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Strategies of Mixed Coalitions in Group Lending and Equitable Distribution of Joint Profit

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Abstract

The objective of this paper is to highlight the usefulness of cooperative game theory as a conceptual framework for approaching problems related to the formation of mixed coalitions within group lending and the fair distribution of joint profit. The focus was on a particular technique, the Shapley value, originally proposed as an a priori solution to a cooperative game. Furthermore, the central idea of this approach is indifference to the desires of others: individualism. We succeed in establishing the threshold level for determining the value obtained for a fair distribution of joint resources. The only element that could limit the applicability of this value to any sharing problem comes from the assumption that coalitions are formed in a random way, which is not always respected in reality.

Jel classification: G21; G23; O12; O17

Keywords: Group Lending, Equitable Distribution, Mixed Coalitions, Joint Profit

1. Introduction

The individual behaviour of low-income populations is usually assessed through household models. The literature distinguishes between theoretical analyses and applications in both developed and developing economies. Turning first to theoretical modelling, four major contributions generally attract attention. The first is that of Chayanov, taken up by Nakajima [1,2]. Based on a marginalist approach to rural economics, this contribution, which is also a pioneering one, draws on the historical context of debates on agriculture in the Union of Soviet Socialist Republics (USSR) to present the internal equilibrium of the household as an optimisation of a production-consumption equilibrium. The second contribution places the separability principle at the centre of rural household models [3]. This results in two types of models. The first type concerns separable or recursive models. These are precisely those constructs that assume that production, consumption and labour decisions within a household are made separately. The central assumption is that markets are perfect for all factors of production and for the good produced. Prices are then exogenous to the household. It should be stressed that perfect markets are a sufficient but not necessary condition for separability [4]. Indeed, separability implies that household production decisions are not affected by consumption and labour resource decisions. However, labour resources and consumption decisions are not independent of production decisions.

Production decisions are considered to be made for the purpose of optimising the first phase, allowing for the solution of factor demands, output supplies and optimal profit. Given the optimal level of profit determined in the first stage, the second stage consumption problem is solved. The leisure demands of all household members and the demands for other products are determined by the results of the first stage. The second type are non-separable or non-recursive models. The idea is that in most cases markets are imperfect, which implies the non-marketing of produced goods or non-marketable factors of production [5]. Since the publication in 1944 of von Neumann and Morgenstern's masterpiece, Theory of Games and Economie Behavior, the mathematical theory of games has maintained a privileged relationship with certain disciplines of the social sciences, as shown by the vast existing literature. By

the very nature of this theory, the most striking applications are most often linked to the modelling of behaviour, categorised as cooperative or non-cooperative, of economic and political agents. Microfinance is seen as an attempt to respond to the dual and potentially contradictory objectives of economic competitiveness and social cohesion.

On the one hand, microfinance initiatives seek to promote entrepreneurship as a source of long-term growth. They therefore aim to find new mechanisms to promote the competitiveness of an economic area, without entering into a logic of "downward" territorial competition, which is not very promising in the long term. On the other hand, the dynamics of financial exclusion in the North are strongly linked to exclusion from the labour market. In the South, the entrepreneur has, in a way, already "proven himself" in the informal sector and the cohesion within his community is used as a means to facilitate access to credit. This difference in context is not only interesting for understanding the divergence of objectives pursued, but has strong implications for the effectiveness of mechanisms imported from the South. In the cooperative case, the attention of theorists has been focused on the search for arbitration procedures with 'desirable' properties that aim to share the gains resulting from their cooperation among the players. Until now, few works have been interested in determining the value of a fair distribution of joint gains, where the phenomenon of precariousness remains dominant.

This work focuses on the question of how to determine the value of a fair distribution of the profit generated by a group loan in the microfinance sector. Members are assumed to be able to talk to each other to form coalitions and engage with each other. Indeed, the assumption made here is that members do not, under any circumstances, make attempts to coordinate their strategic decisions. Furthermore, the central idea of this approach is indifference to the desires of others: individualism. Based on the models of the cooperative and non-cooperative game theory we manage to develop a model whose unique solution is based on the Shapley value which allows to measure the value of a fair distribution of the so-called collective resources.

