ISSN: 2994-6433

Research Article

Journal of Sensor Networks and Data Communications

The Generalized Z-Entropy's Fractal Dimension within the Context of the Rényian Formalism Applied to a Stable $M/G/1$ Queue and the Fractal Dimension's Significance to Revolutionize Big Data Analytics

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Submitted: 2024, Apr 03; **Accepted**: 2024, May 22: **Published**: 2024, Jun 13

Citation: Mageed, I. A. (2024). The Generalized Z-Entropy's Fractal Dimension within the Context of the Rényian Formalism Applied to a Stable $M/G/1$ Queue and the Fractal Dimension's Significance to Revolutionize Big Data Analytics. *J Sen Net Data Comm, 4*(2), 01-11.

Abstract

The current study that explores the Generalised Z-Entropy's fractal dimension within a non-time-dependent M/G/1 queuing system. Through numerical tests, the researchers analyse how the generated fractal dimension aligns with the specifications of the Generalised Z-Entropy. This investigation aims to enhance understanding of the relationship between entropy, complexity, and fractal geometry, with potential implications for Big Data Analytics. By combining information theory and fractal geometry, this work provides a significant generalisation in the literature on the relationship between entropy and complexity. More importantly, it is emphasised how important the fractal dimension is to the advancement of Big Data Analytics (BDAs). In addition, it also highlights the importance of the fractal dimension in advancing Big Data Analytics and mentions that the research includes unresolved questions and outlines the next steps for further investigation.

Keywords: Fractal Dimension(D), Big Data Analytics(BDAs)

1. Introduction

The Shannonian entropy [1], namely $H(X)$ reads as:

$$H(X) = \sum_i p(x_i) ln(p(x_i)) \quad (1)$$

p_i serves as the *ith*-event probability.

The probability of the *ith*-event is given in this expression as p_i. This entropy establishes the definition of information in information theory. There are many methods for measuring information, as well as in this scenario, we argue regarding how entropy and D interact with one another. D[2-7], is a measure that assesses how a fractal pattern expands beyond the area it occupies, indicating the complexity of the pattern in spatial dimensions. D[2-7], involves calculations that consider sticks' number (N) required in coastline coverage and the factor of scaling (ε). By analyzing these factors, the fractal dimension provides insights into the intricate nature of patterns and their representation of complexity in spatial dimensions.

$$N \propto \varepsilon^{-D} \quad (2)$$

$$ln\, N = -D = \frac{lnN}{ln\varepsilon} \quad (3)$$

[8] created a map and rigid sticks for an experiment like Richardson's in the book by using GEs pictures combined with GIMP (c.f., Fig. 1). The practical application of this technique for measuring the fractal dimension was demonstrated on a part of the Grand Canyon.

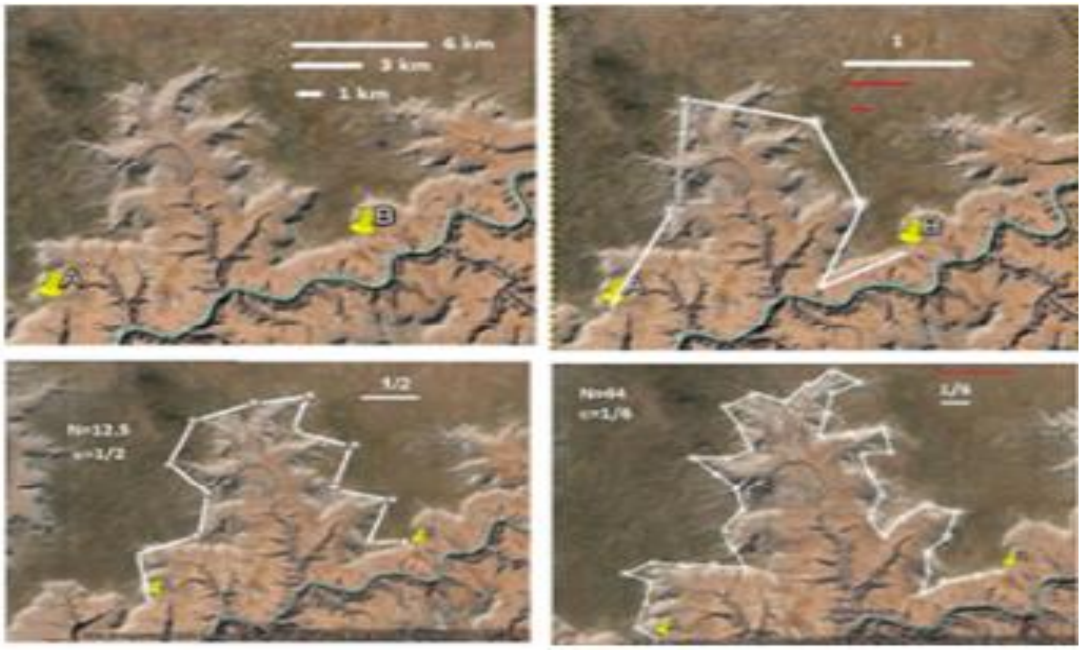

Figure 1 (c.f., [8]).

The visualization of how *N, D* and ε are corelated is illustrated by Fig. 2.

