)\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_{e_1} t)\sin(\omega_{j_1} t)+$$
$$+\cos(\omega_j t)\cos(\omega_{e_1} t)\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_k t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)-$$
$$-\cos(\omega_j t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\sin(\omega_i t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_j t)\cos(\omega_k t)\sin(\omega_i t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)-$$
$$-\cos(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t);$$

$$v\left(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t\right)=$$
$$\cos(\omega_i t)\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_j t)-$$
$$-\cos(\omega_j t)\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_k t)+$$
$$+\cos(\omega_i t)\cos(\omega_k t)\sin(\omega_j t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_j t)\sin(\omega_i t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{j_1} t)\sin(\omega_{i_1} t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{i_1} t)\cos(\omega_{k_1} t)\sin(\omega_{e_1} t)\sin(\omega_{j_1} t)-$$

$$-\cos(\omega_{e_1}t)\cos(\omega_{j_1}t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{i_1}t)\sin(\omega_{k_1}t)-$$
$$-\cos(\omega_{i_1}t)\cos(\omega_{k_1}t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1}t)\sin(\omega_{j_1}t)-$$
$$-\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\sin(\omega_k t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)+$$
$$+\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_k t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)-$$
$$-\cos(\omega_k t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)+$$
$$+\cos(\omega_k t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)-$$
$$-\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1}t)\cos(\omega_{k_1}t)\sin(\omega_i t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)+$$
$$+\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\sin(\omega_i t)\sin(\omega_{e_1}t)\sin(\omega_{k_1}t)-$$
$$-\cos(\omega_i t)\cos(\omega_{e_1}t)\cos(\omega_{k_1}t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)+$$
$$+\cos(\omega_i t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1}t)\sin(\omega_{k_1}t);$$

$$w\left(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t\right)=$$
$$-\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_k t)+$$
$$+\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\sin(\omega_{e_1}t)\sin(\omega_{k_1}t)-$$
$$-\cos(\omega_k t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_i t)\sin(\omega_j t)-$$
$$-\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1}t)\cos(\omega_{k_1}t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)-$$
$$-\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1}t)\cos(\omega_{j_1}t)\sin(\omega_i t)\sin(\omega_{i_1}t)\sin(\omega_{k_1}t)-$$
$$-\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{i_1}t)\cos(\omega_{k_1}t)\sin(\omega_i t)\sin(\omega_{e_1}t)\sin(\omega_{j_1}t)-$$
$$-\cos(\omega_i t)\cos(\omega_k t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\sin(\omega_j t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)+$$
$$+\cos(\omega_i t)\cos(\omega_k t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_j t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)-$$
$$-\cos(\omega_i t)\cos(\omega_{e_1}t)\cos(\omega_{j_1}t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{i_1}t)\sin(\omega_{k_1}t)-$$
$$-\cos(\omega_i t)\cos(\omega_{i_1}t)\cos(\omega_{k_1}t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1}t)\sin(\omega_{j_1}t)+$$
$$+\cos(\omega_j t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\sin(\omega_i t)\sin(\omega_k t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)-$$
$$-\cos(\omega_j t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_i t)\sin(\omega_k t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)-$$
$$-\cos(\omega_i t)\cos(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)+$$
$$+\cos(\omega_{e_1}t)\cos(\omega_{k_1}t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)-$$
$$-\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1}t)\sin(\omega_{k_1}t)-$$
$$-\cos(\omega_k t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t);$$
$$p_1\left(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t\right)=$$
$$\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\sin(\omega_k t)\sin(\omega_{k_1}t)+$$
$$+\cos(\omega_k t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_{k_1}t)+$$

$$\begin{aligned}
&+\cos(\omega_{i}t)\cos(\omega_{j}t)\cos(\omega_{k_1}t)\sin(\omega_{k}t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)+\\
&+\cos(\omega_{k}t)\cos(\omega_{k_1}t)\sin(\omega_{i}t)\sin(\omega_{j}t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)+\\
&+\cos(\omega_{j}t)\cos(\omega_{k}t)\cos(\omega_{e_1}t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_{i}t)\sin(\omega_{i_1}t)+\\
&+\cos(\omega_{i}t)\cos(\omega_{e_1}t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_{j}t)\sin(\omega_{k}t)\sin(\omega_{i_1}t)+\\
&+\cos(\omega_{j}t)\cos(\omega_{k}t)\cos(\omega_{i_1}t)\sin(\omega_{i}t)\sin(\omega_{e_1}t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)+\\
&+\cos(\omega_{i}t)\cos(\omega_{i_1}t)\sin(\omega_{j}t)\sin(\omega_{k}t)\sin(\omega_{e_1}t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)+\\
&+\cos(\omega_{i}t)\cos(\omega_{k}t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\cos(\omega_{k_1}t)\sin(\omega_{j}t)\sin(\omega_{j_1}t)-\\
&-\cos(\omega_{j}t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\cos(\omega_{k_1}t)\sin(\omega_{i}t)\sin(\omega_{k}t)\sin(\omega_{j_1}t)-\\
&-\cos(\omega_{i}t)\cos(\omega_{k}t)\cos(\omega_{j_1}t)\sin(\omega_{j}t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{k_1}t)+\\
&+\cos(\omega_{j}t)\cos(\omega_{j_1}t)\sin(\omega_{i}t)\sin(\omega_{k}t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{k_1}t)-\\
&-\cos(\omega_{i}t)\cos(\omega_{j}t)\cos(\omega_{k}t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_{e_1}t)+\\
&+\cos(\omega_{i}t)\cos(\omega_{j}t)\cos(\omega_{k}t)\cos(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)+\\
&+\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_{i}t)\sin(\omega_{j}t)\sin(\omega_{k}t)\sin(\omega_{e_1}t)+\\
&-\cos(\omega_{e_1}t)\sin(\omega_{i}t)\sin(\omega_{j}t)\sin(\omega_{k}t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t);
\end{aligned}$$

$$\begin{aligned}
&u_1\left(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t\right)=\\
&=\cos(\omega_{i}t)\cos(\omega_{k}t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\sin(\omega_{j}t)\sin(\omega_{k_1}t)-\\
&-\cos(\omega_{j}t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\sin(\omega_{i}t)\sin(\omega_{k}t)\sin(\omega_{k_1}t)+\\
&+\cos(\omega_{i}t)\cos(\omega_{k}t)\cos(\omega_{k_1}t)\sin(\omega_{j}t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)-\\
&-\cos(\omega_{j}t)\cos(\omega_{k_1}t)\sin(\omega_{i}t)\sin(\omega_{k}t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{j_1}t)+\\
&+\cos(\omega_{i}t)\cos(\omega_{j}t)\cos(\omega_{k}t)\cos(\omega_{e_1}t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_{i_1}t)+\\
&+\cos(\omega_{i}t)\cos(\omega_{j}t)\cos(\omega_{k}t)\cos(\omega_{i_1}t)\sin(\omega_{e_1}t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)-\\
&-\cos(\omega_{e_1}t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_{i}t)\sin(\omega_{j}t)\sin(\omega_{k}t)\sin(\omega_{i_1}t)-\\
&-\cos(\omega_{i_1}t)\sin(\omega_{i}t)\sin(\omega_{j}t)\sin(\omega_{k}t)\sin(\omega_{e_1}t)\sin(\omega_{j_1}t)\sin(\omega_{k_1}t)-\\
&-\cos(\omega_{i}t)\cos(\omega_{j}t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\cos(\omega_{k_1}t)\sin(\omega_{k}t)\sin(\omega_{j_1}t)-\\
&-\cos(\omega_{k}t)\cos(\omega_{e_1}t)\cos(\omega_{i_1}t)\cos(\omega_{k_1}t)\sin(\omega_{i}t)\sin(\omega_{j}t)\sin(\omega_{j_1}t)+\\
&+\cos(\omega_{i}t)\cos(\omega_{j}t)\cos(\omega_{j_1}t)\sin(\omega_{k}t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{k_1}t)+\\
&+\cos(\omega_{k}t)\cos(\omega_{j_1}t)\sin(\omega_{i}t)\sin(\omega_{j}t)\sin(\omega_{e_1}t)\sin(\omega_{i_1}t)\sin(\omega_{k_1}t)+\\
&+\cos(\omega_{j}t)\cos(\omega_{k}t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_{i}t)\sin(\omega_{e_1}t)+\\
&+\cos(\omega_{i}t)\cos(\omega_{i_1}t)\cos(\omega_{j_1}t)\cos(\omega_{k_1}t)\sin(\omega_{j}t)\sin(\omega_{k}t)\sin(\omega_{e_1}t)-
\end{aligned}$$

$$-\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1} t)\sin(\omega_i t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_{e_1} t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t);$$

$$v_1\left(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t\right)=$$
$$=\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\sin(\omega_i t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_{k_1} t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_{e_1} t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_k t)\sin(\omega_{i_1} t)-$$
$$-\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_{i_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_{i_1} t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_k t)\cos(\omega_{i_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_{e_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\cos(\omega_{k_1} t)\sin(\omega_{j_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{j_1} t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{j_1} t)-$$
$$-\cos(\omega_{j_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_k t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_j t)\sin(\omega_{e_1} t)+$$
$$+\cos(\omega_j t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_k t)\sin(\omega_{e_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_k t)\cos(\omega_{e_1} t)\sin(\omega_j t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_j t)\cos(\omega_{e_1} t)\sin(\omega_i t)\sin(\omega_k t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t);$$

$$w_1\left(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t\right)=$$
$$=\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{k_1} t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)-$$
$$-\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_j t)\sin(\omega_{i_1} t)+$$
$$+\cos(\omega_j t)\cos(\omega_{e_1} t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_k t)\sin(\omega_{i_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_k t)\cos(\omega_{i_1} t)\sin(\omega_j t)\sin(\omega_{e_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_j t)\cos(\omega_{i_1} t)\sin(\omega_i t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_{j_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_{e_1} t)\cos(\omega_{i_1} t)\cos(\omega_{k_1} t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{j_1} t)-$$

$$-\cos(\omega_j t)\cos(\omega_k t)\cos(\omega_{j_1} t)\sin(\omega_i t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_{j_1} t)\sin(\omega_j t)\sin(\omega_k t)\sin(\omega_{e_1} t)\sin(\omega_{i_1} t)\sin(\omega_{k_1} t)+$$
$$+\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_k t)\sin(\omega_{e_1} t)+$$
$$+\cos(\omega_k t)\cos(\omega_{i_1} t)\cos(\omega_{j_1} t)\cos(\omega_{k_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_{e_1} t)-$$
$$-\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_{e_1} t)\sin(\omega_k t)(\omega_{i_1} t)\sin(\omega_{j_1} t)\sin\sin(\omega_{k_1} t)-$$
$$-\cos(\omega_k t)\cos(\omega_{e_1} t)\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_{i_1} t)\sin(\omega_{j_1} t)\sin(\omega_{k_1} t)\,.$$