2. Contractual Approaches to Group Lending in Microfinance

Economic theory today offers us a fragmented vision of the firm: the two main currents we have discussed are essentially unaware of each other and there are still differences, if not major oppositions, within each of them. The opposition between the contractual theorisation - the firm as a network of contracts - and the competence-based theorisation - the firm as a system of competences - remains, in our opinion, at the centre of the differences in conception. This opposition is very clear if we compare contractualism, faithful to the neo-classical foundations (theories of agency and property rights) and evolutionists. It concerns both the nature of the questions posed and the theoretical foundations. As Dosi et al. state, the firm is structured around two orders of devices: the first refers to the organisation and division of labour and to production activity; the second to all the incentive and control systems and procedures [6]. This gives rise to two types of questions concerning, on the one hand, the 'problem-solving features' of organisations and, on the other, the problems of confrontation between the divergent interests and objectives of the individuals and groups making up the organisation. Contractual approaches deal practically only with the second dimension, and deal with it in their own way, i.e. on the basis of the hypotheses of the economic theory of rational behaviour and the search for parietal optima, by the method of equilibrium (in imperfect information); whereas evolutionary and competence-based analyses focus on the first dimension (the firm as a place of implementation of know-how), and this on the basis of radically different behavioural assumptions, based on bounded rationality and theorisation of individual and collective *knowledge* and learning in a cognitive and evolutionary perspective opposed to the equilibrium method. The problem, however, is that these two orders of questions are closely intertwined. One of the major challenges facing the theory of the firm is thus to understand how these two dimensions are articulated.

2.1 Group Lending: Mixed Coalitions and Opportunism

The consideration of a risky environment due to the imperfect nature of information has led to an important body of contributions focusing on the foundations and modalities of cooperation between individuals. The aim has been to understand the reasons for and forms of organisations in a market economy, including the financial institution. The agency relationship poses much more of an existential problem between the principal and the agent. This is based on governance based on a shareholder model. For Jensen and Meckling, organisations, and in particular the firm, are nothing more than 'legal fictions that serve as a nexus for a set of contractual relationships between individuals [7]. The firm is understood in the literature as a black box (Adams), a contract node (Jensen and meckling), a node of skills and resources. This divergent view of the notion of the firm is still unclear. Coase believes that there is no difference between the firm and the market and that the firm is fiction and has no real existence [8]. The firm is not recognised as an entity in its own right, which can be seen as running counter to the whole historical evolution of the firm in legal and institutional terms [9]. The question of the boundaries of the firm is irrelevant, there is no fundamental difference between firm and market. This point was made very clearly by Alchian and Demestz [10]. In contrast, Simon shows that it is an 'authority' relationship, and different in market relationships from the viewpoint of Jensen and Meckling [7,11].

He assume that the borrower knows exactly what he wants but the lender can ignore his behaviour hence the idea of incomplete contracts. The lender is led to rationalise his credit to limit the risk because it is he who incurs this risk due to the absence of complete information on the borrower [12]. To explain the existence of the risk of opportunism on the credit market, Stiglitz and Weiss study the impact that information asymmetries between lenders (the principal) and borrowers (the agents) can have on the use of the interest rate as a variable for adjusting the supply and demand of credit [13]. However, despite the decision to ration credit, bank failure due to borrower failure is not ruled out, since the projects to be financed always involve risks. Similarly, the asymmetric

distribution of information between the borrower and the lender is not removed. In order to reduce its risk of default due to the borrower's default, the bank selects clients on the basis of various criteria: the client's reputation, which is based on the "long-term relationship" of the two partners, and the client's social and financial situation. The uncertainty in the credit market is even greater in developing countries.

Indeed, in Western countries, banks try to reduce the risk of anti-selection by collecting information on the credit applicant and by demanding both material and financial guarantees to protect themselves from the risk of non-repayment. According to Aryeetey and Udry, the solution of credit rationing and bank exclusion that results from these imperfections is not effective for two reasons [14]. First, particularly secure poor people with projects with high expected returns may not have access to credit because they cannot convince banks of the quality of their project. Second, lenders with access to particularly cheap sources of finance may not be able to access certain local markets because they cannot distinguish between good and bad borrowers. Examples include the Grameen Bank, BIS and BancoSol, or the African case studies: West Africa, Central Africa and Maghreb countries. If we are witnessing the emergence of a microfinance sector within the financial systems of developing countries (DCs) today, it is, on the one hand, in response to the inefficiency or inability of the traditional credit market to meet the needs of some of the stakeholders, and on the other hand, due to the appropriation of credit tools and methodologies that have proved their worth within the informal financial sector by more or less formalised institutions.

