ISSN: 2993-5377

Review Article

Open Access Journal of Applied Science and Technology

Review of the Advanced Waves Inside the Transformer, Antenna and Photon System

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Submitted: 2024, Oct 04; **Accepted:** 2024, Nov 13; **Published:** 2024, Dec 06

Citation: Zhao, S. R. (2024). Review of the Advanced Waves Inside the Transformer, Antenna and Photon System. *OA J Applied Sci Technol, 2*(3), 01-16.

Abstract

Maxwell's equations have two solutions, retarded potential and the other is advanced potential. Physics and electronic industry only accept the retarded wave. Advanced waves are not acceptable. However, a group of scientists believes the advanced wave is an objective existence. These scientists include Wheeler, Feynman, Cramer, etc. The author establishes the mutual energy flow theory of electromagnetic field, which supports the objective existence of advanced wave. This paper reviews the existing advanced wave theory and clarifies the expression form of advanced wave. The retarded wave and advanced wave can be expressed by the same symbol. For example, in quantum mechanics, if the wave function is used to represent the retarded wave, this wave function can also represent the advanced wave, and it is wrong to use the conjugate complex of the wave function to represent the advanced wave. In addition, advanced wave and retarded wave are inseparable. For example, the wave generated by the current element in the waveguide is a Leftward wave and a right wave. The Leftward wave starts as an advanced wave and moves towards the current element. When the Leftward wave crosses the current, it becomes a retarded wave. The rightward wave is the same.

Keywords: Maxwell's Equations, Reciprocity Theorem, Conservation Of Energy, Poynting Theorem, Energy Flow, Transformer, Primary Coil, Secondary Coil, Transmitting Antenna, Receiving Antenna, Retarded Wave, Retarded Potential, Advanced Wave, Advanced Potential, Absorber, Emitter, Photons, Quantum, Electromagnetic Wave, Electromagnetic Field, Transactional Interpretation

1 Introduction

1.1. Solution of Maxwell's Equations

knows that the solutions of Maxwell's equations include retarded wave (or potential) and advanced wave. It is recognized that the retarded wave is the real physical solution. It is generally believed that the advanced wave is a virtual solution, which does not represent the real objective existence.

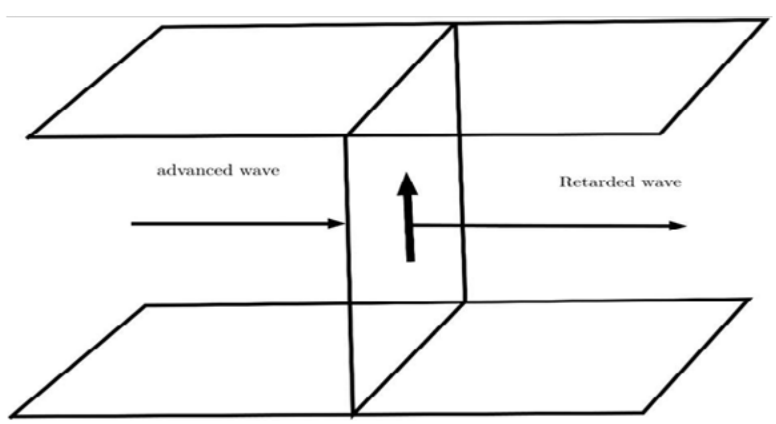

Figure 1: Assume there is a current source inside a wave guide. The current source can produce either a rightward wave or a leftward wave. The above shows a rightward wave. The rightward wave is advanced wave at the left side of the current. The rightward wave is retarded wave on the right side of the current.

1.2. Physical Theory Supporting Advanced Wave

Today's electromagnetic field theory is flawed. It cannot describe photons, and there is no solution to the collapse of waves and the duality of waves and particles. Wheeler Feynman put forward the absorber theory in 1945 [1-2]. This theory advocates that any current source radiates not only retarded waves, but also advanced waves. Wheeler Feynman's absorber theory is based on the principle of action-at-a-distance [15, 17-8]. Before 1980, Stephenson also put forward his own advanced wave theory [16]. Based on the absorber theory, Cramer put forward the transactional interpretation of quantum mechanics 1986 [5-6], arguing that the handshake process between the retarded wave emitted by the light source and the advanced wave emitted by the light sink really determines the electromagnetic radiation.

1.3. Physical Theory Supporting Advanced Wave

Welch proposed the time domain reciprocity theorem in 1960 [18]. This reciprocity theorem involves advanced waves. As a reciprocity theorem, there is a advanced wave, which can be a virtual quantity. In 1987, the author proposed the mutual energy theorem [9, 19, 20]. Starting from the mutual energy theorem, the author developed the electromagnetic mutual energy theory in 2017, including the law of conservation of energy, the principle of mutual energy, the law of energy does not overflow the universe, and the theorem of mutual energy flow [10-13]. According to the theory of mutual energy, the phenomenon of electromagnetic radiation is the interaction between radiation source and sink. The source randomly radiates the retarded wave and the sink randomly radiates the advanced wave. When the retarded wave moves to the sink, if the sink just sends out the advanced wave, the advanced wave can be synchronized with the retarded wave. This synchronization process results in mutual energy flow. Mutual energy flow transfers energy from the source to the sink. So, photon is the mutual energy flow.

1.4. Theory of Advanced Wave

This paper studies how to express the advanced wave. For example, for electromagnetic wave, the retarded wave and the advanced wave can be expressed by the retarded and advanced potential through current elements J(t-r/c), J(t+r/c). But for Schrodinger equation, there is generally no source, so how to express the advanced wave? It is often seen that many authors consider the conjugate complex ψ^* of wave function ψ as advanced wave. But the author found that this is wrong. So how should the advanced wave be expressed for the Schrodinger equation? Therefore, there must be a correct expression of advance wave. Another example. Wheeler Feynman's absorber theory proposes that the current source radiates half retarded wave and half advanced wave, but how to realize this half-retarded wave and half-advanced wave. The author will consider a conical waveguide to study this phenomenon.