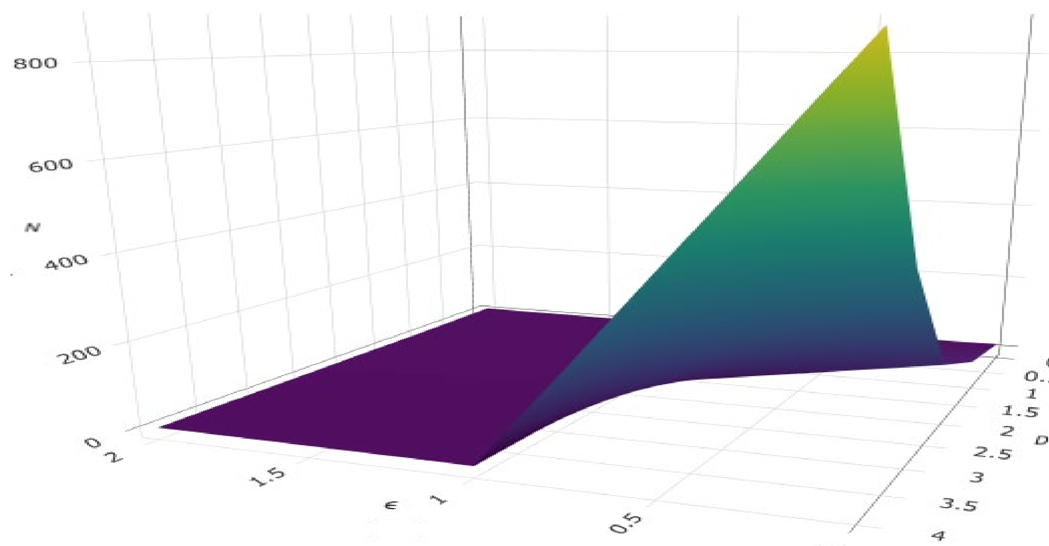

Figure 2: The Correlation between *N, D* and ε.

The present work investigates analysing the behaviour of the generated fractal dimension that matches the GZE specifications. By combining information theory and fractal geometry, this work provides a significant generalisation in the literature on the relationship between entropy and complexity. More importantly, it is emphasised how important the fractal dimension is to the advancement of Big Data Analytics (BDAs). In addition to unresolved questions and the next stage of research, concluding thoughts are given.

2. *D* of Entropies

In [8], the fractal dimension (*D*) associated with different types of entropy measures was determined, for entropies measures [9-12]. These derivations were conducted under proposing that all outcomes have equal probabilities.

The Shannonian fractal dimension [8] reads:

$$D_S = lim_{\varepsilon\to 0}\frac{lnN}{ln\frac{1}{\varepsilon}} \quad (4)$$

Rényian dimension of order $q \in (0.5,1)$ reads [10]

$$D_R = lim_{\varepsilon\to 0}\frac{lnN}{ln\frac{1}{\varepsilon}} \quad (5)$$

As for the Tsallisian case[8], of order $q \in (0.5,1)$,

$$D_T = lim_{\varepsilon\to 0}\frac{\frac{1}{1-q}(N^{1-q}-1)}{ln\frac{1}{\varepsilon}} \quad (6)$$

With $q \in (0.5,1)$, $D_T > 0$

The Kaniadakisian fractal dimension [8] for the entropic index, κ reads as:

$$D_K = lim_{\varepsilon\to 0}\frac{\frac{1}{2k}(N^{k}-N^{-k})}{ln\frac{1}{\varepsilon}} \quad (7)$$

Notably, D_R , D_T and D_T are entropic index impacted. The Koch snowflake (KSF) fractal dimension follows for ($N = 4$ *and* $\varepsilon = 1/3$).

3. Rényian Formalism of Stable *M*/*G*/1 Queueing System

Non-extensivity coins interactions of long range[13].

The Rényi's [9] non-extensive maximum entropy functionals reads:

$$\mathrm{H}_{\mathrm{q,R}}(\mathrm{p}) = \frac{c}{1-q}ln\{\textstyle\sum_{i=1}^{N}(p_i)^q\} \quad (8)$$

respectively, for a constant $c > 0$.

Theorem 1(c.f., [14])

The Rényi's non-extensive maximum entropy solution, $\mathrm{p}_{\mathrm{q,R}}$ (n), for *M*/*G*/1 queue in the stability under normalization, server utilization and Mean Queue Length reads:

$$\mathrm{p}_{\mathrm{q,R}}(\mathrm{n}) = \mathrm{p}_{\mathrm{q,R}}(0)\tau_s^{\frac{1}{q}}x^n \quad \mathrm{n} > 0 \quad (9)$$

Such that

$$\mathrm{p}_{\mathrm{q,R}}(0) = 1-\rho\ ,\rho \text{ is the server utilization} \quad (10)$$

where τ_s and x to be:

$$\tau_s = 2/(1 + C_{s,1,S}^2) \quad (11)$$

with

$$x = \frac{\rho}{\rho+\tau_s^{\frac{1}{q}}(1-\rho)} \quad (12)$$

The Generalized Z-Entropy (GZE) [14] reads as:

$$H_{q,a,b,\mathbb{Z}}(p) = Z_{a,b} = \frac{1}{(1-q)(a-b)}[\left(\sum_n p_{q,\mathbb{Z}}(n)^q\right)^a - \left(\sum_n p_{q,\mathbb{Z}}(n)^q\right)^b] \tag{13}$$