2.2 Group Lending Contracts in Microfinance: Mechanism and Functioning

Two main types of credit contracts govern the lending activity of MFIs: individual contracts that link the lending institution to a single borrower; group contracts that link the institution to a "pool" of borrowers. In the group contract, loans are granted to individuals, but the group is jointly and severally liable for the credit, the guarantee becoming collective. The introduction of group credit contracts (solidarity credit) is considered one of the major innovations in credit risk management in MFIs [15]. Group lending is based on Stiglitz's model, built around the case study of the Grameen Bank, which, together with the Bancosol in Bolivia, is considered to be the pioneer institution of group lending contracts [15,16]. Stiglitz observe that the costs of information gathering by lending institutions are high in rural credit markets [16]. Due to geographical proximity and close social and cultural ties, rural populations have an informational advantage over the institution with regard to selection (anti-selection risk) and control of borrowers (moral hazard risk). With regard to moral hazard, the literature review highlights two main theoretical models: the control incentive model and the repayment incentive model [15-18]. According to the control incentive model, the informational advantage induces lending institutions to transfer management control to the co-signers of the credit contract who internalise the costs. By doing so, institutions minimise their agency costs. Co-signatories have an incentive to control each other for two reasons: collective responsibility and the subordination of future loans to the repayment of current loans. However, the artificial size and composition of groups, as well as collusion within credit groups can limit the effectiveness of delegating control to the borrower group [16,19]. According to Chowdhury, joint control by the group and the MFI can ensure the effectiveness of group credit contracts [17]. The MFI is more efficient if it internalises part of the agency costs (control costs). According to the repayment incentive model, the introduction of a system of social and institutional sanctions encourages the co-signatories of the credit to ensure repayment. The risk of social ostracism of the members of the credit group on the one hand, and the risk of non-eligibility of the group for future credits on the other hand, encourage them to monitor each other, which increases repayment rates [20]. Besley and Coate examine the role played by group credit contracts, specifically their composition, in selecting borrowers and minimising the risk of anti-selection [18]. They show that homogeneous credit groups, formed on the basis of social ties between members , have a low risk of anti-selection, thus improving credit repayment rates. Ghatak shows that collective responsibility is an instrument for selecting members of the borrower pool [21].

Theoretical models, show that the financial efficiency of MFIs is explained by the choice of group credit contracts [15,16,18]. Furthermore, group lending contracts lead to lower transaction costs and increased credit supply and, consequently, to better social efficiency of MFIs [18]. However, the results of the exploratory and empirical studies seem to converge as regards the relationship between the choice of group credit contracts and financial performance, but diverge as regards the relationship between this type of contract and social performance. The exploratory studies highlight a positive impact of the choice of group credit contracts on the social efficiency of MFIs. Morduch's study on the case of the Grameen Bank reveals that the implementation of group loans has improved its social efficiency but very little its financial efficiency [22]. It is not financially self-sufficient and has repayment rates close to 90%. Based on a study of a sample of 1,500 MFIs, Lapenu, et al. conclude that the number of active borrowers is higher for MFIs that choose group lending [23].

In contrast, Hartarska studies a sample of MFIs in Central and Eastern Europe and comes to the conclusion that the type of credit contract has no significant influence on financial (profitability and financial sustainability) and social (number of active borrowers and average loan size) performance [24]. Similarly, Mersland and Strøm find no significant relationship between the type of credit agreement and financial performance in a sample of 278 private microfinance companies [25]. Regarding the social efficiency of MFIs, the results are mixed. They show that the intensity of transactions is significantly important when private companies choose individual credit. The latter has a negative but insignificant influence on the outreach of MFIs (number of active borrowers). On the other hand, based on a sample of 124 MFIs, Cull et al. show that those that provide individual loans are more profitable and

more autonomous than those that provide group loans [26]. They operate according to bank management principles and on a bilateral lender-borrower model. The majority of their loan portfolio is made up of people around the poverty line. Furthermore, they find no significant influence of group lending on the portfolio quality of MFIs. The results of the empirical studies do not show a positive relationship between the group lending contract and the financial performance of the MFIs, and therefore do not go in line with the theoretical results.