2. Advanced Waves

2.2. Maxwell's Equations

As you know, Maxwell's equations in vacuum is,

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0 \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B} \tag{3}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \epsilon_0 \frac{\partial}{\partial t}\mathbf{E} \tag{4}$$

where, **E** is the electric field intensity, **D** is the electric displacement, **H** is the magnetic field intensity, **B** is the magnetic induction intensity, **J** is the current density, and ρ is the charge density. $\mathbf{B} = \mu_0\mathbf{H}$. $\mathbf{D} = \epsilon_0\mathbf{E}$.

2.3. Wave Equation

Considering $\nabla \cdot \mathbf{B} = 0$,

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{5}$$

Considering $\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$, and hence,

$$\mathbf{E} = -\frac{\partial}{\partial t}\mathbf{A} - \nabla\phi \tag{6}$$

ϕ is the scalar potential, consider the Lorenz gauge condition,

$$\nabla \cdot \mathbf{A} = -\mu_0\epsilon_0\frac{\partial}{\partial t}\phi \tag{7}$$

the wave equation can be deduced, considering $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ there is,

$$\nabla^2\phi - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\phi = -\rho/\epsilon_0 \tag{8}$$

Considering Maxwell-Ampere circular law $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D}$, and Lorenz gauge condition, there is

$$\nabla^2\mathbf{A} - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\mathbf{A} = -\mu_0\mathbf{J} \tag{9}$$

The above are two wave equations.

2.4. The Solution of the Wave Equations

There are two solutions to the waveguide equation, one is retarded potential,

$$\mathbf{A}^{(+)}(\mathbf{x}, t) = \frac{\mu_0}{4\pi}\iiint_V \frac{\mathbf{J}(\mathbf{x}', t - r/c)}{r} dV \tag{10}$$

$$\phi^{(+)}(\mathbf{x}, t) = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\mathbf{x}', t - r/c)}{r} dV \tag{11}$$

$$\mathbf{r} = \mathbf{x} - \mathbf{x}', \quad r = \sqrt{|\mathbf{r}|}, \quad c = \frac{1}{\sqrt{\mu_0\epsilon_0}} \tag{12}$$

(12) The other is the advanced potential,

$$\mathbf{A}^{(-)}(\mathbf{x}, t) = \frac{\mu_0}{4\pi}\iiint_V \frac{\mathbf{J}(\mathbf{x}', t + r/c)}{r} dV \tag{13}$$

$$\phi^{(-)}(\mathbf{x}, t) = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\mathbf{x}', t + r/c)}{r} dV \tag{14}$$

For AC current $\mathbf{J} = \mathbf{J}_0 \exp(j\omega t)$

$$\mathbf{A}^{(+)}(\mathbf{x},t) = \frac{\mu_0}{4\pi}\iiint_V \frac{\mathbf{J}_0(\mathbf{x}')\exp\big(j\omega(t-r/c)\big)}{r}dV$$
$$= \frac{\mu_0}{4\pi}\iiint_V \frac{\mathbf{J}_0(\mathbf{x}')\exp(j\omega t)\exp(-j\omega r/c)}{r}dV$$
$$= \frac{\mu_0}{4\pi}\iiint_V \frac{\mathbf{J}_0(\mathbf{x}')\exp(j\omega t)\exp(-jkr)}{r}dV$$
$$= \frac{\mu_0}{4\pi}\iiint_V \frac{\mathbf{J}(\mathbf{x}',t)\exp(-j\mathbf{k}\cdot\mathbf{r})}{r}dV \tag{15}$$

Similarly

$$\phi^{(+)}(\mathbf{x},t) = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\mathbf{x}',t)}{r}\exp(-j\mathbf{k}\cdot\mathbf{r})dV \tag{16}$$

$$k = \frac{\omega}{c}, \quad \mathbf{k} = k\hat{n}, \quad \hat{n} = \frac{\mathbf{r}}{r} \tag{17}$$

Therefore, the retarded potential is to add a retarded factor exp$(-j\ \mathbf{k}\cdot\mathbf{r})$ to the non retarded potential. Similarly, the advanced potential is to add an advanced factor exp $(+J\ \mathbf{k}\cdot\mathbf{r})$ to the non-retarded potential. So, the advanced potential is,

$$\mathbf{A}^{(-)}(\mathbf{x},t) = \frac{\mu_0}{4\pi}\iiint_V \frac{\mathbf{J}(\mathbf{x}',t)\exp(+j\mathbf{k}\cdot\mathbf{r})}{r}dV \tag{18}$$

$$\phi^{(-)}(\mathbf{x},t) = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\mathbf{x}',t)}{r}\exp(+j\mathbf{k}\cdot\mathbf{r})dV \tag{19}$$

2.5. Conjugate Transformation

Both retarded potential and advanced potential satisfy the wave equation and Maxwell's equations. The retarded potential and advanced potential can be transformed into each other by conjugate transformation. The conjugate transformation is defined as follows,

$$\big(\mathbf{E}(t),\mathbf{H}(t),\rho(t),\mathbf{J}(t)\big)$$
$$\rightarrow \big(\mathbf{E}_{new}(-t),-\mathbf{H}_{new}(-t),\rho(-t),-\mathbf{J}(-t)\big) \tag{20}$$

In the frequency domain,

$$\big(\mathbf{E}(\omega),\mathbf{H}(\omega),\rho(\omega),\mathbf{J}(\omega)\big)$$
$$\rightarrow \big(\mathbf{E}^*_{new}(\omega),-\mathbf{H}^*_{new}(\omega),\rho^*(\omega),-\mathbf{J}^*(\omega)\big) \tag{21}$$

Considering the Maxwell's equations in frequency domain,

$$\begin{cases}\nabla \times \mathbf{E} = -j\omega\mathbf{B} \\ \nabla \times \mathbf{H} = \mathbf{J} + j\omega\mathbf{D}\end{cases} \tag{22}$$

Do conjugate transformation and get

$$\begin{cases}\nabla \times \mathbf{E}_{new}^{*} = -j\omega(-\mathbf{B}_{new}^{*}) \\ \nabla \times (-\mathbf{H}_{new}^{*}) = (\ \ \mathbf{J}\ \ )^{*} + j\omega\mathbf{D}^{*}\end{cases} \tag{23}$$

$$\begin{cases}\nabla \times \mathbf{E}_{new} = -j\omega\mathbf{B}_{new} \\ \nabla \times \mathbf{H}_{new} = \mathbf{J} + j\omega\mathbf{D}\end{cases} \tag{24}$$

This shows that the quantity that originally satisfies Maxwell's equations after conjugate transformation still satisfies Maxwell's equations.