Such that $1 > q > 0.5, a > 0, b \in \mathbb{R}\ or\ b > 0, a \in \mathbb{R}\ with\ a \neq b.$

4. New Results

Theorem 2

Engaging [14], (9)-(13), the GZE fractal dimension, $D_{\mathbb{Z}_{a,b}}$ is devised by:

$$D_{\mathbb{Z}_{a,b}} = \frac{1}{(1-q)(a-b)} lim_{\varepsilon\to 0} \frac{1}{ln\frac{1}{\varepsilon}}\left(\left(\left(\rho^q\left(\frac{(1-\rho)\left(\frac{2}{1+C^2_{S,1,S}}\right)^{\frac{1}{q}}}{\left(\rho+(1-\rho)\left(\frac{2}{1+C^2_{S,1,S}}\right)^{\frac{1}{q}}\right)}\right)^{q-1}\left(1-\left(\frac{\rho}{\left(\rho+(1-\rho)\left(\frac{2}{1+C^2_{S,1,S}}\right)^{\frac{1}{q}}\right)}\right)^{(N-1)q}\right)\right)\right)^a - \left(\rho^q\left(\frac{(1-\rho)\left(\frac{2}{1+C^2_{S,1,S}}\right)^{\frac{1}{q}}}{\left(\rho+(1-\rho)\left(\frac{2}{1+C^2_{S,1,S}}\right)^{\frac{1}{q}}\right)}\right)^{q-1}\left(1-\left(\frac{\rho}{\left(\rho+(1-\rho)\left(\frac{2}{1+C^2_{S,1,S}}\right)^{\frac{1}{q}}\right)}\right)^{(N-1)q}\right)\right)^b\right) \tag{14}$$

Proof

By the definition,

$$D_{\mathbb{Z}_{a,b}} = \frac{c}{(1-q)(a-b)}[\left(\sum_{n=1}^{N}(p_{q,R}(0)\tau_s^{\frac{1}{q}}x^n)^q\right)^a - \left(\sum_{n=1}^{N}(p_{q,R}(0)\tau_s^{\frac{1}{q}}x^n)^q\right)^b] \text{ (c.f., (9))} \tag{15}$$

$$\sum_{n=1}^{N}\left(p_{q,R}(0)\tau_s^{\frac{1}{q}}x^n\right)^q = \rho^q\left(\frac{(1-\rho)\left(\frac{2}{1+C^2_{s,1,S}}\right)^{\frac{1}{q}}}{\left(\rho+(1-\rho)\left(\frac{2}{1+C^2_{s,1,S}}\right)^{\frac{1}{q}}\right)}\right)^{q-1}\left(1-\left(\frac{\rho}{\left(\rho+(1-\rho)\left(\frac{2}{1+C^2_{s,1,S}}\right)^{\frac{1}{q}}\right)}\right)^{(N-1)q}\right)$$

Hence, it follows that:

$$D_{\mathbb{Z}_{q,a,b,N,\varepsilon}} = \frac{1}{(1-q)(a-b)} lim_{\varepsilon\to 0} \frac{1}{ln\frac{1}{\varepsilon}}\left(\left(\left(\rho^q\left(\frac{(1-\rho)\left(\frac{2}{1+C^2_{S,1,S}}\right)^{\frac{1}{q}}}{\left(\rho+(1-\rho)\left(\frac{2}{1+C^2_{S,1,S}}\right)^{\frac{1}{q}}\right)}\right)^{q-1}\left(1-\left(\frac{\rho}{\left(\rho+(1-\rho)\left(\frac{2}{1+C^2_{S,1,S}}\right)^{\frac{1}{q}}\right)}\right)^{(N-1)q}\right)\right)\right)^a - \left(\rho^q\left(\frac{(1-\rho)\left(\frac{2}{1+C^2_{S,1,S}}\right)^{\frac{1}{q}}}{\left(\rho+(1-\rho)\left(\frac{2}{1+C^2_{S,1,S}}\right)^{\frac{1}{q}}\right)}\right)^{q-1}\left(1-\left(\frac{\rho}{\left(\rho+(1-\rho)\left(\frac{2}{1+C^2_{S,1,S}}\right)^{\frac{1}{q}}\right)}\right)^{(N-1)q}\right)\right)^b\right) \tag{16}$$

Hence, (14) follows.

Accordingly, let's discuss the following cases:
Case 1: $C_s^2 = 3, \rho = 0.5$

The information-theoretic impact on $D_{\mathbb{Z}_{q,1,-2,4,\frac{1}{3}}}$ is visualized by Fig. 3.