3. Conceptual Framework of the Analysis and Modelling

Group lending is probably the best known of these mechanisms. It consists of financing independent projects by making the members of a group jointly responsible for repayment. This lending technique has the advantage of securing the loan by creating a kind of collateral. The literature on information asymmetries has provided explanations for the illustration of these contracts, as identified in the work of Aghion and Morduch [15]. The main idea is to make individuals co-responsible in order to transfer agency costs to the group level. Group contracts can thus make it possible to manage both anti-selection and moral hazard problems. Indeed, individuals with the least risky projects have an incentive to form a group together. If the terms of the loan are such that individuals with the riskiest projects have no interest in grouping together, or cannot group together with a less risky project, then lending to groups allows full disclosure of information and is socially optimal . These models assume that borrowers know each other perfectly. This system then corresponds to loans to the same extended family, or members of the same village. But the projects may not have truly independent achievements, thus calling into question the advantage of peer selection.

The second advantage of group lending is that it reduces the cost of moral hazard, and thus generates better lending conditions for borrowers. Indeed, by making borrowers co-responsible, individuals will jointly choose to make the effort, thus decreasing the moral hazard rent to be paid, as shown by Stiglitz for example [16]. Again, it is assumed that borrowers have the means to influence the decision of other group members. In 2021, 72% of MFIs surveyed by the Microbanking Bulletin used one or more group lending methodologies (MBB, 2021). While the formulation of Shapley's value (SV) dates back to 1953, it was not until the 1960s that applications of this (a priori) solution of a cooperative game to various problems in the social science world began to emerge. Shubik proposes SV as a mechanism for distributing the profit of a complex organisation among its departments and demonstrates, through a series of examples, how this approach is a stimulus to innovation [27]. Aumann and Shapley's extension of SV to non-atomic games has made it possible to tackle problems where the agents are infinitesimal [28].

3.1. Formation of Mixed Coalitions in Groups: A Review of Cooperative Game Theory

Microcredit is no longer an experiment, but is proving to be one of the most effective development instruments [25]. The clientele of these MFIs is mainly composed of micro and small entrepreneurs (MSEs) and promoters of income generating activities (IGAs) [29]. As the name suggests, solidarity microcredit brings together a group of individuals who can enter into agreements to maximise their joint profit. It is therefore assumed that there are no legal, sociological, economic or psychological barriers to cooperation. This usually takes the form of a coordination of strategies or an arrangement to share the gains from the project. It is also assumed that the gains are valued in monetary terms, that all members have a constant utility for the money, that interpersonal comparisons of utilities are possible and that side payments are allowed. Solidarity microcredit, which is a formalised way to analyse situations of interdependence between economic agents, can also be seen as a set of two parts.

The first part is concerned with the description of the group rules and the process of grouping members, while the second part focuses on the search for solutions with certain characteristics. A coalition can be described in three different ways depending on the degree of detail desired in the presentation of the group rules and the actions available to the members. Thus, the extended form proceeds to a mathematical description of these rules and of the complete dynamic sequence of actions of each member. The normal form simply examines the actions or strategies of members and the payoffs associated with each. In the context of cooperative games, one is primarily interested in the process of coalition formation between members and, therefore, needs to assess the potential strength of each. In the following paragraphs, a quick review of the essential notions of these two poles of interest will be made.