2.6. Conversion of Advanced Potential and Retarded Potential

We prove that after conjugate transformation, the retarded wave becomes the advanced wave and the advanced wave becomes the retarded wave. Consider,

$$\mathbf{E}^{(+)} = -j\omega\mathbf{A}^{(+)} - \nabla\phi^{(+)} \tag{25}$$

$$\mathbf{E}^{(+)} = -j\omega\frac{\mu_0}{4\pi}\iiint_V \frac{\mathbf{J}(\mathbf{x}', t)\exp(-j\mathbf{k}\cdot\mathbf{r})}{r}dV - \nabla\frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\mathbf{x}', t)}{r}\exp(-j\mathbf{k}\cdot\mathbf{r}) \tag{26}$$

$$B^{(+)} = \nabla \times \mathbf{A}^{(+)} \tag{27}$$

$$B^{(+)} = \nabla \times \frac{\mu_0}{4\pi}\iiint_V \frac{\mathbf{J}(\mathbf{x}', t)\exp(-j\mathbf{k}\cdot\mathbf{r})}{r}dV \tag{28}$$

The conjugate transformation of the above formula is obtained

$$\left(\mathbf{E}_{new}^{(+)}\right)^{*} = -j\omega\frac{\mu_0}{4\pi}\iiint_V \frac{-\mathbf{J}(\mathbf{x}', t)^{*}\quad\exp(-j\mathbf{k}\cdot\mathbf{r})}{r}dV - \nabla\frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\mathbf{x}', t)^{*}}{r}\exp(-j\mathbf{k}\cdot\mathbf{r}) \tag{29}$$

The complex conjugate of the above formula is obtained

$$\left(\mathbf{E}_{new}^{(+)}\right) = -j\omega\frac{\mu_0}{4\pi}\iiint_V \frac{\mathbf{J}(\mathbf{x}',t) \quad \exp(+j\mathbf{k}\cdot\mathbf{r})}{r}dV$$

$$-\nabla\frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\mathbf{x}',t)}{r}\exp(+j\mathbf{k}\cdot\mathbf{r}) \tag{30}$$

Conjugate transformation of magnetic field

$$-\boldsymbol{B}_{new}^{(+)*} = \nabla\times\frac{\mu_0}{4\pi}\iiint_V \frac{-\mathbf{J}^*(\mathbf{x}',t)\exp(-j\mathbf{k}\cdot\mathbf{r})}{r}dV \tag{31}$$

The complex conjugate of the above formula is obtained,

$$\boldsymbol{B}_{new}^{(+)} = \nabla\times\frac{\mu_0}{4\pi}\iiint_V \frac{\mathbf{J}(\mathbf{x}',t)\exp(+j\mathbf{k}\cdot\mathbf{r})}{r}dV \tag{32}$$

It can be seen from the above that after conjugate transformation, the original retarded wave $\mathbf{E}^{(+)}, \boldsymbol{B}^{(+)}$, after the conjugate transform become $\mathbf{E}_{new}^{(+)}, \boldsymbol{B}_{new}^{(+)}$, Which are advanced wave.

2.7. Lorentz Reciprocity Theorem and Mutual Energy Theorem

We know Lorentz reciprocity theorem [3][4],

$$\iiint_{V_1} \mathbf{E}_2(\omega)\cdot\mathbf{J}_1(\omega)dV = \iiint_{V_2} \mathbf{E}_1(\omega)\cdot\mathbf{J}_2(\omega)dV \tag{33}$$

In this theorem, two field quantities $\xi_1 = [\mathbf{E}_1, \mathbf{H}_1, \mathbf{J}_1]^T, \xi_2 = [\mathbf{E}_2, \mathbf{H}_2, \mathbf{J}_2]^T, T$ is matrix transfer. They are all retarded waves. Obtained from the conjugate transformation of the quantity of subscript 2 in Lorentz reciprocity theorem,

$$\iiint_{V_1} \mathbf{E}_{new2}^*(\omega)\cdot\mathbf{J}_1(\omega)dV = \iiint_{V_2} \mathbf{E}_1(\omega)\cdot\left(-\mathbf{J}^*(\omega)\right)dV \tag{34}$$

Remove the subscript new and get,

$$-\iiint_{V_1} \mathbf{E}_2^*(\omega)\cdot\mathbf{J}_1(\omega)dV = \iiint_{V_2} \mathbf{E}_1(\omega)\cdot\mathbf{J}_2^*(\omega)dV \tag{35}$$

This is the law of conservation of energy or the mutual energy theorem [9][20][19] , which is also Rumsey's new reciprocity theorem [14]. The following can be obtained by inverse Fourier transform through the transform of $\omega \to t$,

$$-\int_{t=-\infty}^{\infty} d\,t\iiint_{V_1} \mathbf{E}_2(t)\cdot\mathbf{J}_1(t+\tau)dV = \int_{t=-\infty}^{\infty} d\,t\iiint_{V_2} \mathbf{E}_1(t+\tau)\cdot\mathbf{J}_2(t)dV \tag{36}$$

This is de Hoop's correlation reciprocity theorem [7]. If $\tau = 0$ is taken, Welch's time domain reciprocity theorem [18] can be obtained,

$$-\int_{t=-\infty}^{\infty} d\,t\iiint_{V_1} \mathbf{E}_2(t)\cdot\mathbf{J}_1(t)dV = \int_{t=-\infty}^{\infty} d\,t\iiint_{V_2} \mathbf{E}_1(t)\cdot\mathbf{J}_2(t)dV \tag{37}$$

It can be seen that the four theorems, Welch reciprocity theorem, Rumsey reciprocity theorem, de Hoop reciprocity theorem and the author's mutual energy theorem are a formula that can be connected by Fourier transform. The author thinks it is the energy theorem, others think it is just a reciprocity theorem. As the reciprocity theorem, the two waves in this formula correspond to the subscript 1 or 2, only one of them is a real physical wave, and the other can be virtual or mathematical. As an energy theorem, it is required that the quantities corresponding to the two subscripts 1 and 2 are all physically real quantities. Because this formula involves the advanced wave. If this theorem is an energy theorem, we must first admit that the advanced wave is a real physical objective existence. The author supports that the advanced wave is a real physicalobjective existence. If it is the reciprocity theorem, even if it does not recognize that the advanced wave is a physical objective existence, this theorem can be applied as some kinds of Green's function.