Thus, we obtained 8 functions from 16 combinations of products of cosines and sines in different combinations for 7 reference angular frequencies varying over time. Using well-known trigonometry formulas, we represent the product of sines and cosines of seven different angles as the sum of cosines and sines of various combinations with different signs. From 7 reference requencies ω_i, ω_j, ω_k, ω_{e_1}, ω_{i_1}, ω_{j_1}, ω_{k_1} we obtain $2^{8-2} = 2^6 = 64$ positive combination frequencies, which we denote by the number of the corresponding combination of the 7- dimensional bipolar vector:

$$\Omega_1 = \omega_i + \omega_j + \omega_k + \omega_{e_1} + \omega_{i_1} + \omega_{j_1} + \omega_{k_1}\,,\ \Omega_2 = \omega_i + \omega_j + \omega_k + \omega_{e_1} + \omega_{i_1} + \omega_{j_1} - \omega_{k_1}\,, \tag{7}$$
$$\Omega_3 = \omega_i + \omega_j + \omega_k + \omega_{e_1} + \omega_{i_1} - \omega_{j_1} + \omega_{k_1}\,, \quad , \quad \Omega_{63} = \omega_i - \omega_j - \omega_k - \omega_{e_1} - \omega_{i_1} - \omega_{j_1} + \omega_{k_1}\,,$$
$$\Omega_{64} = \omega_i - \omega_j - \omega_k - \omega_{e_1} - \omega_{i_1} - \omega_{j_1} - \omega_{k_1}\,.$$

When transferred to a high-frequency carrier, we also will obtain negative frequencies for combination frequencies (7). However, when receiving a signal, demodulation is carried out at zero carrier frequency and therefore we will operate only with positive combination frequencies.

According to the method [9, 10], to obtain the channel matrix (1), we represent the octonion in algebraic form (2) as an 8×8 matrix:

$$\mathbf{O} = \begin{bmatrix} s & x & y & z & s_1 & x_1 & y_1 & z_1 \\ -x & s & -z & y & -x_1 & s_1 & z_1 & -y_1 \\ -y & z & s & -x & -y_1 & -z_1 & s_1 & x_1 \\ -z & -y & x & s & -z_1 & y_1 & -x_1 & s_1 \\ -s_1 & x_1 & y_1 & z_1 & s & -x & -y & -z \\ -x_1 & -s_1 & z_1 & -y_1 & x & s & z & -y \\ -y_1 & -z_1 & -s_1 & x_1 & y & -z & s & x \\ -z_1 & y_1 & -x_1 & -s_1 & z & y & -x & s \end{bmatrix}. \tag{8}$$

We decompose matrix (8) into basis matrices [12]:

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \ \mathbf{I} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}, \tag{9}$$

$$\mathbf{J}=\begin{bmatrix}0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&0&0&0&0&0&-1&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\end{bmatrix},\ \mathbf{K}=\begin{bmatrix}0&0&0&1&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&1&0&0&0\end{bmatrix}$$

$$\mathbf{E}_1=\begin{bmatrix}0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\\-1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&0&-1&0&0&0&0\end{bmatrix},\ \mathbf{I}_1=\begin{bmatrix}0&0&0&0&0&1&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&-1&0\\0&1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&-1&0&0&0&0&0\end{bmatrix},$$

$$\mathbf{J}_1=\begin{bmatrix}0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&1&0&0&0&0&0\\0&0&0&-1&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\end{bmatrix},\ \mathbf{K}_1=\begin{bmatrix}0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&1&0&0&0&0\\0&0&1&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\end{bmatrix}.$$

The basis matrices of the octonion are orthogonal, since $\mathbf{II}^{\mathrm{T}}=\mathbf{I}^{\mathrm{T}}\mathbf{I}=\mathbf{E}$, $\mathbf{JJ}^{\mathrm{T}}=\mathbf{J}^{\mathrm{T}}\mathbf{J}=\mathbf{E}$, $\mathbf{KK}^{\mathrm{T}}=\mathbf{K}^{\mathrm{T}}\mathbf{K}=\mathbf{E}$, $\mathbf{I}_1\mathbf{I}_1^{\mathrm{T}}=\mathbf{I}_1^{\mathrm{T}}\mathbf{I}_1=\mathbf{E}$, $\mathbf{J}_1\mathbf{J}_1^{\mathrm{T}}=\mathbf{J}_1^{\mathrm{T}}\mathbf{J}_1=\mathbf{E}$, $\mathbf{K}_1\mathbf{K}_1^{\mathrm{T}}=\mathbf{K}_1^{\mathrm{T}}\mathbf{K}_1=\mathbf{E}$. Octonion (8) can be written through the sum of basis matrices (9), as

$$\mathbf{O}=s\mathbf{E}+x\mathbf{I}+y\mathbf{J}+z\mathbf{K}+s_1\mathbf{E}_1+x_1\mathbf{I}_1+y_1\mathbf{J}_1+z_1\mathbf{K}_1\,. \tag{10}$$

Matrix determinant (8) $|\mathbf{O}|=\left(s^2+x^2+y^2+z^2+s_1^2+x_1^2+y_1^2+z_1^2\right)^4$. The octonion matrix (8) will be orthogonal when the octonion is normalized with modulo $|o|=\sqrt{s^2+x^2+y^2+z^2+s_1^2+x_1^2+y_1^2+z_1^2}$.

The basis matrices (9) satisfy the same multiplication rules presented in Table 1. Let us write these rules for basis matrices also in the form of a table.

Table 2: Multiplication Operations of Basis Matrices (9) of the Octonion.

×	**E**	**I**	**J**	**K**	$\mathbf{E}_1$	$\mathbf{I}_1$	$\mathbf{J}_1$	$\mathbf{K}_1$
E	**E**	**I**	**J**	**K**	$\mathbf{E}_1$	$\mathbf{I}_1$	$\mathbf{J}_1$	$\mathbf{K}_1$
I	**I**	–**E**	**K**	–**J**	$\mathbf{I}_1$	$-\mathbf{E}_1$	$-\mathbf{K}_1$	$\mathbf{J}_1$
J	**J**	–**K**	–**E**	**I**	$\mathbf{J}_1$	$\mathbf{K}_1$	$-\mathbf{E}_1$	$-\mathbf{I}_1$
K	**K**	**J**	–**I**	–**E**	$\mathbf{K}_1$	$-\mathbf{J}_1$	$\mathbf{I}_1$	$-\mathbf{E}_1$.
$\mathbf{E}_1$	$\mathbf{E}_1$	$-\mathbf{I}_1$	$-\mathbf{J}_1$	$-\mathbf{K}_1$	–**E**	**I**	**J**	**K**
$\mathbf{I}_1$	$\mathbf{I}_1$	$\mathbf{E}_1$	$-\mathbf{K}_1$	$\mathbf{J}_1$	–**I**	–**E**	–**K**	**J**
$\mathbf{J}_1$	$\mathbf{J}_1$	$\mathbf{K}_1$	$\mathbf{E}_1$	$-\mathbf{I}_1$	–**J**	**K**	–**E**	–**I**
$\mathbf{K}_1$	$\mathbf{K}_1$	$-\mathbf{J}_1$	$\mathbf{I}_1$	$\mathbf{E}_1$	–**K**	–**J**	**I**	–**E**

According to the methodology [8-10], let us represent the information transmission system as a model in state space using the dynamics equation [13]:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)\,, \qquad (11)$$

where **A** – is the state transition matrix, $\mathbf{x}(t)$ – is the state vector, $\mathbf{x}(t)$ – is the time derivative of the state vector.

The state transition matrix is determined by the imaginary part of the octonion matrix (8) in which the elements at the imaginary parts $x, y, z, s_1, x_1, y_1, z_1$ are replaced by angular frequencies $\omega_i, \omega_j, \omega_k, \omega_{e_1}, \omega_{i_1}, \omega_{j_1}, \omega_{k_1}$ on the orthogonal coordinate axes **I, J, K,** $\mathbf{E}_1$, $\mathbf{I}_1$, $\mathbf{J}_1$, $\mathbf{K}_1$ in 8D space:

$$\mathbf{A} = \begin{bmatrix} 0 & \omega_i & \omega_j & \omega_k & \omega_{e_1} & \omega_{i_1} & \omega_{j_1} & \omega_{k_1} \\ -\omega_i & 0 & -\omega_k & \omega_j & -\omega_{i_1} & \omega_{e_1} & \omega_{k_1} & -\omega_{j_1} \\ -\omega_j & \omega_k & 0 & -\omega_i & -\omega_{j_1} & -\omega_{k_1} & \omega_{e_1} & \omega_{i_1} \\ -\omega_k & -\omega_j & \omega_i & 0 & -\omega_{k_1} & \omega_{j_1} & -\omega_{i_1} & \omega_{e_1} \\ -\omega_{e_1} & \omega_{i_1} & \omega_{j_1} & \omega_{k_1} & 0 & -\omega_i & -\omega_j & -\omega_k \\ -\omega_{i_1} & -\omega_{e_1} & \omega_{k_1} & -\omega_{j_1} & \omega_i & 0 & \omega_k & -\omega_j \\ -\omega_{j_1} & -\omega_{k_1} & -\omega_{e_1} & \omega_{i_1} & \omega_j & -\omega_k & 0 & \omega_i \\ -\omega_{k_1} & \omega_{j_1} & -\omega_{i_1} & -\omega_{e_1} & \omega_k & \omega_j & -\omega_i & 0 \end{bmatrix}. \qquad (12)$$

Let's call (12) *a matrix of octonion reference frequencies*. Let's write matrix (12) using basis matrices (9):

$$\mathbf{A} = \omega_i\mathbf{I} + \omega_j\mathbf{J} + \omega_k\mathbf{K} + \omega_{e_1}\mathbf{E}_1 + \omega_{i_1}\mathbf{I}_1 + \omega_{j_1}\mathbf{J}_1 + \omega_{k_1}\mathbf{K}_1\,.$$

Matrix determinant (12) $|\mathbf{A}| = \left(\omega_i^2 + \omega_j^2 + \omega_k^2 + \omega_{e_1}^2 + \omega_{i_1}^2 + \omega_{j_1}^2 + \omega_{k_1}^2\right)^4$. The state transition matrix (12) is orthogonal when normalized to the eighth root of the determinant. As can be seen from equation (11), the matrix of octonion reference frequencies is a differential operator in time. The solution to the linear homogeneous matrix differential equation (11) will be the exponential of matrix (12) when the angular frequencies change over time:

$$\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t) = e^{\mathbf{A}t} = \exp\left\{(\omega_i\mathbf{I} + \omega_j\mathbf{J} + \omega_k\mathbf{K} + \omega_{e_1}\mathbf{E}_1 + \omega_{i_1}\mathbf{I}_1 + \omega_{j_1}\mathbf{J}_1 + \omega_{k_1}\mathbf{K}_1)t\right\}.$$

In this regard, the matrix $\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)$ is called the *fundamental matrix*.