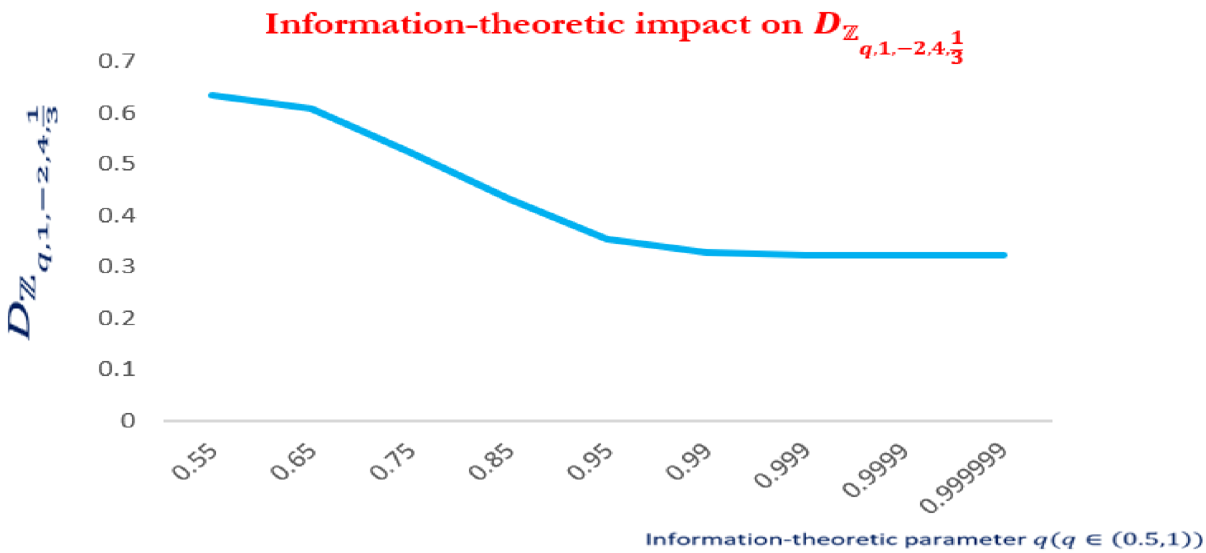

Figure 3:The influence of q on $D_{\mathbb{Z}_{q,1,-2,4,\frac{1}{3}}}$.

Approaching the instability zone, $C_s^2 \to 1$

$$lim_{a\to 0, b\to 0, C_s^2 \to 1} D_{\mathbb{Z}_{q,a,b,N,\varepsilon}} = \frac{1}{(1-q)} lim_{\varepsilon\to 0} \frac{1}{ln\frac{1}{\varepsilon}} ln(\rho^{qN}(1-\rho\ )^{q-1})$$

$$lim_{\ C_{s,}^2 \to 1} D_{\mathbb{Z}_{q,0,0,4,{}^1/_3}} = \frac{1}{(1-q)ln3} ln(\rho^{4N}(1-\rho\ )^{q-1}) = \frac{1}{(1-q)ln3} ln(\rho^{4q}(1-\rho\ )^{q-1}) = \frac{4qln\rho + (q-1)ln(1-\rho)}{(1-q)ln3}$$

$$\lim_{C_s^2 \to 1} D_{\mathbb{Z}_{0.5,0,0,4,{}^1/_3}} = \frac{4ln\rho - ln(1-\rho)}{ln3} = (ln(1-\rho)\rho^4)^{ln3}$$

More fundamentally,

$$\lim_{a\to 0} D_{\mathbb{Z}_{q,a,0,4,{}^1/_3}} = \frac{1}{(1-q)ln3} ln\left(\rho^q \left(\frac{(1-\rho)(\tau_s)^{\frac{1}{q}}}{\left(\rho + (1-\rho)(\tau_s)^{\frac{1}{q}}\right)}\right)^{q-1} \left(\frac{\rho}{\left(\rho + (1-\rho)(\tau_s)^{\frac{1}{q}}\right)}\right)^{4q}\right)$$

For $\tau_s = 0.5 = \rho$, Clearly, we have $D_{\mathbb{Z}_{1,0,0,4,{}^1/_3}} \to \infty$. Figs 4,5 and 6 portray these key findings.