3.1.1 Coordination of Coalition Actions and Formalisation

The normal form simply examines the actions or strategies of the members and the payoffs associated with each. In the context of cooperative games, one is primarily interested in the process of coalition formation between players and, therefore, needs to assess the potential strength of each. This way of describing a group was provided by a concept formulated by Von Neumann in 1928 and called the characteristic function of the game. Formally, let Γ a cooperative group with n members and $I = \{1,\ldots,n\}$ the set of members. A subset S included in I is called a coalition. By allowing, for convenience, the existence of an empty coalition, there will be a total of 2^n possible coalitions. Let $P(I)$ be the set of parts of Ithen the characteristic function is defined as a real-valued function $v : P(I) \Longrightarrow$ IR such that $v(S)$ represents the maximum payoff that a coalition can guarantee to its members, Von Neumann and Morgenstern interpreted the term guarantee in the following way: the worst case for a coalition S is that the members $I \setminus S$ form an anti-coalition whose objective is to minimise the payoff of S. In this case, v(S) is given by $\boldsymbol{v(S)} = \mathbf{max}_{x \in X_S} \ \mathbf{min}_{y \in X_{I/S}} \sum_{i \in s} \boldsymbol{p_i(x,y)}$; $p_i(x,y)$ is the expected gain of member i when the mixed strategy $x \in X_s$,$y \in X_{I/S}$ strategy is employed. This interpretation offers a considerable technical advantage in that the problem of finding $v(S)$ is reduced to that of solving a non-cooperative game between S and $I \setminus S$ and whose procedure is well known. Nevertheless, and as pointed out by McKinsey and Luce and Raiffa, the characteristic function thus defined provides a pessimistic representation of

reality [30,31]. Indeed, it always postulates the formation of highly antagonistic coalitions (S and $I \setminus S$) which does not necessarily reflect the process of coalition formation observed in rational members by definition.

3.1.2 Individual Actions and Collective Rationality

The characteristic function v has the following two properties:

$$\boldsymbol{v(\emptyset) = 0 \;\; ; v(SUT) \geq v(S) + v(T) \quad \forall S,T \supseteq I,\; I \cap T}$$

While the first property merely formalises the idea that the payoff of an empty coalition is zero, the second property, superadditivity, states that a coalition of members is at least as efficient as all of its disjoint sub-coalitions. A game where v is additive $\boldsymbol{v(S \cup T) = v(S) + v(T)}$ is said to be inessential i.e. there is no incentive to form coalitions of more than one member. An imputation is a set of n numerical values ($X1,...,Xn$) representing the payoffs of the members at the end of an investment and satisfying the following two conditions:

$$\boldsymbol{x_i \geq v\{i\}\ \forall i \in I\ ;}$$

$$\boldsymbol{\sum_{i \in I} X_i = v(I)} \quad \boldsymbol{v(I)} \text{ the total gain of the group}$$

The first condition means that no member will accept a sharing that gives him less than he could gain by acting alone (individual rationality). On the other hand, a satisfactory sharing $\sum_{i \in I} X_i < v(I)$ will not be accepted by the members because it amounts to wasting the quantity $v(I) - \sum_{i \in I} X_i$. On the other hand, if we assume that satisfactory $\sum_{i \in I} X_i < v(I)$ this will amount, in Ekeland's words, to "selling the bear's skin before killing it" (the solution is not feasible) [32]. Thus, $\sum_{i \in I} X_i = v(I)$

This condition is called collective rationality. Individual rationality being a basic ingredient of game theory, and indeed of the decision making approach in the social sciences in general, has allowed the first condition to escape criticism which has focused on the second. Luce and Raiffa have expressed reservations about the validity of the transition from individual to group rationality [31]. Their argument is that the latter is neither a postulate nor a result of cooperative game theory. On the other hand, if one accepts this hypothesis, why not extend it to all coalitions S ($\sum_{i \in S} X_i > v(S)$) rather than just the grand condition I ? The reason is, as we shall see, primarily technical. To recap, an n-person cooperative game with side payments is a triplet $< I, v, X >$ Or $I = \{1,\ldots,n\}$ is the set of members, v a superadditive function of $P(I)$ in $\mathbb{R}$ verifying $v((\emptyset)) = 0$ and X is the set of imputations such as that $X_i > v(\{i\}), \forall i \in I$ and $\sum_{i \in I} X_i = v(I)$

The uniqueness of a solution is always a desirable and reassuring property, especially when it comes to sharing the benefits of a group investment among the members involved.

3.2 Members' Interactions with Joint Resource Sharing

In general, a cooperative game described by a characteristic function does not generate a unique imputation and, consequently, the interest of theorists has turned to the search for means or procedures that allow, in the absence of a unique solution, to exclude a number of imputations on the basis of intuitively justified criteria. In what follows, I will outline only those solutions that have, like Shapley's value, been applied in the field of equitable sharing of welfare as a group resource, so that I can refer to them on occasion.