To obtain the expression for the fundamental matrix, we use formulas (5) - (6) for the exponential of the octonion. To reduce the size of the fundamental matrix record, we denote the 7 reference frequencies as $\omega_7 \leftrightarrow (\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1})$. In this representation, functions (6) will be determined by arguments $(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)$ written in the form (ω_7,t), and the fundamental matrix will take the form:

$$\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)=\mathbf{\Phi}(\omega_7,t)=$$

$$=\begin{bmatrix} p(\omega_7,t) & u(\omega_7,t) & v(\omega_7,t) & w(\omega_7,t) & p_1(\omega_7,t) & u_1(\omega_7,t) & v_1(\omega_7,t) & w_1(\omega_7,t) \\ -u(\omega_7,t) & p(\omega_7,t) & -w(\omega_7,t) & v(\omega_7,t) & -u_1(\omega_7,t) & p_1(\omega_7,t) & w_1(\omega_7,t) & -v_1(\omega_7,t) \\ -v(\omega_7,t) & w(\omega_7,t) & p(\omega_7,t) & -u(\omega_7,t) & -v_1(\omega_7,t) & -w_1(\omega_7,t) & p_1(\omega_7,t) & u_1(\omega_7,t) \\ -w(\omega_7,t) & -v(\omega_7,t) & u(\omega_7,t) & p(\omega_7,t) & -w_1(\omega_7,t) & v_1(\omega_7,t) & -u_1(\omega_7,t) & p_1(\omega_7,t) \\ -p_1(\omega_7,t) & u_1(\omega_7,t) & v_1(\omega_7,t) & w_1(\omega_7,t) & p(\omega_7,t) & -u(\omega_7,t) & -v(\omega_7,t) & -w(\omega_7,t) \\ -u_1(\omega_7,t) & -p_1(\omega_7,t) & w_1(\omega_7,t) & -v_1(\omega_7,t) & u(\omega_7,t) & p(\omega_7,t) & w(\omega_7,t) & -v(\omega_7,t) \\ -v_1(\omega_7,t) & -w_1(\omega_7,t) & -p_1(\omega_7,t) & u_1(\omega_7,t) & v(\omega_7,t) & -w(\omega_7,t) & p(\omega_7,t) & u(\omega_7,t) \\ -w_1(\omega_7,t) & v_1(\omega_7,t) & -u_1(\omega_7,t) & -p_1(\omega_7,t) & w(\omega_7,t) & v(\omega_7,t) & -u(\omega_7,t) & p(\omega_7,t) \end{bmatrix}. \tag{13}$$

Let us write the fundamental matrix (13) in terms of the basis matrices as

$$\mathbf{\Phi}(\omega_7,t)=p(\omega_7,t)\mathbf{E}+u(\omega_7,t)\mathbf{I}+v(\omega_7,t)\mathbf{J}+w(\omega_7,t)\mathbf{K}+$$
$$+p_1(\omega_7,t)\mathbf{E}_1+u_1(\omega_7,t)\mathbf{I}_1+v_1(\omega_7,t)\mathbf{J}_1+w_1(\omega_7,t)\mathbf{K}_1\,. \tag{14}$$

The fundamental matrix (13) is orthogonal, since

$$\mathbf{\Phi}(\omega_7,t)\mathbf{\Phi}^{\mathrm{T}}(\omega_7,t)=\mathbf{\Phi}^{\mathrm{T}}(\omega_7,t)\mathbf{\Phi}(\omega_7,t)=\mathbf{E}\,.$$

Thus, we have obtained the fundamental matrix (13), the arguments of which are 7 reference frequencies ω_i, ω_j, ω_k, ω_{e_1}, ω_{i_1}, ω_{j_1}, ω_{k_1}.

Since the elements of the fundamental matrix (13) are decomposed into a sum of elements with combination frequencies (7), we will represent the fundamental matrix of reference frequencies as a sum of combination frequencies. We use trigonometry formulas and present the products of sines and cosines (6) as the sum of sines and cosines of combination frequencies (7). Each combination of products of 8 elements of sines and cosines represents the sum of 64 frequencies with a common factor of 1/64. According to expressions (6), we summarize the obtained results.

When summing up the results of representing 16 products of sines and cosines for each of the eight functions (6), some elements are reduced. As a result, the functions for combination frequencies are divided into two groups of frequencies. When adding the elements of each group, we obtain a representation of the matrix elements with a factor of 1/16. The common factor for all matrices of the two groups will be 1/8.

The first group of combination frequencies includes frequencies with frequency numbers: 1 2 5 6 11 12 15 16 19 20 23 24 25 26 29 30 33 34 37 38 43 44 47 48 51 52 55 56 57 58 61 62. The second group includes frequencies with frequency numbers: 3 4 7 8 9 10 13 14 17 18 21 22 27 28 31 32 35 36 39 40 41 42 45 46 49 50 53 54 59 60 63 64.

The first group includes frequencies that form elements of single-frequency matrices corresponding to the basis matrices **E, I**, $\mathbf{J}_1$, $\mathbf{K}_1$ and, accordingly, have a structure:

$$\mathbf{E}+\mathbf{I}+\mathbf{J}_1+\mathbf{K}_1=\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}. \quad (15)$$

The second group includes frequencies that form elements of single-frequency matrices corresponding to the basis matrices $\mathbf{J}$, $\mathbf{K}$, $\mathbf{E}_1$, $\mathbf{I}_1$ and, accordingly, have the structure:

$$\mathbf{J}+\mathbf{K}+\mathbf{E}_1+\mathbf{I}_1=\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 \end{bmatrix}. \quad (16)$$

As can be seen from the structures of matrices (15) and (16), their elements do not intersect when superimposed and form a total matrix (10) as the sum of all basis matrices (9). This property of single-frequency matrices also contributes to the separation of information elements in space when receiving an information vector.

Let us represent the fundamental single-frequency matrices of the 1st frequency group through the basis matrices $\mathbf{E}$, $\mathbf{I}$, $\mathbf{J}_1$, $\mathbf{K}_1$:

$$\mathbf{\Phi}(\Omega_1,t)=\frac{1}{8}\Big[\big(\cos(\Omega_1 t)+\sin(\Omega_1 t)\big)\mathbf{E}+\big(\cos(\Omega_1 t)-\sin(\Omega_1 t)\big)\mathbf{I}+ \quad (17)$$
$$+\big(\cos(\Omega_1 t)-\sin(\Omega_1 t)\big)\mathbf{J}_1+\big(-\cos(\Omega_1 t)-\sin(\Omega_1 t)\big)\mathbf{K}_1\Big];$$
$$\mathbf{\Phi}(\Omega_2,t)=\frac{1}{8}\Big[\big(-\cos(\Omega_2 t)+\sin(\Omega_2 t)\big)\mathbf{E}+\big(\cos(\Omega_2 t)+\sin(\Omega_2 t)\big)\mathbf{I}+$$
$$+\big(\cos(\Omega_2 t)+\sin(\Omega_2 t)\big)\mathbf{J}_1+\big(\cos(\Omega_2 t)-\sin(\Omega_2 t)\big)\mathbf{K}_1\Big];$$
$$\mathbf{\Phi}(\Omega_5,t)=\frac{1}{8}\Big[\big(\cos(\Omega_5 t)-\sin(\Omega_5 t)\big)\mathbf{E}+\big(-\cos(\Omega_5 t)-\sin(\Omega_5 t)\big)\mathbf{I}+$$
$$+\big(-\cos(\Omega_5 t)-\sin(\Omega_5 t)\big)\mathbf{J}_1+\big(-\cos(\Omega_5 t)+\sin(\Omega_5 t)\big)\mathbf{K}_1\Big];$$
$$\mathbf{\Phi}(\Omega_6,t)=\frac{1}{8}\Big[\big(\cos(\Omega_6 t)+\sin(\Omega_6 t)\big)\mathbf{E}+\big(\cos(\Omega_6 t)-\sin(\Omega_6 t)\big)\mathbf{I}+$$
$$+\big(\cos(\Omega_6 t)-\sin(\Omega_6 t)\big)\mathbf{J}_1+\big(-\cos(\Omega_6 t)-\sin(\Omega_6 t)\big)\mathbf{K}_1\Big];$$
$$\mathbf{\Phi}(\Omega_{11},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{11} t)+\sin(\Omega_{11} t)\big)\mathbf{E}+\big(\cos(\Omega_{11} t)-\sin(\Omega_{11} t)\big)\mathbf{I}+$$
$$+\big(-\cos(\Omega_{11} t)+\sin(\Omega_{11} t)\big)\mathbf{J}_1+\big(\cos(\Omega_{11} t)+\sin(\Omega_{11} t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{12},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{12}t)-\sin(\Omega_{12}t)\big)\mathbf{E}+\big(-\cos(\Omega_{12}t)-\sin(\Omega_{12}t)\big)\mathbf{I}+$$

$$+\big(\cos(\Omega_{12}t)+\sin(\Omega_{12}t)\big)\mathbf{J}_1+\big(\cos(\Omega_{12}t)-\sin(\Omega_{12}t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{15},t)=\frac{1}{8}\Big[\big(-\cos(\Omega_{15}t)+\sin(\Omega_{15}t)\big)\mathbf{E}+\big(\cos(\Omega_{15}t)+\sin(\Omega_{15}t)\big)\mathbf{I}+$$

$$+\big(-\cos(\Omega_{15}t)-\sin(\Omega_{15}t)\big)\mathbf{J}_1+\big(-\cos(\Omega_{15}t)+\sin(\Omega_{15}t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{16},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{16}t)+\sin(\Omega_{16}t)\big)\mathbf{E}+\big(\cos(\Omega_{16}t)-\sin(\Omega_{16}t)\big)\mathbf{I}+$$