3. Theory and Application of Advanced Wave

In order to study the properties of advanced wave, biconical waveguide is considered see Figure 1. Considering the retarded wave and advanced wave in the double cone, it is assumed that there are the following double cones. Assume that there is a current element

$$\mathbf{J} = \hat{y}\delta(x-0)\delta(y-0)\delta(z-0)\delta(t-0) \tag{38}$$

at the coordinate origin **O** and time $t = 0$. This current will produce a leftward wave and a rightward wave. The rightward wave moves from $x = -\infty$ to $x = 0$, at the time from $t = -\infty$ to $t = 0$, that is the advanced wave. After that, the rightward wave is converted into a retarded wave. The retarded wave occurs from $x = 0$ to $x = \infty$, the time is from $t = 0$ to $t = \infty$. Of course, this current also generates a leftward wave, which begins as the advanced wave from $t = -\infty$, $x = \infty$ to $t = 0$ and $x = 0$. After sweeping the current **J**, the leftward wave becomes a retarded wave and moves from $t = 0$, $x = 0$, $t = \infty$, $x = -\infty$. Figure 1.

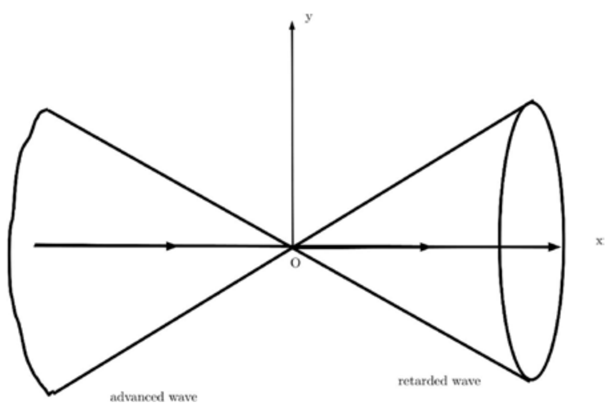

Figure 1: The Rightward and Leftward Waves in Conical Space.

A special case is the transmission line model shown in the figure 2 below. There is a current J in the middle of the transmission line. This current generates a rightward wave and a leftward wave. The rightward wave begin as the advanced wave from $x = -\infty$ $t = -\infty$ on the left to $t = 0$, $x = 0$, that is, at the current, and then the rightward wave is transformed into a retarded wave. The wave continues to move to $t = \infty$, $x = \infty$. Of course, there will also leftward wave. The current will produce both righward wave and leftward wave and they are composed of retarded wave and advanced wave. Wheeler and Feynman put forward the absorber theory [1-2]. Wheeler Feynman believes that current will produce half retarded wave and half advanced wave. The author further details Wheeler Feynman's theory and holds that for a conical waveguide, the current will produce a rightward wave (positive wave) and a leftward wave (negative wave). Among them, the positive wave and negative wave respectively include a advanced wave process and a retarded wave process. It is noticed that if the conical angle increased to 2π, the conical waveguide has fill full the whole space. If the conical angle increase to 4π, we have obtained two 3D space, one is for positive time, the other is for negative time.

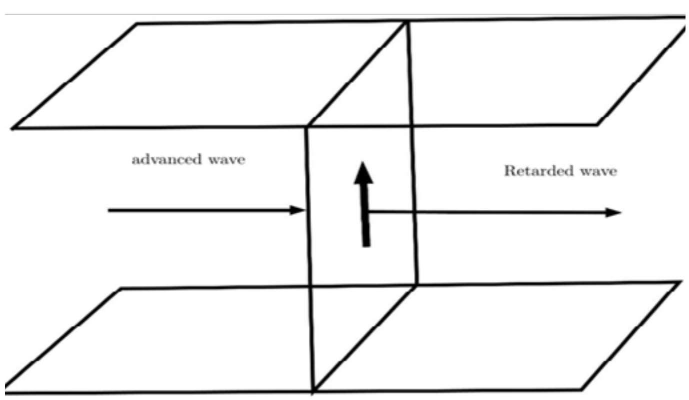

Figure 2: On Both Sides Of The Current In The Waveguide, The Both Positive And Negative Waves Change The Advanced Wave To The Retarded Wave.

If the opening solid angle of the cone is equal to 4π, the above biconical space is the three-dimensional real space. At this time, we can actually think that there are two three-dimensional spaces, one is the three-dimensional space of t ≥ 0, and the other is the three-dimensional space of t < 0. The wave in three-dimensional space with t < 0 is the advanced wave. t > 0 The space of is a retarded wave. The retarded potential can be written as,

$$\mathbf{A}^{(+)} = \frac{\mu}{4\pi}\iiint \frac{1}{r}\mathbf{J}(x, t - r/c)dV, \qquad t \geq 0 \tag{39}$$

The advanced wave can be written,

$$\mathbf{A}^{(-)} = \frac{\mu}{4\pi}\iiint \frac{1}{r}\mathbf{J}(x, t + r/c)dV \qquad t \geq 0 \tag{40}$$

But we can also use the following formula to express the retarded potential and advanced potential at the same time,

$$\mathbf{A} = \frac{\mu}{4\pi}\iiint \frac{1}{r}\mathbf{J}(x, t - r/c)dV \qquad -\infty < t < \infty \tag{41}$$

However, the current occurs when t = 0.

$$\mathbf{J} = \delta(t - 0)\delta(\mathbf{x} - \mathbf{0}) \tag{42}$$

And time t can be positive and negative. Positive time represents retarded potential and negative time represents advanced potential. In this way, we can avoid the trouble of writing the retarded potential and advanced potential into two formulas. For the vector potential function in electromagnetics, even if it is limited to t ≥ 0, we have a way to distinguish the retarded potential from the advanced potential (10, 11), but for the Schrodinger equation of quantum mechanics, there is no better way to represent the advanced wave.