3.2.1 The Core as a Necessary and Sufficient Condition

To be able to rule out certaine charge, one would have to define criteria that make a coalition prefer one charge to another. Intuitively, it is clear that a coalition will reject a particular imputation if there is another that will provide a greater gain to its members. Formally, an imputation is said to be dominated by x if both of the following conditions are met:

(i) $\sum_{i \in I} X_i =< v(S)$ (*Feasibility*)

(ii) $X_i > y_i \; \forall i \in S$ (preferability)

The set of imputations that are not dominated by any coalition is called the core of the game. A characterisation of the imputations (if any) that belong to the core is given by the following theorem: An imputation $x = (X1,...,Xn)$ belongs to the core of a cooperative game (group) $< I, v, X >$ if and only if : $\sum_{i \in I} X_i < v(S)\ \forall S \subseteq I$

The kernel, proposed by Gillies (1959), has the (good) property of satisfying all coalitions insofar as none of them can increase its payoff. On the other hand, the core can be empty. Indeed, let us consider a 3-member group whose characteristic function is :

$$v(\{l,2,3\}) = 1 \;\; ; \;\; v((\emptyset)) = 0 \; ; \;\; v(U,2\}) = v(U,3\}) = v(\{2,3\}) = a$$
$$0 < a < 1 \;\; ; \;\; v((\emptyset)) = 0 \;\; ; \;\; i = 1,2,3$$

If $a > 2/3$, the kernel is empty, because no matter how you divide 1, there will always be two members who have less than between them and therefore form a blocking coalition.

However, trust cannot be decreed, it must be granted, it must be earned [33,34]. It is even a social construction [33].

3.2.2 Measuring Group Stability in the Face of Member Coalition: the Von Neumann-Morgenstern Solution

Von Neumann and Morgenstern proposed a solution concept based on the notion of stability, which is stated as follows:

3.2.2.1 Explanation of the Concept of Imputation

Definition: A set of allocations V of a cooperative group $< I, v, X >$ is stable:

(1) *if x and y* $\in V$ So y is not dominated by x *and* x is not dominated by y (internal stability); (2) *if* $y \notin V$ *so it* $\exists\ x \in V$ *such as* y *is dominated by* x (external stability).

A von Neumann-Morgenstern solution is any stable set. Generally, this solution is not unique.

3.2.2.2 Measuring the Value of Members' Dissatisfaction with Coalition Issues

The idea of the nucleus, a solution introduced by Schmeidler,

"is to minimise the maximum dissatisfaction or, in other words, to minimise the most glaring complaint" [32,35]. To define this rigorously, some technical preliminaries are necessary.

either $<$ ***and*** $\leq$ ***two binary relations on*** R^p $(P \in N)$ ***defined by r***:

$$x < y \leftrightarrow [Ej\epsilon \{1,...,p\}: (X_i = y_j \text{ for } i < j) \text{ and } x_i < y_j]$$
$$x \leq y \leftrightarrow [x < y \text{ or } x = y]$$

The relationship $\leq$ is a total order relation called lexicographic order. On the other hand, we define component by component an application θ: $R^P \rightarrow R^P$ called ranking application :

$$\theta_1(X) = max\{X_j / 1 \leq i \leq P\} = X_{i_1}$$
$$\theta_1(X) = max\{X_j / 1 \leq i \leq P, i \neq i_1, i \neq i_2 \ldots, i \neq i_{j-1}\} = X_{i_j}$$

A nucleolus of a set is coopératif $< I, v, X >$ an imputation $(\bar{X}_i) i \in I$ such that, for any imputation (X_i) $i \in I$ we have:

$$\theta'(v - \bar{X}) \leq \theta(v - x) \quad if\ v(S) - x(S) > 0,$$

the S coalition will feel aggrieved by the imputation xand beneficiary if $v(S) - x(S) < 0$.

$v(S)$- $x(S)$ is a measure of the coalition's dissatisfaction S with the imputation X and $\bar{X}$ minimises the maximum dissatisfaction. Two things make the nucleus particularly interesting. Firstly, it is unique and secondly, if the nucleus is not empty, then the nucleolus belongs to the nucleus. This makes it possible to select an imputation of the nucleus, as it were, if it contained several. In the economic literature on social welfare, there is an analogy to the nucleus offered by Rawls, who proposes, in the absence of reaching the Pareto optimum in some cases, to maximise the utility function of the poorest individual [36].