$$+\big(-\cos(\Omega_{16}t)+\sin(\Omega_{16}t)\big)\mathbf{J}_1+\big(\cos(\Omega_{16}t)+\sin(\Omega_{16}t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{19},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{19}t)+\sin(\Omega_{19}t)\big)\mathbf{E}+\big(\cos(\Omega_{19}t)-\sin(\Omega_{19}t)\big)\mathbf{I}+$$

$$+\big(-\cos(\Omega_{19}t)+\sin(\Omega_{19}t)\big)\mathbf{J}_1+\big(\cos(\Omega_{19}t)+\sin(\Omega_{19}t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{20},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{20}t)-\sin(\Omega_{20}t)\big)\mathbf{E}+\big(-\cos(\Omega_{20}t)-\sin(\Omega_{20}t)\big)\mathbf{I}+$$

$$+\big(\cos(\Omega_{20}t)+\sin(\Omega_{20}t)\big)\mathbf{J}_1+\big(\cos(\Omega_{20}t)-\sin(\Omega_{20}t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{23},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{23}t)-\sin(\Omega_{23}t)\big)\mathbf{E}+\big(-\cos(\Omega_{23}t)-\sin(\Omega_{23}t)\big)\mathbf{I}+$$

$$+\big(\cos(\Omega_{23}t)+\sin(\Omega_{23}t)\big)\mathbf{J}_1+\big(\cos(\Omega_{23}t)-\sin(\Omega_{23}t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{24},t)=\frac{1}{8}\Big[\big(-\cos(\Omega_{24}t)-\sin(\Omega_{24}t)\big)\mathbf{E}+\big(-\cos(\Omega_{24}t)+\sin(\Omega_{24}t)\big)\mathbf{I}+$$

$$+\big(\cos(\Omega_{24}t)-\sin(\Omega_{24}t)\big)\mathbf{J}_1+\big(-\cos(\Omega_{24}t)-\sin(\Omega_{24}t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{25},t)=\frac{1}{8}\Big[\big(-\cos(\Omega_{25}t)-\sin(\Omega_{25}t)\big)\mathbf{E}+\big(-\cos(\Omega_{25}t)+\sin(\Omega_{25}t)\big)\mathbf{I}+$$

$$+\big(-\cos(\Omega_{25}t)+\sin(\Omega_{25}t)\big)\mathbf{J}_1+\big(\cos(\Omega_{25}t)+\sin(\Omega_{25}t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{26},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{26}t)-\sin(\Omega_{26}t)\big)\mathbf{E}+\big(-\cos(\Omega_{26}t)-\sin(\Omega_{26}t)\big)\mathbf{I}+$$

$$+\big(-\cos(\Omega_{26}t)-\sin(\Omega_{26}t)\big)\mathbf{J}_1+\big(-\cos(\Omega_{26}t)+\sin(\Omega_{26}t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{29},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{29}t)-\sin(\Omega_{29}t)\big)\mathbf{E}+\big(-\cos(\Omega_{29}t)-\sin(\Omega_{29}t)\big)\mathbf{I}+$$

$$+\big(-\cos(\Omega_{29}t)-\sin(\Omega_{29}t)\big)\mathbf{J}_1+\big(-\cos(\Omega_{29}t)+\sin(\Omega_{29}t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{30},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{30}t)+\sin(\Omega_{30}t)\big)\mathbf{E}+\big(\cos(\Omega_{30}t)-\sin(\Omega_{30}t)\big)\mathbf{I}+$$

$$+\big(\cos(\Omega_{30}t)-\sin(\Omega_{30}t)\big)\mathbf{J}_1+\big(-\cos(\Omega_{30}t)-\sin(\Omega_{30}t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{33},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{33}t)-\sin(\Omega_{33}t)\big)\mathbf{E}+\big(-\cos(\Omega_{33}t)-\sin(\Omega_{33}t)\big)\mathbf{I}+$$

$$+\big(-\cos(\Omega_{33}t)-\sin(\Omega_{33}t)\big)\mathbf{J}_1+\big(-\cos(\Omega_{33}t)+\sin(\Omega_{33}t)\big)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{34},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{34}t)+\sin(\Omega_{34}t)\big)\mathbf{E}+\big(\cos(\Omega_{34}t)-\sin(\Omega_{34}t)\big)\mathbf{I}+$$

$$+\left(\cos(\Omega_{34}t)-\sin(\Omega_{34}t)\right)\mathbf{J}_1+\left(-\cos(\Omega_{34}t)-\sin(\Omega_{34}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{37},t)=\frac{1}{8}\Big[\left(-\cos(\Omega_{37}t)-\sin(\Omega_{37}t)\right)\mathbf{E}+\left(-\cos(\Omega_{37}t)+\sin(\Omega_{37}t)\right)\mathbf{I}+$$
$$+\left(-\cos(\Omega_{37}t)+\sin(\Omega_{37}t)\right)\mathbf{J}_1+\left(\cos(\Omega_{37}t)+\sin(\Omega_{37}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{38},t)=\frac{1}{8}\Big[\left(\cos(\Omega_{38}t)-\sin(\Omega_{38}t)\right)\mathbf{E}+\left(-\cos(\Omega_{38}t)-\sin(\Omega_{38}t)\right)\mathbf{I}+$$
$$+\left(-\cos(\Omega_{38}t)-\sin(\Omega_{38}t)\right)\mathbf{J}_1+\left(-\cos(\Omega_{38}t)+\sin(\Omega_{38}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{43},t)=\frac{1}{8}\Big[\left(\cos(\Omega_{43}t)-\sin(\Omega_{43}t)\right)\mathbf{E}+\left(-\cos(\Omega_{43}t)-\sin(\Omega_{43}t)\right)\mathbf{I}+$$
$$+\left(\cos(\Omega_{43}t)+\sin(\Omega_{43}t)\right)\mathbf{J}_1+\left(\cos(\Omega_{43}t)-\sin(\Omega_{43}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{44},t)=\frac{1}{8}\Big[\left(-\cos(\Omega_{44}t)-\sin(\Omega_{44}t)\right)\mathbf{E}+\left(-\cos(\Omega_{44}t)+\sin(\Omega_{44}t)\right)\mathbf{I}+$$
$$+\left(\cos(\Omega_{44}t)-\sin(\Omega_{44}t)\right)\mathbf{J}_1+\left(-\cos(\Omega_{44}t)-\sin(\Omega_{44}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{47},t)=\frac{1}{8}\Big[\left(\cos(\Omega_{47}t)+\sin(\Omega_{47}t)\right)\mathbf{E}+\left(\cos(\Omega_{47}t)-\sin(\Omega_{47}t)\right)\mathbf{I}+$$
$$+\left(-\cos(\Omega_{47}t)+\sin(\Omega_{47}t)\right)\mathbf{J}_1+\left(\cos(\Omega_{47}t)+\sin(\Omega_{47}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{48},t)=\frac{1}{8}\Big[\left(\cos(\Omega_{48}t)-\sin(\Omega_{48}t)\right)\mathbf{E}+\left(-\cos(\Omega_{48}t)-\sin(\Omega_{48}t)\right)\mathbf{I}+$$
$$+\left(\cos(\Omega_{48}t)+\sin(\Omega_{48}t)\right)\mathbf{J}_1+\left(\cos(\Omega_{48}t)-\sin(\Omega_{48}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{51},t)=\frac{1}{8}\Big[\left(-\cos(\Omega_{51}t)+\sin(\Omega_{51}t)\right)\mathbf{E}+\left(\cos(\Omega_{51}t)+\sin(\Omega_{51}t)\right)\mathbf{I}+$$
$$+\left(-\cos(\Omega_{51}t)-\sin(\Omega_{51}t)\right)\mathbf{J}_1+\left(-\cos(\Omega_{51}t)+\sin(\Omega_{51}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{52},t)=\frac{1}{8}\Big[\left(\cos(\Omega_{52}t)+\sin(\Omega_{52}t)\right)\mathbf{E}+\left(\cos(\Omega_{52}t)-\sin(\Omega_{52}t)\right)\mathbf{I}+$$
$$+\left(-\cos(\Omega_{52}t)+\sin(\Omega_{52}t)\right)\mathbf{J}_1+\left(\cos(\Omega_{52}t)+\sin(\Omega_{52}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{55},t)=\frac{1}{8}\Big[\left(\cos(\Omega_{55}t)+\sin(\Omega_{55}t)\right)\mathbf{E}+\left(\cos(\Omega_{55}t)-\sin(\Omega_{55}t)\right)\mathbf{I}+$$
$$+\left(-\cos(\Omega_{55}t)+\sin(\Omega_{55}t)\right)\mathbf{J}_1+\left(\cos(\Omega_{55}t)+\sin(\Omega_{55}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{56},t)=\frac{1}{8}\Big[\left(\cos(\Omega_{56}t)-\sin(\Omega_{56}t)\right)\mathbf{E}+\left(-\cos(\Omega_{56}t)-\sin(\Omega_{56}t)\right)\mathbf{I}+$$
$$+\left(\cos(\Omega_{56}t)+\sin(\Omega_{56}t)\right)\mathbf{J}_1+\left(\cos(\Omega_{56}t)-\sin(\Omega_{56}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{57},t)=\frac{1}{8}\Big[\left(\cos(\Omega_{57}t)-\sin(\Omega_{57}t)\right)\mathbf{E}+\left(-\cos(\Omega_{57}t)-\sin(\Omega_{57}t)\right)\mathbf{I}+$$
$$+\left(-\cos(\Omega_{57}t)-\sin(\Omega_{57}t)\right)\mathbf{J}_1+\left(-\cos(\Omega_{57}t)+\sin(\Omega_{57}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{58},t)=\frac{1}{8}\Big[\left(\cos(\Omega_{58}t)+\sin(\Omega_{58}t)\right)\mathbf{E}+\left(\cos(\Omega_{58}t)-\sin(\Omega_{58}t)\right)\mathbf{I}+$$
$$+\left(\cos(\Omega_{58}t)-\sin(\Omega_{58}t)\right)\mathbf{J}_1+\left(-\cos(\Omega_{58}t)-\sin(\Omega_{58}t)\right)\mathbf{K}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{61},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{61}t)+\sin(\Omega_{61}t)\big)\mathbf{E}+\big(\cos(\Omega_{61}t)-\sin(\Omega_{61}t)\big)\mathbf{I}+$$
$$+\big(\cos(\Omega_{61}t)-\sin(\Omega_{61}t)\big)\mathbf{J}_1+\big(-\cos(\Omega_{61}t)-\sin(\Omega_{61}t)\big)\mathbf{K}_1\Big];$$
$$\mathbf{\Phi}(\Omega_{62},t)=\frac{1}{8}\Big[\big(-\cos(\Omega_{62}t)+\sin(\Omega_{62}t)\big)\mathbf{E}+\big(\cos(\Omega_{62}t)+\sin(\Omega_{62}t)\big)\mathbf{I}+$$
$$+\big(\cos(\Omega_{62}t)+\sin(\Omega_{62}t)\big)\mathbf{J}_1+\big(\cos(\Omega_{62}t)-\sin(\Omega_{62}t)\big)\mathbf{K}_1\Big].$$