The figure 2 tells us that the advanced wave and the retarded wave often form a wave moving to the left or to the right. A wave moving to the left can contain an advanced wave and a retarded wave. A wave moving to the right can contain an advanced wave and a retarded wave. This is of great significance for the author to realize transactional interpretation of Cramer's quantum mechanics In the future [5-6].

3.2. Advanced Wave Corresponding to Schrodinger Equation

In the electromagnetic field theory, the transformation of the retarded wave into the advanced wave is the conjugate transformation formula (20, 21). If we use ψ (x, t) to represent the retarded wave, many people think ψ(x, t)* is the advanced wave in quantum mechanics. The author believes that this is wrong. To be exact, ψ (x, t)* is a time reversal wave. For Schrodinger equation, we often consider the following conjugate transformation for Schrodinger equation,

$$\Psi(\mathbf{x}, t) \to \Psi_{new}^{*}(\mathbf{x}, t), \frac{\partial}{\partial t} \to -\frac{\partial}{\partial t} \tag{43}$$

We know Schrodinger equation is,

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{x}, t) - V\Psi(\mathbf{x}, t) = i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x}, t) \tag{44}$$

Considering the conjugate transform,

$$-\frac{\hbar^2}{2m}\nabla^2\Psi_{new}^{*}(\mathbf{x}, t) - V\Psi_{new}^{*}(\mathbf{x}, t) = \Upsilon\ i\hbar\frac{\partial}{\partial t}\Psi_{new}^{*}(\mathbf{x}, t) \tag{45}$$

Find the complex conjugate on both sides of the above formula,

$$-\frac{\hbar^2}{2m}\nabla^2\Psi_{new}(\mathbf{x}, t) - V\Psi_{new}(\mathbf{x}, t) = (i\hbar)^{*}\frac{\partial}{\partial t}\Psi_{new}(\mathbf{x}, t) \tag{46}$$

Note that $(i\hbar)^* = -i\hbar$. Therefore $\Psi_{new}(x, t)$ does satisfy the Schrodinger equation. Therefore, the conjugate transformation (43) is an advanced wave for the Schrodinger equation. Therefore, the advanced wave keep can be obtained by conjugate transformation for Schrodinger equation. When we allow the time to be negative, we can use $\Psi(x, t)$, $t < 0$ to represent the advanced wave. So, for Schrodinger equation, we have at least one way to represent the advanced wave. In short, for the Schrodinger equation, the wave function $\Psi(x, t)$ can represent either the retarded wave or the advanced wave, but it depends on the sign of time t.

3.3. Inner Product

We consider the inner product in the path integral of quantum mechanics.

$$(\psi_2, \psi_1) = \iiint_V \psi_2^* \psi_1 dV \qquad (47)$$

ψ_1 is the wave emitted by point I, ψ_2 it's a wave from point F. I stands for initial point and F stands for final point. ψ_1 and ψ_2 are two wave functions moving in the same direction. It can be two retarded waves or two advanced waves. Of course, it can also be one retarded wave and one advanced wave. The two retarded waves emitted from the same point can be synchronized, and the two advanced waves emitted from the same point can be synchronized. However, two retarded waves from the different points cannot be synchronized, and two advanced waves from different points cannot be synchronized. Because of the two waves here ψ_1 and ψ_2 are waves from different points, so they must be a retarded wave and an advanced wave. Let's assume ψ_2 is retarded wave, ψ_2 is a advanced wave. There is ψ_2^* in the above formula, here ψ_2 has the conjugate sign *. It is not because the conjugate sign, ψ_2 is the advance wave. Here Ψ_2 is the advanced wave is because ψ_1 is a retarded wave, and ψ_2 is a wave emitted from different points. It can only be a advanced wave is because ψ_1 is a retarded wave, and ψ_2 is a wave emitted from different points. It can only be a advanced wave, otherwise it cannot be synchronized with the retarded wave 1. The conjugate sign * belong to the inner product formula. See the following figure 3.

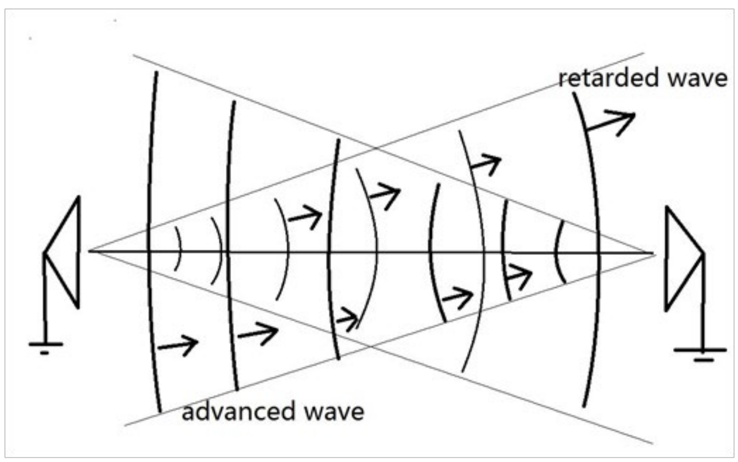

Figure 3: In Vacuum, The Wave Emitted From Two Different Points Can Only Be Synchronized With A Retarded Wave And An Advanced Wave.

3.4. The Waves of a Plane Sheet Current

A plane sheet current can produce two kinds of waves: retarded wave and advanced wave. The magnetic field can be obtained by using the Maxwell-Ampere circuital law. In Figure 4 example shows that the current is point direction. Magnetic field close to the current is at the direction $-x$. The electric field has two possibilities. If E has the direction of $-\hat{z}$, the Poynting vector $E \times H^*$ is at y direction, this is the retarded wave. But the electric field can also at the direction $\hat{z}$. In this case the Poynting vector is at the direction $-y$, this is the advanced wave. This is correct, if current direction is same with the electric field, that means the power ($\iint E \cdot J^* dS$) is positive, the current will consume the power. If the electric field has opposite direction with the current. This power of the current is negative, that means the current produce some power. The current produces the power will create a retarded wave. The current consumes the power will produce the advanced wave. Hence this figure clear tells us a current can produce the retarded wave and the advanced wave in the same time.