4. Determining the Equilibrium Threshold in the Distribution of Joint Resources

Shapley value differs from the other solutions, outlined in the previous section, in that it is an a priori valuation of the group rather than the result of the group. This characteristic makes it a solution that, in a way, dispenses with investment. To better understand its motivation.

4.1 The Axiomatic Approach

To determine this value, Shapley adopted the axiomatic approach of selecting, on a conceptual or intuitive basis, the set of desirable properties it should have. In doing so, Shapley stated three axioms and proved the existence of a unique vector $\varnothing(v) = (\varnothing_1(v), \ldots, \varnothing_n(v))$ that satisfies them, where $\varnothing_i(v)$ is the value of the group for member i. The axioms are as follows:

Symmetry axiom: For any automorphism π *of the group* $< I, v, X >$, $\varnothing_i(v) = (\varnothing_{\pi_i})(v)$ $\forall\ i \in I$ *or* π is a permutation of I *in* I. This axiom means that the value assigned to member i depends only on its strategic strength (not on its wording).

Efficiency axiom: $\sum_{i \in I} \varnothing_i(v) = v(I)$ This axiom corresponds to optimality in the Pareto sense.

Linearity axiom: If v and w are two characteristic functions related to the same set of I of members, then $\varnothing_i(v + w) = \varnothing_i(v) + \varnothing_i(w)$ $\forall\ i \in I$

The explicit formula for the Shapley value is given by the following theorem:

The unique vector $\varnothing(v) = (\varnothing_1(v), \ldots, \varnothing_n(v))$ that satisfies these three axioms is given by :

$$\emptyset_i(v) = \sum_{s \ni i} \frac{(n-s)!\,(s-1)!}{n!} (v(s) - v\left(\frac{s}{\{i\}}\right))$$

where n and s represent the total number of members and the number of members belonging to the coalition respectively S. A probabilistic interpretation has been given to this value. Indeed, let us suppose that $i \in S$. The coalition S {i} could have been formed *of* $(s - 1)$! ways and the other members $(n - s)$ can be arranged in $(n - s)$! ways. Thus, the number of ways that player i can join the coalition $S \setminus \{i\}$ is the $(n - s)$! $(s - 1)$!. On the other hand, the grand coalition Ican be formed in n ! ways. Assuming each of them equiprobable then $\frac{(n-s)!(s-1)!}{n!}$ is nothing more than the probability that the member i joins the coalition S. Furthermore,$(v(S) - v(S \setminus \{i\}))$ measures the contribution of the member i to the coalition S and, therefore, $\varnothing_i(v)$ represents an average of its marginal contributions to all coalitions it is likely to join. Uniqueness, symmetry (meaning that two members with the same strategic strength will receive the same payoff) and efficiency make Shapley's value particularly attractive for addressing the problem of equitable sharing of common resources among several economic agents. Indeed, the uniqueness is reassuring at the psychological level insofar as the agent would no longer have to ask himself the question of whether there is a better distribution of these resources, all the more so as this distribution is Pareto-optimal (efficiency axiom) and equitable. Let us point out that this value exists even if the core is empty, and that it coincides with its centre of gravity if the action of the group is convex, i.e. if the characteristic function satisfies the following condition:

$$v(S) + v(T) < v(S \cup T) + v(S \cap T) \forall\ S, T \in P(I).$$

On the other hand, while the first two axioms were favourably received, this was not the case for the linearity axiom, as shown by the criticism of Luce and Raiffa [31]. Their argument is that if a group u is composed of two characteristic functions v and w ($u = v + w$), it cannot be expected to be invested as if it were the sum of two different groups in the sense that it will have its own structure which will determine the payoffs which may differ *from v and w*.

4.2 Reasoning Through the Non-Atomic Games Approach

Aumann and Shapley extended this notion of value to non-atomic (or atomistic) actions, the group characterised by the presence of a continuum of members (rather than a finite number) and whose decisions taken by each member do not influence the others [28]. The analogy with the classical microeconomic model, where a 'large' number of producers and consumers is assumed, is clear. In the following paragraph, I will only introduce the minimum of technical preliminaries necessary to explicitly determine the value of non-atomic shares. Let (I, C) be a measurable space, where I is the set of members and C a ∂-algebra of measurable subsets of I.