Let us represent through the basis matrices $\mathbf{J}$, $\mathbf{K}$, $\mathbf{E}_1$, $\mathbf{I}_1$ the fundamental single-frequency matrices of the 2nd frequency group:

$$\mathbf{\Phi}(\Omega_3,t)=\frac{1}{8}\Big[\big(\cos(\Omega_3t)-\sin(\Omega_3t)\big)\mathbf{J}+\big(-\cos(\Omega_3t)-\sin(\Omega_3t)\big)\mathbf{K}+ \tag{18}$$
$$+\big(-\cos(\Omega_3t)-\sin(\Omega_3t)\big)\mathbf{E}_1+\big(-\cos(\Omega_3t)+\sin(\Omega_3t)\big)\mathbf{I}_1\Big];$$
$$\mathbf{\Phi}(\Omega_4,t)=\frac{1}{8}\Big[\big(\cos(\Omega_4t)+\sin(\Omega_4t)\big)\mathbf{J}+\big(\cos(\Omega_4t)-\sin(\Omega_4t)\big)\mathbf{K}+$$
$$+\big(\cos(\Omega_4t)-\sin(\Omega_4t)\big)\mathbf{E}_1+\big(-\cos(\Omega_4t)-\sin(\Omega_4t)\big)\mathbf{I}_1\Big];$$
$$\mathbf{\Phi}(\Omega_7,t)=\frac{1}{8}\Big[\big(\cos(\Omega_7t)+\sin(\Omega_7t)\big)\mathbf{J}+\big(\cos(\Omega_7t)-\sin(\Omega_7t)\big)\mathbf{K}+$$
$$+\big(\cos(\Omega_7t)-\sin(\Omega_7t)\big)\mathbf{E}_1+\big(-\cos(\Omega_7t)-\sin(\Omega_7t)\big)\mathbf{I}_1\Big];$$
$$\mathbf{\Phi}(\Omega_8,t)=\frac{1}{8}\Big[\big(-\cos(\Omega_8t)+\sin(\Omega_8t)\big)\mathbf{J}+\big(\cos(\Omega_8t)+\sin(\Omega_8t)\big)\mathbf{K}+$$
$$+\big(\cos(\Omega_8t)+\sin(\Omega_8t)\big)\mathbf{E}_1+\big(\cos(\Omega_8t)-\sin(\Omega_8t)\big)\mathbf{I}_1\Big];$$
$$\mathbf{\Phi}(\Omega_9,t)=\frac{1}{8}\Big[\big(-\cos(\Omega_9t)+\sin(\Omega_9t)\big)\mathbf{J}+\big(\cos(\Omega_9t)+\sin(\Omega_9t)\big)\mathbf{K}+$$
$$+\big(-\cos(\Omega_9t)-\sin(\Omega_9t)\big)\mathbf{E}_1+\big(-\cos(\Omega_9t)+\sin(\Omega_9t)\big)\mathbf{I}_1\Big];$$
$$\mathbf{\Phi}(\Omega_{10},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{10}t)+\sin(\Omega_{10}t)\big)\mathbf{J}+\big(\cos(\Omega_{10}t)-\sin(\Omega_{10}t)\big)\mathbf{K}+$$
$$+\big(-\cos(\Omega_{10}t)+\sin(\Omega_{10}t)\big)\mathbf{E}_1+\big(\cos(\Omega_{10}t)+\sin(\Omega_{10}t)\big)\mathbf{I}_1\Big];$$
$$\mathbf{\Phi}(\Omega_{13},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{13}t)+\sin(\Omega_{13}t)\big)\mathbf{J}+\big(\cos(\Omega_{13}t)-\sin(\Omega_{13}t)\big)\mathbf{K}+$$
$$+\big(-\cos(\Omega_{13}t)+\sin(\Omega_{13}t)\big)\mathbf{E}_1+\big(\cos(\Omega_{13}t)+\sin(\Omega_{13}t)\big)\mathbf{I}_1\Big];$$
$$\mathbf{\Phi}(\Omega_{14},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{14}t)-\sin(\Omega_{14}t)\big)\mathbf{J}+\big(-\cos(\Omega_{14}t)-\sin(\Omega_{14}t)\big)\mathbf{K}+$$
$$+\big(\cos(\Omega_{14}t)+\sin(\Omega_{14}t)\big)\mathbf{E}_1+\big(\cos(\Omega_{14}t)-\sin(\Omega_{14}t)\big)\mathbf{I}_1\Big];$$
$$\mathbf{\Phi}(\Omega_{17},t)=\frac{1}{8}\Big[\big(-\cos(\Omega_{17}t)+\sin(\Omega_{17}t)\big)\mathbf{J}+\big(\cos(\Omega_{17}t)+\sin(\Omega_{17}t)\big)\mathbf{K}+$$
$$+\big(-\cos(\Omega_{17}t)-\sin(\Omega_{17}t)\big)\mathbf{E}_1+\big(-\cos(\Omega_{17}t)+\sin(\Omega_{17}t)\big)\mathbf{I}_1\Big];$$
$$\mathbf{\Phi}(\Omega_{18},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{18}t)+\sin(\Omega_{18}t)\big)\mathbf{J}+\big(\cos(\Omega_{18}t)-\sin(\Omega_{18}t)\big)\mathbf{K}+$$
$$+\big(-\cos(\Omega_{18}t)+\sin(\Omega_{18}t)\big)\mathbf{E}_1+\big(\cos(\Omega_{18}t)+\sin(\Omega_{18}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{21},t)=\frac{1}{8}\Big[\big(-\cos(\Omega_{21}t)-\sin(\Omega_{21}t)\big)\mathbf{J}+\big(-\cos(\Omega_{21}t)+\sin(\Omega_{21}t)\big)\mathbf{K}+$$
$$+\big(\cos(\Omega_{21}t)-\sin(\Omega_{21}t)\big)\mathbf{E}_1+\big(-\cos(\Omega_{21}t)-\sin(\Omega_{21}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{22},t)=\frac{1}{8}\Big[\big(-\cos(\Omega_{22}t)+\sin(\Omega_{22}t)\big)\mathbf{J}+\big(\cos(\Omega_{22}t)+\sin(\Omega_{22}t)\big)\mathbf{K}+$$
$$+\big(-\cos(\Omega_{22}t)-\sin(\Omega_{22}t)\big)\mathbf{E}_1+\big(-\cos(\Omega_{22}t)+\sin(\Omega_{22}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{27},t)=\frac{1}{8}\Big[\big(-\cos(\Omega_{27}t)+\sin(\Omega_{27}t)\big)\mathbf{J}+\big(\cos(\Omega_{27}t)+\sin(\Omega_{27}t)\big)\mathbf{K}+$$
$$+\big(\cos(\Omega_{27}t)+\sin(\Omega_{27}t)\big)\mathbf{E}_1+\big(\cos(\Omega_{27}t)-\sin(\Omega_{27}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{28},t)=\frac{1}{8}\Big[\big(-\cos(\Omega_{28}t)-\sin(\Omega_{28}t)\big)\mathbf{J}+\big(-\cos(\Omega_{28}t)+\sin(\Omega_{28}t)\big)\mathbf{K}+$$
$$+\big(-\cos(\Omega_{28}t)+\sin(\Omega_{28}t)\big)\mathbf{E}_1+\big(\cos(\Omega_{28}t)+\sin(\Omega_{28}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{31},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{31}t)+\sin(\Omega_{31}t)\big)\mathbf{J}+\big(\cos(\Omega_{31}t)-\sin(\Omega_{31}t)\big)\mathbf{K}+$$
$$+\big(\cos(\Omega_{31}t)-\sin(\Omega_{31}t)\big)\mathbf{E}_1+\big(-\cos(\Omega_{31}t)-\sin(\Omega_{31}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{32},t)=\frac{1}{8}\Big[\big(-\cos(\Omega_{32}t)+\sin(\Omega_{32}t)\big)\mathbf{J}+\big(\cos(\Omega_{32}t)+\sin(\Omega_{32}t)\big)\mathbf{K}+$$
$$+\big(\cos(\Omega_{32}t)+\sin(\Omega_{32}t)\big)\mathbf{E}_1+\big(\cos(\Omega_{32}t)-\sin(\Omega_{32}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{35},t)=\frac{1}{8}\Big[\big(-\cos(\Omega_{35}t)-\sin(\Omega_{35}t)\big)\mathbf{J}+\big(-\cos(\Omega_{35}t)+\sin(\Omega_{35}t)\big)\mathbf{K}+$$
$$+\big(-\cos(\Omega_{35}t)+\sin(\Omega_{35}t)\big)\mathbf{E}_1+\big(\cos(\Omega_{35}t)+\sin(\Omega_{35}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{36},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{36}t)-\sin(\Omega_{36}t)\big)\mathbf{J}+\big(-\cos(\Omega_{36}t)-\sin(\Omega_{36}t)\big)\mathbf{K}+$$
$$+\big(-\cos(\Omega_{36}t)-\sin(\Omega_{36}t)\big)\mathbf{E}_1+\big(-\cos(\Omega_{36}t)+\sin(\Omega_{36}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{39},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{39}t)-\sin(\Omega_{39}t)\big)\mathbf{J}+\big(-\cos(\Omega_{39}t)-\sin(\Omega_{39}t)\big)\mathbf{K}+$$
$$+\big(-\cos(\Omega_{39}t)-\sin(\Omega_{39}t)\big)\mathbf{E}_1+\big(-\cos(\Omega_{39}t)+\sin(\Omega_{39}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{40},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{40}t)+\sin(\Omega_{40}t)\big)\mathbf{J}+\big(\cos(\Omega_{40}t)-\sin(\Omega_{40}t)\big)\mathbf{K}+$$
$$+\big(\cos(\Omega_{40}t)-\sin(\Omega_{40}t)\big)\mathbf{E}_1+\big(-\cos(\Omega_{40}t)-\sin(\Omega_{40}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{41},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{41}t)+\sin(\Omega_{41}t)\big)\mathbf{J}+\big(\cos(\Omega_{41}t)-\sin(\Omega_{41}t)\big)\mathbf{K}+$$
$$+\big(-\cos(\Omega_{41}t)+\sin(\Omega_{41}t)\big)\mathbf{E}_1+\big(\cos(\Omega_{41}t)+\sin(\Omega_{41}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{42},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{42}t)-\sin(\Omega_{42}t)\big)\mathbf{J}+\big(-\cos(\Omega_{42}t)-\sin(\Omega_{42}t)\big)\mathbf{K}+$$
$$+\big(\cos(\Omega_{42}t)+\sin(\Omega_{42}t)\big)\mathbf{E}_1+\big(\cos(\Omega_{42}t)-\sin(\Omega_{42}t)\big)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{45},t)=\frac{1}{8}\Big[\big(\cos(\Omega_{45}t)-\sin(\Omega_{45}t)\big)\mathbf{J}+\big(-\cos(\Omega_{45}t)-\sin(\Omega_{45}t)\big)\mathbf{K}+$$