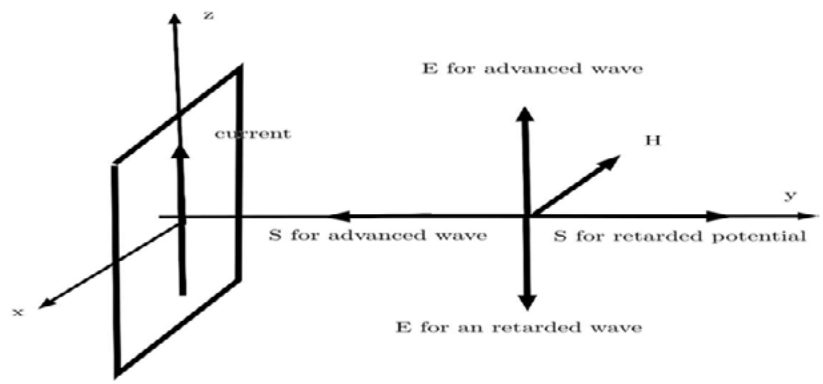

Figure 4: A Current Can Produce Retarded Wave and Advanced Wave in the Same Time

4 Mutual Energy Theory

4.1. Law of Conservation of Energy

The above formula (37) can be rewritten as,

$$\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2}\int_{t=-\infty}^{\infty}d\,t\iiint_{V_j}\mathbf{E}_i\,(t)\cdot\mathbf{J}_j(t)dV=0 \tag{48}$$

The above formula can be extended to,

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{t=-\infty}^{\infty}d\,t\iiint_{V_j}\mathbf{E}_i\,(t)\cdot\mathbf{J}_j(t)dV=0 \tag{49}$$

The above formula is the energy conservation law of N current elements. The author believes that the above formula is self-evident. If charge i gives charge j some energy, the energy of charge j increases and the energy of charge i decreases, but the total energy remains unchanged. Therefore, it can be applied as an additional axiom of Maxwell's electromagnetic field theory.

4.2. Mutual Energy Principle

In electromagnetic theory, it is assumed that there are two current elements **J**i, i = 1,2. Maxwell's equations can be written

$$L\xi_i=\tau_i,\qquad i=1,2 \tag{50}$$

As, where,

$$L=\begin{bmatrix}-\epsilon_0\dfrac{\partial}{\partial t}, & \nabla\times\\ -\nabla\times & -\mu_0\dfrac{\partial}{\partial t}\end{bmatrix},\quad \xi_i=[\mathbf{E}_i,\mathbf{H}_i]^T,\quad \tau_i=[\mathbf{J}_i,0]^T \tag{51}$$

A mathematical formula similar to Green's function can be proved,

$$-\oiint_{\Gamma}(\mathbf{E}_1\times\mathbf{H}_2+\mathbf{E}_2\times\mathbf{H}_1)\cdot\hat{n}d\Gamma$$

$$=\iiint_V(\,\xi_1\cdot L\xi_2+\xi_2\cdot L\xi_1$$

$$+\mathbf{E}_1\cdot\frac{\partial}{\partial t}\mathbf{D}_2+\mathbf{E}_2\cdot\frac{\partial}{\partial t}\mathbf{D}_1+\mathbf{H}_1\cdot\frac{\partial}{\partial t}\mathbf{B}_2+\mathbf{H}_2\cdot\frac{\partial}{\partial t}\mathbf{B}_1)dV \tag{52}$$

By substituting it the Maxwell's equations Eq. (50) into the above formula, we obtain,

$$-\oiint_{\Gamma}(\mathbf{E}_1\times\mathbf{H}_2+\mathbf{E}_2\times\mathbf{H}_1)\cdot\hat{n}d\Gamma$$
$$=\iiint_V(\mathbf{E}_1\cdot\mathbf{J}_2+\mathbf{E}_2\cdot\mathbf{J}_1+\mathbf{E}_1\cdot\frac{\partial}{\partial t}\mathbf{D}_2+\mathbf{E}_2\cdot\frac{\partial}{\partial t}\mathbf{D}_1+\mathbf{H}_1\cdot\frac{\partial}{\partial t}\mathbf{B}_2+\mathbf{H}_2\cdot\frac{\partial}{\partial t}\mathbf{B}_1)dV \quad (53)$$

The above formula is called the principle of mutual energy. There are two waves in the principle of mutual energy, one is retarded wave and the other is advanced wave. These two waves must exist at the same time. Just now we deduced the principle of mutual energy from Maxwell's equations. In fact, it is not difficult to deduce Maxwell's equations from the principle of mutual energy. However, the principle of mutual energy always corresponds to two sets of Maxwell's equations, one of which is the solution of retarded wave and the other is the solution of advanced wave.

The author believes that the solutions satisfying a set of Maxwell's equations can be retarded waves or advanced waves, which are not the solutions of the principle of mutual energy. The theory of mutual energy holds that only the solution of the principle of mutual energy corresponds to the physical radiation phenomenon. Although a single retarded wave or a advanced wave satisfies Maxwell's equations, it is still an invalid solution.

This wave is emitted but has no physical effect. This is because either there is a time reversal wave for this wave to offset it [10], or the energy of this wave is reactive power, so the energy is returned while radiating.
Therefore, the solution of the principle of mutual energy is much less than that of Maxwell's equations. The principle of mutual energy plays a role in selecting part of Maxwell's equations as electromagnetic radiation phenomenon. Therefore, the principle of mutual energy further restricts the solutions of Maxwell's equations. Therefore, compared with Maxwell's equations, the principle of mutual energy has its own significance as a new axiom independent of Maxwell's equations. Therefore, the author calls the principle of mutual energy rather than some theorem. Of course, the principle of mutual energy can replace the two main Maxwell's equations (50).