The elements of C are called coalitions. A game v on I is a real-valued function on C satisfyingv($(\varnothing)$)= 0. v is a monotone action if $\forall S,T,\in C, S\subset T \Rightarrow v(S)\leq v(T)$. The action v is boundedly variable if it is the difference between two monotone actions. The space of all actions with bounded variation is called BV (Bounded Variation). Let Q be any group space, Q^+ then denote the cone of monotone elements of Q. An application is said to be positive if it returns Q^+ in BV^+. The subspace of BV formed by functions of bounded and additive sets is called FA.

A measure μ is said to be non-atomic if

$$\forall\, \boldsymbol{S \in C, |\mu(S)| > 0 \Rightarrow E \quad T \subset S, T \in C} \text{ and } \boldsymbol{0 < |\mu(T)| < |\mu(S)|.}$$

Let NA be the space of non-atomic measures on(I,C). The subspace of $B\ V$ generated by all powers of $NA^+$is called pNA. On the other hand, let G be the group of automorphisms on (I,C). Each $\theta \in G$ induces a linear application θ_* of BV into itself defined by$(\theta_*\ ,v)(S) = v(\theta\, S)$, $\forall\, S \in C$. A subspace Q of $B\ V$ is said to be symmetric if $\theta_*\ Q = Q$, $\forall\, \theta \in G$. Finally, let Q a symmetric subspace of BV. A value on Q is then a positive linear application $\varnothing$ of Q in FA which satisfies the following two axioms:
Efficiency **axiom**: $(\varnothing(v)\,(I) = v(I) \quad \forall\ v \in Q$
Symmetry axiom : $\varnothing\, \theta_* = \theta_* \varnothing\ \forall\ \theta \in G$

The explicit formula for this value is given by the following theorem: Theorem: There is a unique value $\varnothing$ on pNA. Let μ be a vector of measures in NA and f a continuously differentiable function on the domain R of μ where $f(0) = 0$. Then, $fo\mu \in pNA$ and when R is full dimensional.

$$\emptyset(\boldsymbol{fo\mu}) = \sum_{j=1}^{S} \boldsymbol{\mu_j(s)} \int_0^1 \boldsymbol{f_j(t_\mu(I))\, dt}\ \textbf{; Where} \quad \boldsymbol{f_j = \frac{\partial f}{\partial x_j}}$$

Each active member receives the maximum marginal productivity when all members are employed, and the beneficiary (owner) receives the rest. The aim is therefore not to maximise profit, but to fulfil a social purpose by satisfying economic needs. Identifying trust is not an easy task. The difficulty with this concept lies in its vague nature. As Arrow says, it is an 'invisible institution', which partly explains, in standard economic theory, if not its rejection, at least its substitution by contractual arrangements, the product of individual calculations and interests.

5. Conclusion

Shapley's value gives a little more than 0.5 to the coalition members and a little less than 0.5 to the non-coalition members Game theory has the merit of having provided a language and a framework of thought allowing for in-depth reflection on the concept of rationality. The main conclusion of this reflection is that the confrontation between maximising individuals generates extremely complex and multiple situations which can only be resolved, in the mathematical sense, by calling upon factors external to these individuals, such as social structures (which are reflected in the rules of the game), nonsense or conventions, or notions which do not fall within the framework of what is usually understood by rationality, such as equity or reputation. To characterise the foundations of this "joint production" of supply and demand - a concept already highlighted by Gadrey - the authors of this current propose the notion of "public space of proximity", referring to a voluntary collaboration of people around a social problem and reinforced by a "space of relational reciprocity", as well as to its purposes aiming to promote "social relationships of solidarity" and to consolidate social cohesion. If local services are an important field of experimentation for the solidarity economy, financial services are also the place for such solidarity practices. By combining non-exclusively economic motivations and non-contractual forms of action, solidarity finance operations are another illustration of the re-integration of the market through reciprocity. Based on the concept of "embedding" in interpersonal relations and networks, Granovetter also offers a theoretical perspective on institutions that differs from that proposed by Williamson in that they are social constructions and not mere economic solutions [37- 44].

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