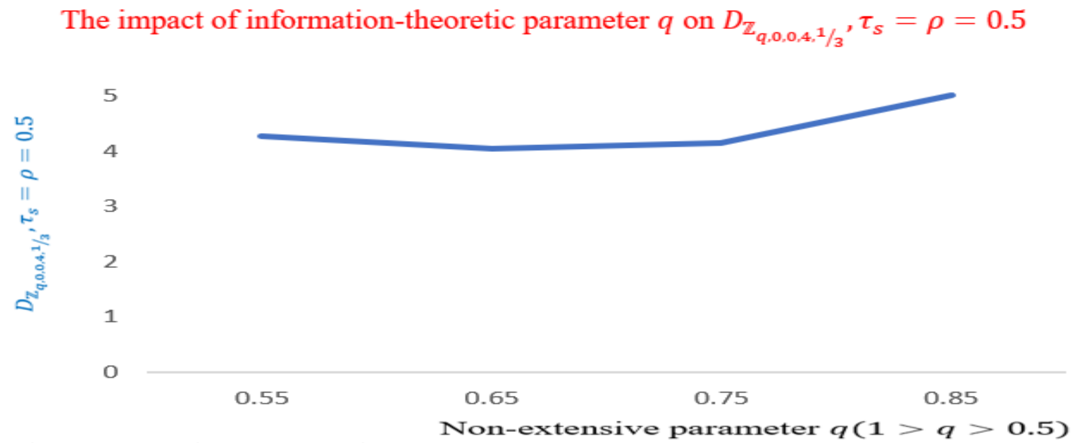

Figure 4

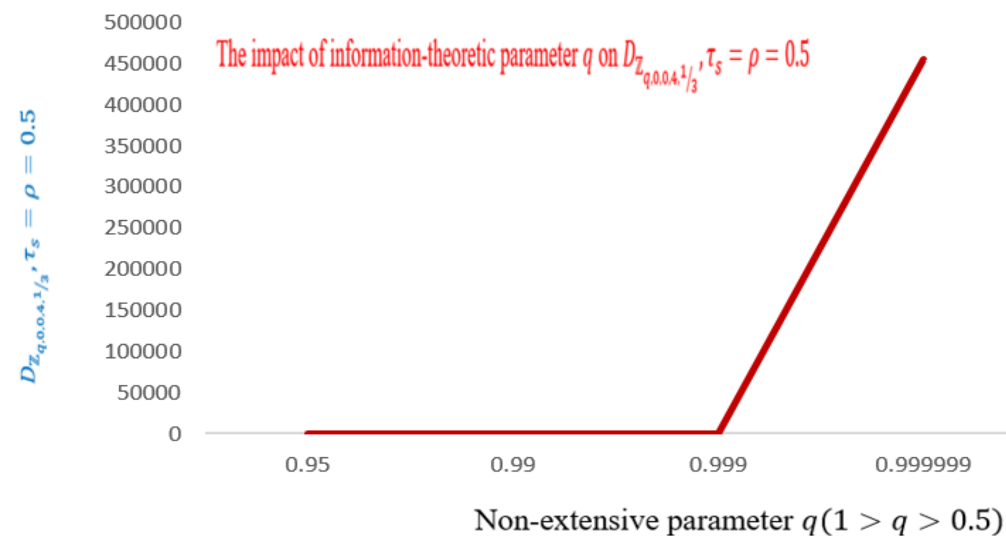

Figure 5

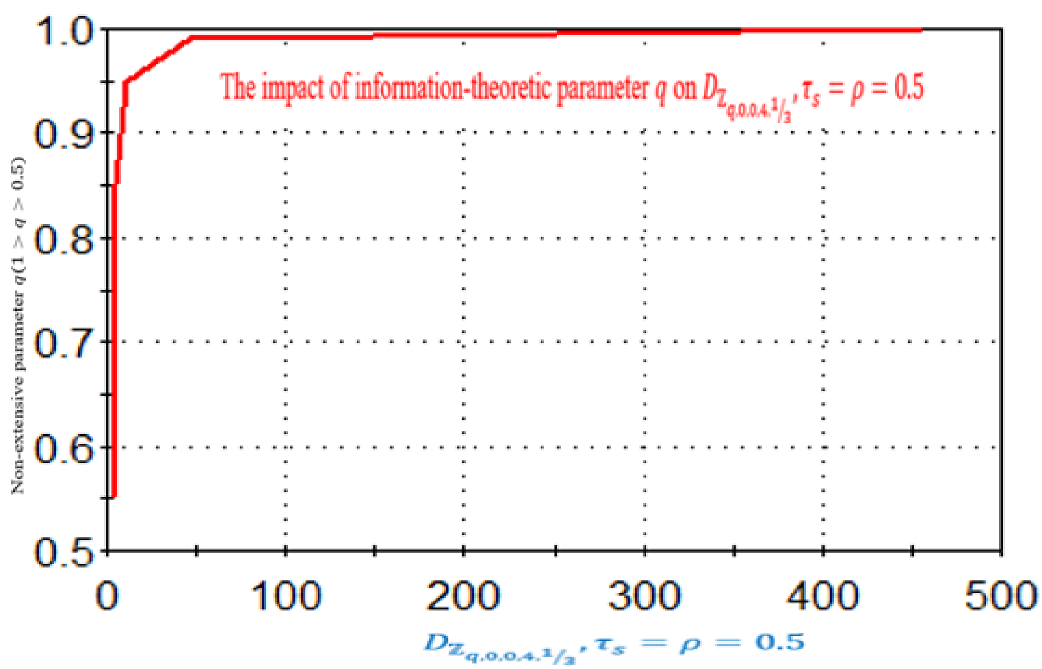

Figure 6

Notably, q impacts $D_{\mathbb{Z}_{q,0,0,4,{}^1/_3}}$, as $D_{\mathbb{Z}_{q,0,0,4,{}^1/_3}}$ decreases while q is in the extensivity phase and starts to increase drastically when q is non-extensive.