The principle of mutual energy can be extended to the case of N current elements,

$$-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\oiint_{\Gamma}(\mathbf{E}_i\times\mathbf{H}_j)\cdot\hat{n}d\Gamma=\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\iiint_V\left(\mathbf{E}_i\cdot\mathbf{J}_j+\mathbf{E}_i\cdot\frac{\partial}{\partial t}\mathbf{D}_j+\mathbf{H}_i\cdot\frac{\partial}{\partial t}\mathbf{B}_j\right)dV \quad (54)$$

The above mutual energy principle is equivalent to N sets of Maxwell's equations,

$$L\xi_i=\tau_i, \qquad i=1,2\cdots\cdots N \quad (55)$$

The above equivalence means that they are necessary and sufficient conditions for each other. Note that the requirement is N sets of Maxwell's equations, $N \geq 2$. That is $N \neq 1$. We know that $N = 1$ is still the solution of Maxwell's equations, but it is not the solution of mutual energy principle. The solution of the mutual energy principle requires at least $N = 2$, so there can be a retarded wave and an advanced wave. The retarded wave and advanced wave must be synchronized to be the solution of the mutual energy principle. Therefore, the solution of the principle of mutual energy is much less than that of Maxwell's equations. The solution of the principle of mutual energy is the real physical solution of the problem of electromagnetic radiation. The solution of Maxwell's equations is only a possible solution or a probabilistic solution.

The following figure 5 shows the relationship between the principle of mutual energy and Maxwell's equations.

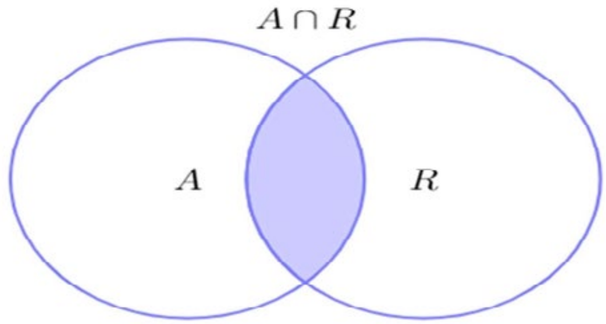

Figure 5: A is All the Solutions of Advanced Wave. R is All the Solutions of the Retarded Wave. A ∩ R is the Solutions of the Mutual Energy Principle. A ∪ R is the Solutions for Maxwell's Equations.

The number of solutions of the mutual energy principle is much less than that of the Maxwell's equations. That means the mutual energy principle does not exactly same as Maxwell's equations. The mutual energy principle Eq.(53) is a independent physics law compare to Maxwell's equations. The mutual energy principle describes a new phenomenon of physics.

4.3. The Proof that the Advanced Wave is Real

Considering that,

$$\int_{-\infty}^{\infty} d\,t \iiint_V \left(\mathbf{E}_i \cdot \mathbf{J}_j + \mathbf{E}_i \cdot \frac{\partial}{\partial t} \mathbf{D}_j + \mathbf{H}_i \cdot \frac{\partial}{\partial t} \mathbf{B}_j \right) dV = \int_{-\infty}^{\infty} d\,t \frac{\partial}{\partial t} U = U(\infty) - U(-\infty) = 0 \tag{56}$$

Where

$$U = \mathbf{E}_i \cdot \mathbf{D}_j + \mathbf{H}_i \cdot \mathbf{B}_j \tag{57}$$

is the mutual energy in the space. U (∞) is the energy the process has finished. U (−∞) is the energy before the process. These two energies should be equal.

Substitute (49) and (56) to the time integral of the mutual energy principle [54], we obtained,

$$\sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \int_{-\infty}^{\infty} d\,t \oiint_{\Gamma} \left(\mathbf{E}_i \times \mathbf{H}_j \right) \cdot \hat{n} d\Gamma = 0 \tag{58}$$

The above formula tells us there is not mutual energy can flow to the outside of the surface Γ. Here Γ is any surface surrounding all the sources and sinks. We can take the surface Γ as sphere surface with infinite radius. In this case, if for ξ_i, ξ_j, one is retarded wave and the other is advanced wave, it can be proved that the above formula can be zero, that is because a advanced wave and a retarded wave do not reach the sphere in the same time. One is in the past and is in the future. In case ξ_i, ξ_j are two retarded wave or two advanced waves, in general, the above formula cannot be zero. This also means the advanced wave must exist otherwise, the mutual energy principle cannot be established.

Many scientists and engineer deny the existence of the advanced wave, they will deny the mutual energy principle(54) and the energy conservation law (49) too. The author set the energy conservation law and the mutual energy principle as axioms of electromagnetic field theory, in this case the advanced wave can be proved as a real objective existence. The first person introduce this kind proof is Welch. He applied this method to prove his time-domain reciprocity from Maxwell's equations [18]. Even he can prove his time domain reciprocity theorem, he still cannot call his theorem a energy theorem. That is because inside the frame of Maxwell's theory, the advanced wave is real or not cannot be sure. The author support the concept of the advanced wave is real thing. Hence the author add more axioms to the theory of electromagnetic field.

4.4. The Second Proof that the Advanced Wave is Real

The concept of advanced waves is very difficult to be accept, since it violates causality. Here, a second proof about the existence of the advance waves is offered. In case, there is only two current elements J_1, J_2. For example, J_1 is the current intensity for the primary coil of a transformer; J_2 is the current intensity for the secondary coil of the transformer. The energy conservation law is (35), see Figure 6,

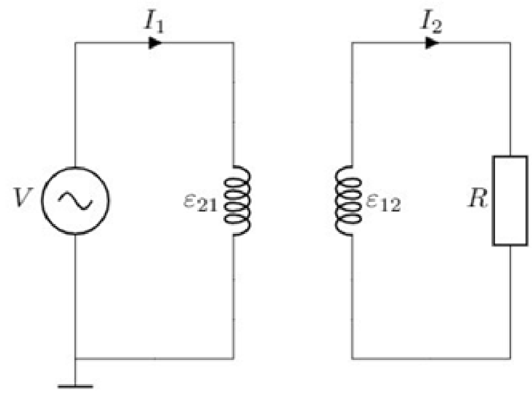

Figure 6: A Transformer with a Primary Coil and Secondary Coil.