$$+\left(\cos(\Omega_{45}t)+\sin(\Omega_{45}t)\right)\mathbf{E}_1+\left(\cos(\Omega_{45}t)-\sin(\Omega_{45}t)\right)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{46},t)=\frac{1}{8}\Big[\left(-\cos(\Omega_{46}t)-\sin(\Omega_{46}t)\right)\mathbf{J}+\left(-\cos(\Omega_{46}t)+\sin(\Omega_{46}t)\right)\mathbf{K}+$$
$$+\left(\cos(\Omega_{46}t)-\sin(\Omega_{46}t)\right)\mathbf{E}_1+\left(-\cos(\Omega_{46}t)-\sin(\Omega_{46}t)\right)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{49},t)=\frac{1}{8}\Big[\left(-\cos(\Omega_{49}t)-\sin(\Omega_{49}t)\right)\mathbf{J}+\left(-\cos(\Omega_{49}t)+\sin(\Omega_{49}t)\right)\mathbf{K}+$$
$$+\left(\cos(\Omega_{49}t)-\sin(\Omega_{49}t)\right)\mathbf{E}_1+\left(-\cos(\Omega_{49}t)-\sin(\Omega_{49}t)\right)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{50},t)=\frac{1}{8}\Big[\left(-\cos(\Omega_{50}t)+\sin(\Omega_{50}t)\right)\mathbf{J}+\left(\cos(\Omega_{50}t)+\sin(\Omega_{50}t)\right)\mathbf{K}+$$
$$+\left(-\cos(\Omega_{50}t)-\sin(\Omega_{50}t)\right)\mathbf{E}_1+\left(-\cos(\Omega_{50}t)+\sin(\Omega_{50}t)\right)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{53},t)=\frac{1}{8}\Big[\left(\cos(\Omega_{53}t)-\sin(\Omega_{53}t)\right)\mathbf{J}+\left(-\cos(\Omega_{53}t)-\sin(\Omega_{53}t)\right)\mathbf{K}+$$
$$+\left(\cos(\Omega_{53}t)+\sin(\Omega_{53}t)\right)\mathbf{E}_1+\left(\cos(\Omega_{53}t)-\sin(\Omega_{53}t)\right)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{54},t)=\frac{1}{8}\Big[\left(-\cos(\Omega_{54}t)-\sin(\Omega_{54}t)\right)\mathbf{J}+\left(-\cos(\Omega_{54}t)+\sin(\Omega_{54}t)\right)\mathbf{K}+$$
$$+\left(\cos(\Omega_{54}t)-\sin(\Omega_{54}t)\right)\mathbf{E}_1+\left(-\cos(\Omega_{54}t)-\sin(\Omega_{54}t)\right)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{59},t)=\frac{1}{8}\Big[\left(-\cos(\Omega_{59}t)-\sin(\Omega_{59}t)\right)\mathbf{J}+\left(-\cos(\Omega_{59}t)+\sin(\Omega_{59}t)\right)\mathbf{K}+$$
$$+\left(-\cos(\Omega_{59}t)+\sin(\Omega_{59}t)\right)\mathbf{E}_1+\left(\cos(\Omega_{59}t)+\sin(\Omega_{59}t)\right)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{60},t)=\frac{1}{8}\Big[\left(\cos(\Omega_{60}t)-\sin(\Omega_{60}t)\right)\mathbf{J}+\left(-\cos(\Omega_{60}t)-\sin(\Omega_{60}t)\right)\mathbf{K}+$$
$$+\left(-\cos(\Omega_{60}t)-\sin(\Omega_{60}t)\right)\mathbf{E}_1+\left(-\cos(\Omega_{60}t)+\sin(\Omega_{60}t)\right)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{63},t)=\frac{1}{8}\Big[\left(-\cos(\Omega_{63}t)+\sin(\Omega_{63}t)\right)\mathbf{J}+\left(\cos(\Omega_{63}t)+\sin(\Omega_{63}t)\right)\mathbf{K}+$$
$$+\left(\cos(\Omega_{63}t)+\sin(\Omega_{63}t)\right)\mathbf{E}_1+\left(\cos(\Omega_{63}t)-\sin(\Omega_{63}t)\right)\mathbf{I}_1\Big];$$

$$\mathbf{\Phi}(\Omega_{64},t)=\frac{1}{8}\Big[\left(-\cos(\Omega_{64}t)-\sin(\Omega_{64}t)\right)\mathbf{J}+\left(-\cos(\Omega_{64}t)+\sin(\Omega_{64}t)\right)\mathbf{K}+$$
$$+\left(-\cos(\Omega_{64}t)+\sin(\Omega_{64}t)\right)\mathbf{E}_1+\left(\cos(\Omega_{64}t)+\sin(\Omega_{64}t)\right)\mathbf{I}_1\Big].$$

Single-frequency matrices are orthogonal.

Thus, we obtained single-frequency octonion fundamental matrices (17) and (18), which, when summed, are equal to the seven-frequency fundamental matrix (14):

$$\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)=\mathbf{\Phi}(\omega_7,t)=\frac{1}{8}\sum_{m=1}^{64}\mathbf{\Phi}(\Omega_m,t)\,. \quad (19)$$

3. Transmission and Receiption of Information

3.1 Modulation of Seven-Frequency Octonion Carrier

We will modulate the seven-frequency carrier (13) or in representation (14), as in [9, 10], by multiplying the bipolar information vector $\mathbf{x}(0)=\begin{bmatrix}x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7\end{bmatrix}^{\mathrm{T}}$ by the fundamental matrix, which acts as a channel matrix in the MIMO:

$$\mathbf{y}(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)=\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)\mathbf{x}(0)\,. \tag{20}$$

When representing the information transmission system by a dynamic model in state space (11), the vector x(0) will be the initial state of the dynamic model [13]. As a modulating information vector x(0), we consider a bipolar binary vector $\mathbf{x}(0) = [\pm1\ \pm1\ \pm1\ \pm1\ \pm1\ \pm1\ \pm1\ \pm1]^{\mathrm{T}}$ with 256 possible equally probable initial states. Euclidean norm or length of an information vector $\|x(0)\| = \sqrt{8} = 2\sqrt{2} = 2{,}828$. If we consider the information vector as rectangular pulses with amplitude A=1 at duration T=1, then the power of each pulse will be equal to 1, and the sum of powers will be P=8.

As was said, 64 reference frequencies, with a positive first element are divided into two groups of 32 combination frequencies. We write combinations of the 1st group of frequencies in the form of a matrix $\mathbf{X}_1$ of 7-dimensional unit vectors with a positive or negative sign of the corresponding reference frequencies:

$$\begin{array}{c} \begin{array}{cccccccccccccccc} 1 & 2 & 5 & 6 & 11 & 12 & 15 & 16 & 19 & 20 & 23 & 24 & 25 & 26 & 29 & 30\cdots \end{array} \\ \mathbf{X}_1=\left[\begin{array}{cccccccccccccccc} 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&-1&-1&-1&-1&-1&-1&-1&-1\\ 1&1&1&1&-1&-1&-1&-1&1&1&1&1&-1&-1&-1&-1\\ 1&1&-1&-1&1&1&-1&-1&1&1&-1&-1&1&1&-1&-1\\ 1&1&1&1&-1&-1&-1&-1&-1&-1&-1&-1&1&1&1&1\\ 1&-1&1&-1&1&-1&1&-1&1&-1&1&-1&1&-1&1&-1 \end{array}\right.\ddots \\ \begin{array}{cccccccccccccccc} \cdots 33 & 34 & 37 & 38 & 43 & 44 & 47 & 48 & 51 & 52 & 55 & 56 & 57 & 58 & 61 & 62 \end{array} \\ \ddots\left.\begin{array}{cccccccccccccccc} 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\ -1&-1&-1&-1&-1&-1&-1&-1&-1&-1&-1&-1&-1&-1&-1&-1\\ 1&1&1&1&1&1&1&1&-1&-1&-1&-1&-1&-1&-1&-1\\ 1&1&1&1&-1&-1&-1&-1&1&1&1&1&-1&-1&-1&-1\\ 1&1&-1&-1&1&1&-1&-1&1&1&-1&-1&1&1&-1&-1\\ 1&1&1&1&-1&-1&-1&-1&-1&-1&-1&-1&1&1&1&1\\ 1&-1&1&-1&1&-1&1&-1&1&-1&1&-1&1&-1&1&-1 \end{array}\right]. \end{array}$$

The top line above the matrix shows the combination frequency numbers for the 64 combinations. Matrix $\mathbf{X}_1^{\mathrm{T}}$ is a pseudoinverse matrix, since $\frac{1}{32}\mathbf{X}_1\mathbf{X}_1^{\mathrm{T}}=\mathbf{E}$.