There is no clearly defined scaling-dimension since the Apollonian gasket is only roughly self-similar [15-17]. However, any "triangular" region enclosed within three circles appeared to be a curved Sierpinski gasket. Remember that scaling it by a factor of 2 requires 3 copies of the Sierpinski gasket as shown by Figs. 7 and 8(c.f., [17]). Consequently, we would expect that the Apollonian gasket's fractal dimension will be near to:

$$D = \frac{ln3}{ln2} \approx 1.585 \tag{17}$$

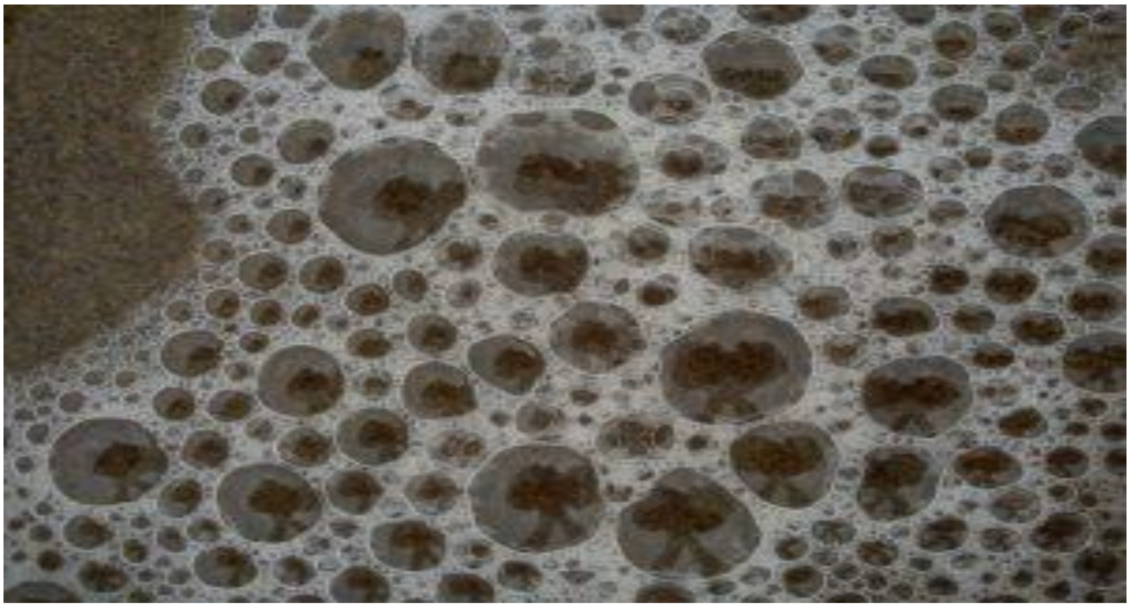

Figure 7: Bubbles are arranged in a fractal way to form foam [17].

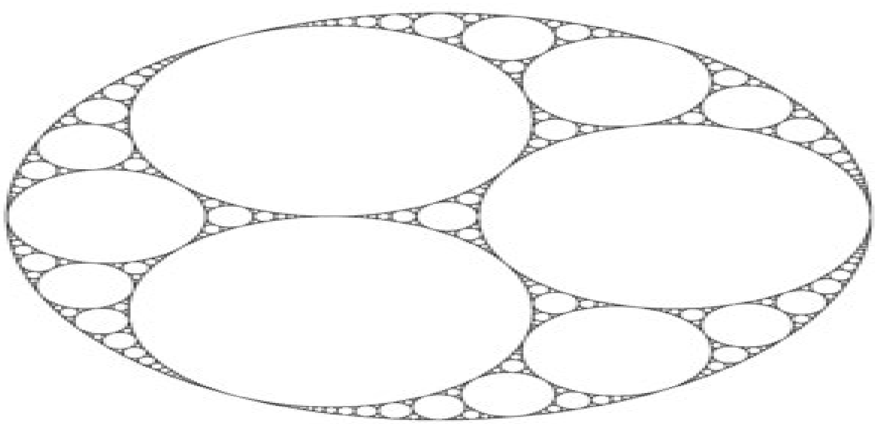

Figure 8: A fractal that can be used to simulate soap bubble foam is the Apollonian gasket [17].

Case 2: $C_s^2 = 1, \rho = 2$(instability)

Sierpiniski Gasket(SG), $D_{\mathbb{Z}_{q,1,0,3,\frac{1}{2}}}$

Mathematically speaking, the un-defindedness of SG at many points is based on the fact of attaining complex values at these points, for example:

$$(-1\ )^{0.55} = \left(cos\frac{\pi}{2} + isin\frac{\pi}{2}\right)^{0.55} = cos\frac{0.55\pi}{2} + isin\frac{0.55\pi}{2} \qquad \text{(De Moivre Theorem)}$$

So, we are in a situation of a complex valued SG fractal dimension. After some mathematical manipulation, one gets:

$$D_{\mathbb{Z}_{0.55,1,0,3,\frac{1}{2}}}(C_s^2 = 1, \rho = 2)$$
$$= 0.4602098599\left(1.464085696 - \left(cos\frac{0.55\pi}{2} + isin\frac{0.55\pi}{2}\right)\right) = 0.2136291295 + i(0.00693904008)$$

Notably, the fluctuations of the derived values of SG fractal dimension between decreasing and the drastic decreasing along the path of approaching sufficiently large values of q while approaching the extensivity zone, q >1. For q = 1, we arrive at invite value for the corresponding SG fractal dimension. Clearly, this shows the significant information-theoretic impact in both non-extensive and extensive phases. This paper provides another revolutionary approach to the traditional definition of both Apollonian and SG dimensions, while mine includes several respective parameters, including queueing and information-theoretic parameters.

5. D Applications to PDAs

To estimate the fractal dimension of datasets, one popular method is the box-counting method [18]. By using a box locality index (BLI) data structure, the information needed for calculating *D* is encoded, making the computation scalable for large datasets using distributed computing methods like MapReduce and Spark. By relating the size of the boxes to the number of points, the fractal dimension (*D*) can be determined. Fig. 9 provides examples of this method using three sample datasets to illustrate the process.

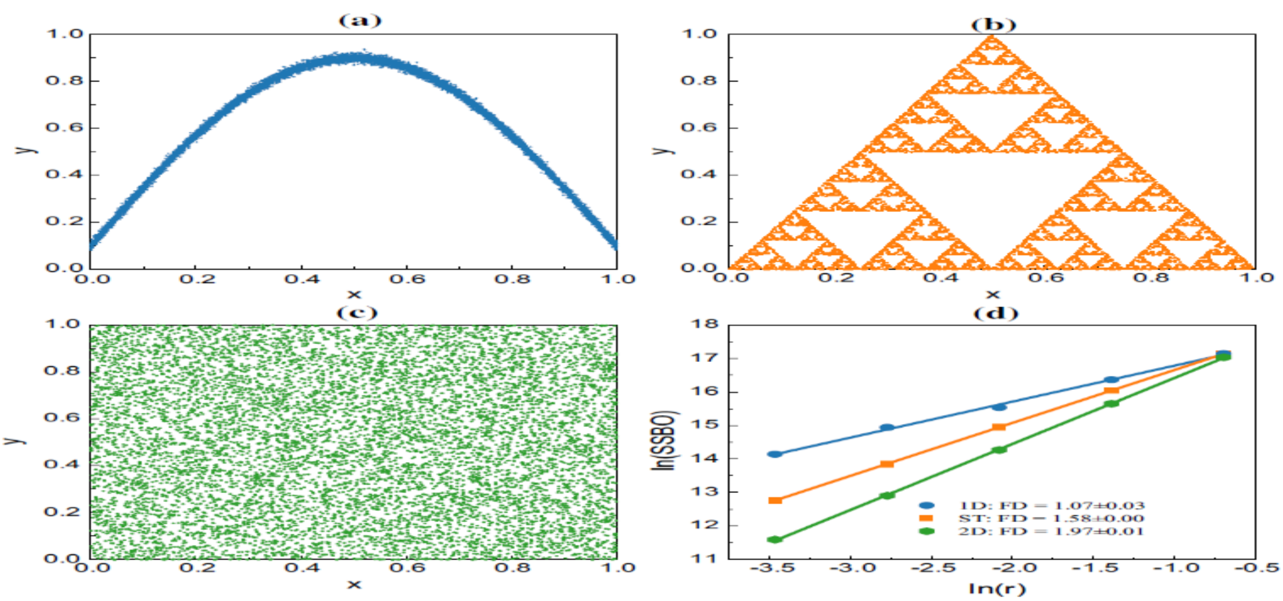

Figure 9: The box-counting plots help estimate the fractal dimensions of these datasets by analyzing the slopes of the fitted lines[18].

A detailed discussion of the challenges associated with online clustering in high-dimensional data and the limitations of existing approaches to this end is found in [19]. The proposed FractStream approach [19] aimed to reduce search complexity, execution time, and memory usage; experimental studies on various datasets demonstrate its effectiveness and efficiency.

To determine a dataset's fractal dimension, a multi-layered, nested grid structure can be constructed as demonstrated in [19]. The number of data points in each grid in the bottom layer of the grid structure is used to compute *D*. Moreover, the sliding window approach is applied to generate cluster partitions on evolving data streams, as Fig. 10 [19] illustrates.