We write combinations of the 2nd group of frequencies in the form of a matrix $\mathbf{X}_2$ of 7-dimensional unit vectors with a positive or negative sign of the corresponding reference frequencies:

$$
\mathbf{X}_2 = \begin{array}{c}
\begin{array}{cccccccccccccccc}
3 & 4 & 7 & 8 & 9 & 10 & 13 & 14 & 17 & 18 & 21 & 22 & 27 & 28 & 31 & 32
\end{array} \\
\left[\begin{array}{cccccccccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
-1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1
\end{array}\right.\cdots
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cccccccccccccccc}
35 & 36 & 39 & 40 & 41 & 42 & 45 & 46 & 49 & 50 & 53 & 54 & 59 & 60 & 63 & 64
\end{array} \\
\cdots\left.\begin{array}{cccccccccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
-1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1
\end{array}\right].
\end{array}
$$

Matrix $\mathbf{X}_2$ differs from matrix $\mathbf{X}_1$ by the opposite sign of the 6th row. Since the matrix $\mathbf{X}_1^{\mathrm{T}}$ is pseudo-inverse, according to (7) it converts the combination frequencies $\Omega_1, \Omega_2, \ldots, \Omega_{64}$ into reference frequencies $\omega_i, \omega_j, \omega_k, \omega_{e_1}, \omega_{i_1}, \omega_{j_1}, \omega_{k_1}$. By multiplying the values of combination frequencies of the first or second group, in accordance with their numbers, by this matrix, we obtain reference frequencies for all combination frequencies.

In accordance with (19), the modulated output vector (20) when equalities (7) are satisfied will be equal to the sum of the modulated output vectors of single-frequency octonion carriers:

$$
\mathbf{y}(\omega_i, \omega_j, \omega_k, \omega_{e_1}, \omega_{i_1}, \omega_{j_1}, \omega_{k_1}, t) = \frac{1}{8}\sum_{m=1}^{64} \mathbf{\Phi}(\Omega_m, t)\mathbf{x}(0) = \frac{1}{8}\sum_{m=1}^{64} \mathbf{y}(\Omega_m, t)\,. \tag{21}
$$

As an example, Figure 1 shows modulated signals at the output of single-frequency channel matrices with frequencies of group 1: Ω_1, Ω_2, and group 2: Ω_3, Ω_4, when multiplied by the information vector x(0) = [−1 1 −1 −1 1 −1 −1 1].

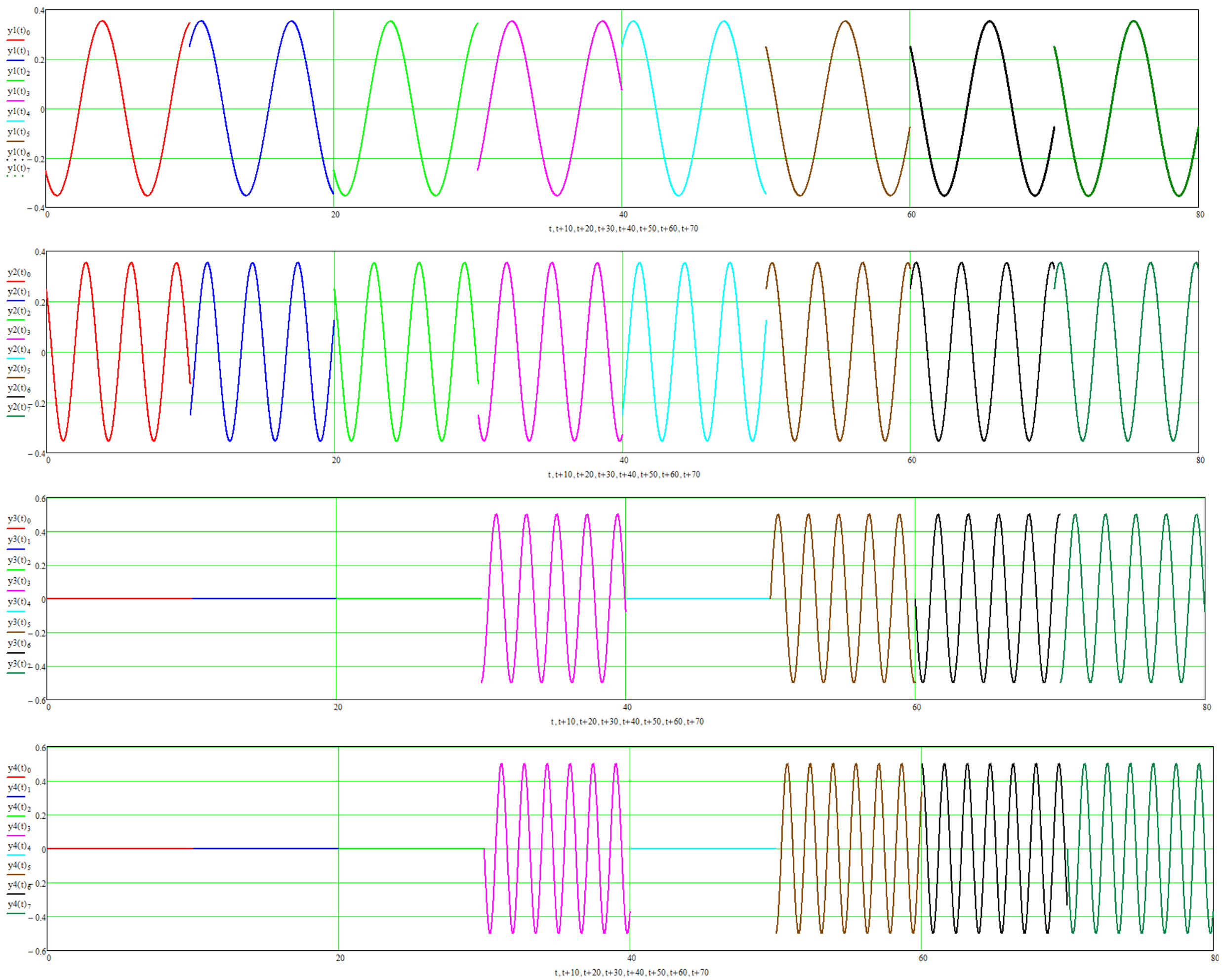
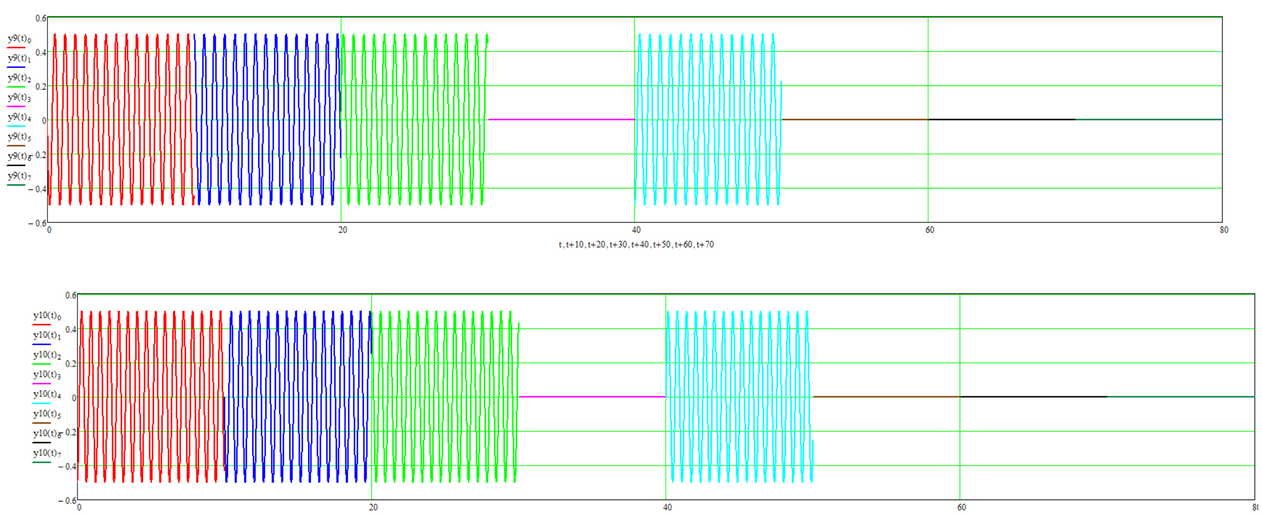


Figure 1: Elements of the Output Vector with Frequencies of Group 1 Ω_1, Ω_2 and Frequencies of Group 2 Ω_3, Ω_4.

Figure 2 shows the elements of the output vector with frequencies of the 1st group Ω_9, Ω_{10} and 2nd group Ω_{11}, Ω_{12} when multiplied by the same vector $\mathbf{x}(0)$.



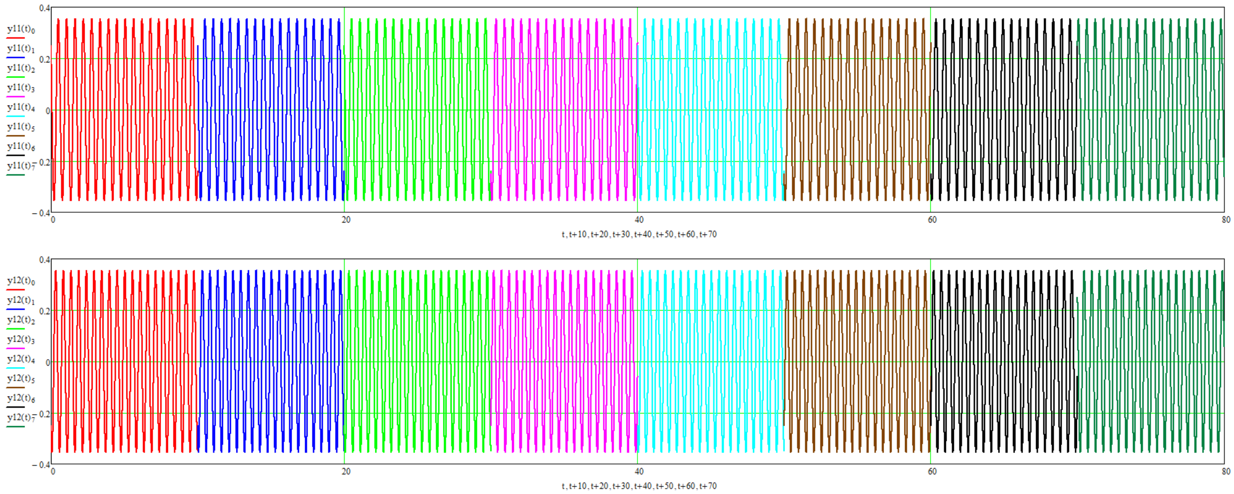


Figure 2: Elements of the Output Vector with Frequencies of Group 1: Ω_9, Ω_{10} and Frequencies of Group 2: Ω_{11}, Ω_{12}.

Figure 3 shows the elements of the output vector (21), obtained by summing all the vectors at the outputs of single-frequency matrices of the first group (17) and the second group (18), multiplied by the information vector $\mathbf{x}(0) = [-1\ 1\ -1\ -1\ 1\ -1\ -1\ 1]$.