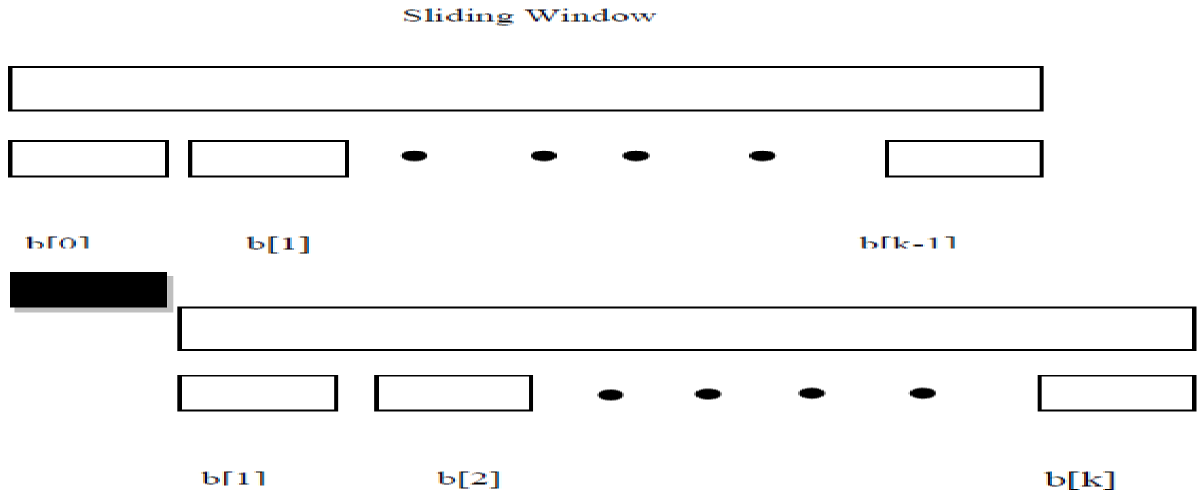

Figure 10: In the context of data analytics and machine learning, sliding window and basic windows are techniques used for analyzing data streams or time series data. A sliding window refers to a fixed-size window that moves along the data stream, allowing for continuous analysis of a subset of data. On the other hand, basic windows are non-overlapping windows of fixed size that partition the data stream into distinct segments for analysis. These techniques are commonly employed to extract meaningful patterns and insights from streaming or time-dependent data.

A clustering algorithm for a growing data stream based on correlation fractal dimension was created in the specified context [19]. A progressive fractal cluster is eliminated to make room for new clusters if its weight drops below a predetermined threshold, which is determined by periodically assessing the weight of the clusters.

The behavior of clusters changes over time when performing online clustering with a window of 1000 data points. This evolution of data can be segmented into intervals, as shown in Fig. 11 [19], to analyze the changes in cluster composition.

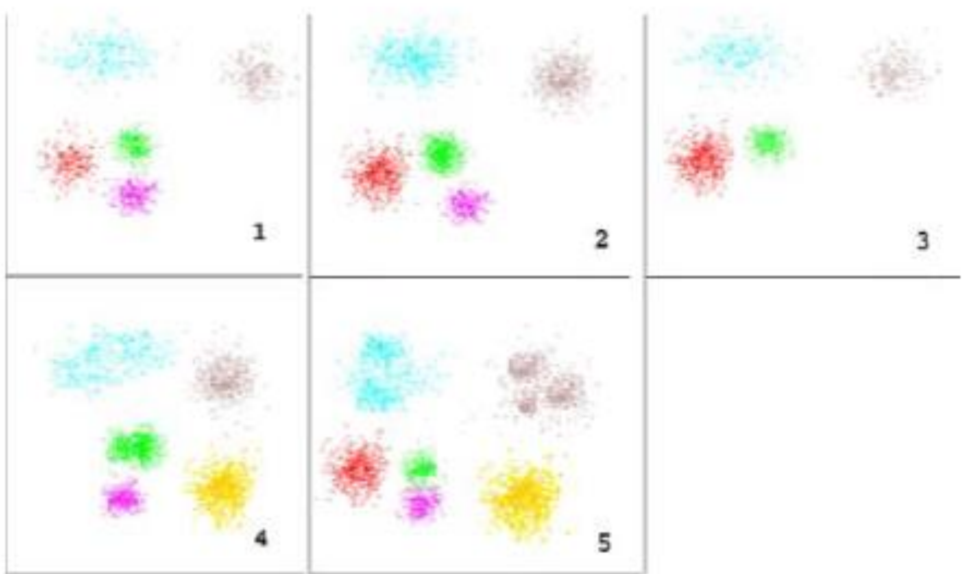

Figure 11: The evolution of clusters over time in the context of online clustering.

In the area of big data applications, disturbances like COVID-19, pollution, or policy changes have a huge effect on economic and financial systems [20]. For expanding the use of big data in financial and economic systems, it is imperative to investigate how these disruptions affect associated time series. The complexity of these time series is analysed using the Generalised Weierstrass-Mandelbrot Function (GWMF) [20], which demonstrates how disturbances in the form of exponential functions can produce multifractal characteristics. Additionally, the model replicates long memory and irregularity, which are evaluated by multifractal analysis and the R/S approach.

Research on how disturbances affect time series produced by the actual part of GWMF, or $C(t,\mu)$, and how to replicate multifractal features in time series is scarce[19]. Furthermore, there is little theoretical evidence to support the claim that time series produced by WMF naturally possess D.

6. Summary

This paper explores the relationship between $D_{\mathbb{Z}_{q,a,b,N,\varepsilon}}$ and the information-theoretic queueing parameters. Numerical experiments analyze the behavior of the derived fractal index to evidence that this work represents a significant advancement in unifying information theory and fractal geometry.

An explanation is given to confirm the influential role of fractal dimension in developing and revolutionizing BDAs. The current paper has several emerging open problems.

- **Open Problem One**

Based on the findings of this paper, is it feasible to undertake their approach much further to find the fractal dimension theory of Ismail's Entropy, namely IE(c.f., [21,22]), which is by default the ultimate generalization of numerous in literature?

- **Open Problem Two**

Based on the possibility to unlock open problem one, can we find any mathematical approach to decide the threshold of the involved universal parameters of IE. If so, what will be the expected form of the mathematical relations involved?

- **Open Problem Three**

Can we extend the case to investigate possible applicability of other fractal dimensions in literatures, such as Sierpinski Gasket and Koch Snowflake?

Future research aims to determine the fractal dimensions of other entropies in literature and compare them to further advance the field of Information Theoretic Fractal Geometry (ITFG). Notably, the search is ongoing to possibly answer the proposed sophisticated research questions.

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