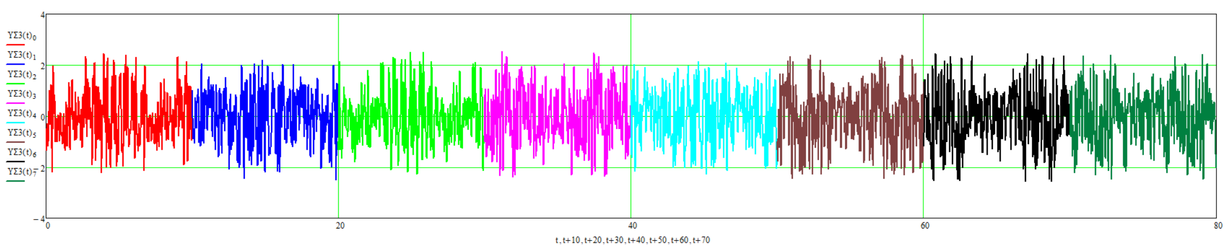


Figure 3. Elements of The Output Vector When Summing All Frequencies

According to the obtained single-frequency matrices of the first group (17) and the second group (18), the elements of the matrices can take the following values: $\cos(\Omega_m t)+\sin(\Omega_m t)$, $-\cos(\Omega_m t)+\sin(\Omega_m t)$, $-\cos(\Omega_m t)-\sin(\Omega_m t)$, $\cos(\Omega_m t)-\sin(\Omega_m t)$, where m is the frequency number from 1 to 64. These values correspond to the possible initial phases of the carrier frequencies: $\varphi_1 = \frac{\pi}{4} = 45°$, $\varphi_2 = \frac{3\pi}{4} = 135°$, $\varphi_3 = \frac{5\pi}{4} = 225°$, $\varphi_4 = \frac{7\pi}{4} = 315°$, as with QPSK.

The power of each multi-frequency element is 1. Hence, the power of the output vector will be equal to 8 and correspond to the power of the information vector $\mathbf{x}(0)$. Since the signal is formed on the basis of hypercomplex numbers, it is closed in a certain area with a constant modulus value and, as a consequence, has a minimum crest factor.

Multi-frequency signal elements, after amplification, are transmitted sequentially over the air using a single antenna.

3.2 Demodulation of a Seven-Frequency Octonion Carrier

Multi-frequency pulses pass through the communication channel and interference in the form of white noise is added to them. Let's imagine the noise as an 8D vector:

$$\mathbf{n}(t) = \begin{bmatrix} n_1(t) & n_2(t) & n_3(t) & n_4(t) & n_5(t) & n_6(t) & n_7(t) & n_8(t) \end{bmatrix}^{\mathrm{T}}.$$

Since noise affects signal elements at different times, they are not correlated. If white noise has a constant dispersion σ^2 and, accordingly, a constant power spectral density N_0, then in 8D space the interference vector will have circular symmetry.

According to model (1), when using multi-frequency fundamental matrix (13) as a channel matrix, we write the received signal as

$$\mathbf{s}(\omega_i, \omega_j, \omega_k, \omega_{e_1}, \omega_{i_1}, \omega_{j_1}, \omega_{k_1}, t) = \mathbf{\Phi}(\omega_i, \omega_j, \omega_k, \omega_{e_1}, \omega_{i_1}, \omega_{j_1}, \omega_{k_1}, t)\mathbf{x}(0) + \mathbf{n}(t). \qquad (22)$$

Since the fundamental matrix is decomposed into the sum of single-frequency matrices (19), (21), the received signal can also be written as

$$\mathbf{s}(\omega_i,\omega_j,\omega_k,\omega_{e_1},\omega_{i_1},\omega_{j_1},\omega_{k_1},t)=\frac{1}{8}\sum_{m=1}^{64}\mathbf{\Phi}(\Omega_m,t)\mathbf{x}(0)+\mathbf{n}(t)=\frac{1}{8}\sum_{m=1}^{64}\mathbf{y}(\Omega_m,t)+\mathbf{n}(t).$$

Figure 4 shows the signal in Figure 3 plus noise that exceeds the signal power by 6 dB. In fact, it is impossible to find signs of a signal by the appearance of the sum of a multi-frequency signal and noise, which increases its secrecy.

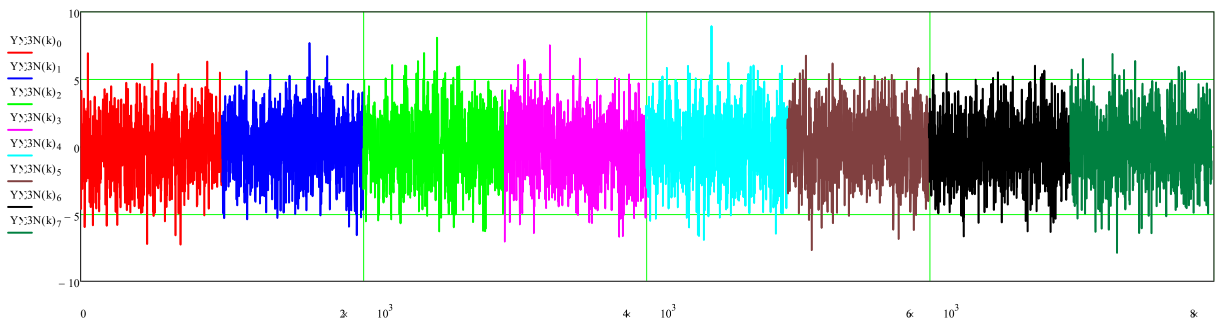


Figure 4: Signal with Noise After Passing Through the Communication Channel

Each multi-frequency pulse contains data on all 8 elements of the vector $\mathbf{x}(0)$ and the data is distributed by frequency. If the frequency spectra do not intersect, the interference power with a constant spectral density N_0 will be 64 times less in one signal band. Also, with circular symmetry of the interference, its power will be 8 times less on the orthogonal axes of 8D space, i.e. for an individual element of the vector $\mathbf{x}(0)$. As shown above, the transmitter power is distributed in a similar way along orthogonal axes and frequencies. Thus, the signal-to-noise ratio (SNR) for each signal element remains unchanged.

According to the theory of optimal signal reception with additive white noise, we will demodulate the signal separately at each frequency Ω_m, where m=1,2,···,64, using transposed basis matrices that make up single-frequency matrices (17) and (18). The demodulation procedure will be the same as in [9], only the dimension will increase.

Figure 5 shows the results of accumulating the signal shown in Figure 3 in the presence of noise, as shown in Figure 4, at a frequency of Ω_1. As can be seen, the influence of interference has decreased due to the accumulation of energy along 8 orthogonal axes.

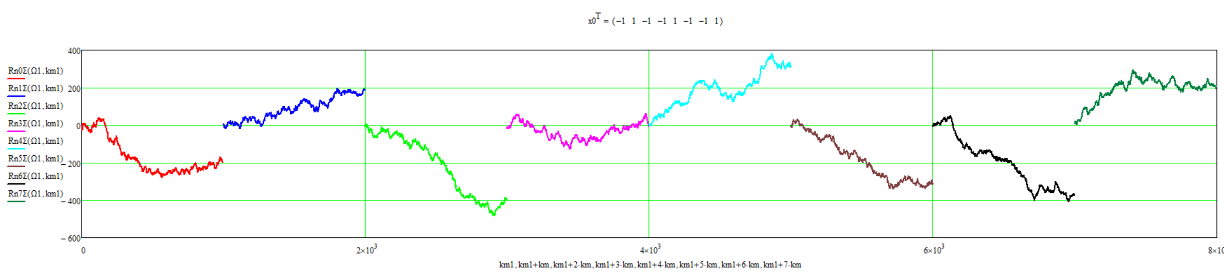


Figure 5: Result of Signal Energy Accumulation at Frequency Ω_1

Figure 6 shows the results of signal energy accumulation at all 64 frequencies. Since at each frequency we get a gain in SNR of 8 times when accumulated along orthogonal axes, the total gain in SNR will be equal to 8· 64 = 512 times or 27 dB.

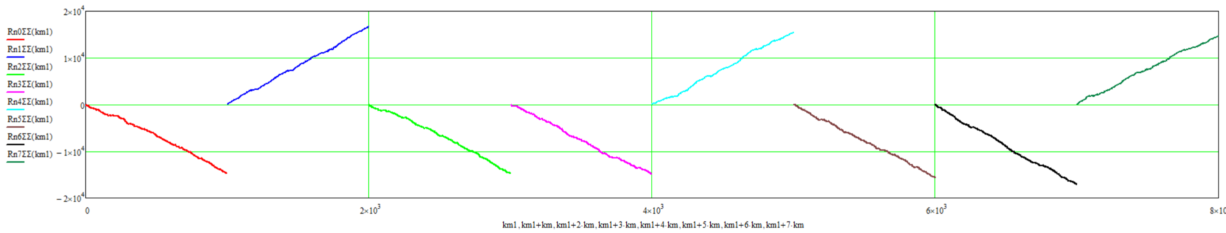


Figure 6: Result of Signal Energy Accumulation at 64 Frequencies

The resulting gain can be used to increase the speed of information transmission or noise protection against intentional interference. Note also that when transmitting each information signal at each frequency and along orthogonal axes, it increases resistance to fading in time and frequency. In this case, as can be seen from formulas (17), (18) and Figure 3, each element of the multi-frequency signal differs in shape from the others.

The decision on the value of the signal received after demodulation will also be made using the maximum likelihood criterion. As shown in [9], the distance between the resulting vectors can be calculated using the scalar product between the vectors or the squared Frobenius norm between the matrices obtained from the vectors.

V. Conclusion

Thus, the use of octonion will allow increasing the space dimension to 8 and forming 64 frequency channels, which will increase the MIMO channel capacity by 8 times along orthogonal axes and 64 times along frequency channels with only one antenna for transmission and reception.

The resulting corresponding gain in signal/noise ratio can be used to increase noise immunity while reducing the information transmission rate.

Since each information pulse is transmitted in each multi-frequency pulse, the proposed transmission system will be more resistant to signal fading in frequency and time.

The proposed transmission system using MIMO can also be used to increase throughput in wired communication systems.

It is known that hypercomplex numbers of a higher order are formed by doubling previous orders; then, apparently, there can be an infinite number of such numbers. Moreover, there can also be an infinite number of imaginary numbers. Since imaginary numbers represent spatial orthogonal coordinates with natural frequencies, it can be assumed that the dimension of hypercomplex multi-frequency space can be very large.

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