$$-\iiint_{V_1} \mathbf{E}_2^*(\omega) \cdot \mathbf{J}_1(\omega) dV = \iiint_{V_2} \mathbf{E}_1(\omega) \cdot \mathbf{J}_2^*(\omega) dV \tag{59}$$

$$-\int_{C_1} \mathbf{E}_2^*(\omega) \cdot d\mathbf{l} I_1(\omega) = \int_{C_2} \mathbf{E}_1(\omega) \cdot d\mathbf{l} I_2^*(\omega) \tag{60}$$

$$-\mathcal{E}_2^* I_1(\omega) = \mathcal{E}_1 I_2^*(\omega) \tag{61}$$

there is

$$\mathcal{E}_2 = -\frac{\partial}{\partial t} M_{1,2} I_2 = -j\omega M_{1,2} I_2 \tag{62}$$

and

$$\mathcal{E}_1 = -\frac{\partial}{\partial t} M_{2,1} I_1 = -j\omega M_{2,1} I_1 \tag{63}$$

Substitute the above two formula to Eq. (60), there is,

$$-\left(-j\omega M_{1,2} I_2\right)^* I_1(\omega) = \left(-j\omega M_{2,1} I_1\right) I_2^*(\omega) \tag{64}$$

or

$$\left(M_{1,2}\right)^* I_2{}^* I_1 = \left(M_{2,1}\right) I_1 I_2^* \tag{65}$$

or

$$\left(M_{1,2}\right)^* = M_{2,1} \tag{66}$$

The above formula is the energy conservation law for the transformer system include a primary coil and secondary coil. It is also suitable to the antenna system which includes a transmitting antenna and a receiving antenna. For a transformer,

$$M_{2,1} = \frac{\mu_0}{4\pi}\int_{C_2}\int_{C_1}\frac{dI_1 \cdot dI_2}{r} \quad (66)$$

$$M_{1,2} = \frac{\mu_0}{4\pi}\int_{C_1}\int_{C_2}\frac{dI_1 \cdot dI_2}{r} \quad (67)$$

$$M_{2,1} = M_{1,2} = real \quad (68)$$

Considering the above formula, Eq. (65) can be satisfied. If the we move the secondary coil to a distance away, the retardation should be considered. In this case, see Figure 7,

Figure 7: A transformer with a Primary Coil and Secondary Coil but the secondary Coil is Far Away from the Primary Coil. In this Case the Primary Coil Become a Transmitting Antenna. The Secondary Coil become a Receiving Antenna.

$$M_{2,1} = \frac{\mu_0}{4\pi}\int_{C_2}\int_{C_1}\frac{d\mathbf{l}_1 \cdot dl_2}{r}\exp(-j\mathbf{k}\cdot\mathbf{r}) \quad (69)$$

exp(−jk · r) is the result to considered the vector potential of A1 inside of $M_{1,2}$ has to consider as retarded potential

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi}\int_{C_1}\frac{I_1 d\mathbf{l}_1}{r}\exp(-jkr) \quad (70)$$

In this case $M_{1,2}$ should considered as,

$$M_{1,2} = \frac{\mu_0}{4\pi}\int_{C_1}\int_{C_2}\frac{d\mathbf{l}_2 \cdot dl_1}{r}\exp(+j\mathbf{k}\cdot\mathbf{r}) \quad (71)$$

In this case the vector potential insider of has to consider as advanced potential,

$$\mathbf{A}_2 = \frac{\mu_0}{4\pi}\int_{C_2}\frac{I_2 d\mathbf{l}_2}{r}\exp(+j\mathbf{k}\cdot\mathbf{r}) \quad (72)$$

Eq. (69, 71) can satisfy Eq. (65). This means if the primary coil (or transmitting antenna) sends the retarded wave, the secondary coil (a receiving antenna) has to send the advanced wave, otherwise the energy conservation law (65) or (60) cannot be satisfied. In the author's mutual energy theory, the energy conservation law is applied as axiom, hence the secondary coil (receiving antenna) should send the advanced wave. The advanced wave is a real thing. Hence, if we accept the two axioms: the energy conservation law and the mutual energy principle, we have at least two method to show that the advanced waves are real objective existence.

4.5. Mutual Energy Flow

The surface Γ in the principle of mutual energy (53) can be taken arbitrarily. If Γ is taken to surround only one current element, combined with the mutual energy theorem (37), we can prove the mutual energy flow theorem [10],

$$-\int_{t=-\infty}^{\infty} d\,t \iiint_{V_1} \mathbf{E}_2\,(t)\cdot\mathbf{J}_1(t)dV = (\xi_1,\xi_2) = \int_{t=-\infty}^{\infty} d\,t \iiint_{V_2} \mathbf{E}_1\,(t)\cdot\mathbf{J}_2(t)dV \tag{73}$$

where,

$$(\xi_1,\xi_2) = \int_{t=-\infty}^{\infty} d\,t \oiint_{\Gamma} (\mathbf{E}_1\times\mathbf{H}_2 + \mathbf{E}_2\times\mathbf{H}_1)\cdot\hat{n}d\Gamma \tag{74}$$

Where Γ is any surface that can divide the region into Γ and . The split surface Γ can be a sphere or an infinite plane. The above formula can also be transformed into Fourier frequency domain, so the integration of time can be omitted,

$$-\iiint_{V_1} \mathbf{E}_2^*\,(t)\cdot\mathbf{J}_1(t)dV = (\xi_2,\xi_1) = \iiint_{V_2} \mathbf{E}_1\,(t)\cdot\mathbf{J}_2^*(t)dV \tag{75}$$

where,

$$(\xi_2,\xi_1) = \oiint_{\Gamma} (\mathbf{E}_1\times\mathbf{H}_2^* + \mathbf{E}_2^*\times\mathbf{H}_1)\cdot\hat{n}d\Gamma \tag{76}$$

The mutual energy flow theorem can also be written in terms of the bra-ket vectors of quantum mechanics,

$$-\langle\mathbf{E}_2|\mathbf{J}_1\rangle = \langle\xi_2|\xi_1\rangle = \langle\mathbf{J}_2|\mathbf{E}_1\rangle \tag{77}$$

where,

$$\langle\mathbf{E}_2|\mathbf{J}_1\rangle = \iiint_{V_1} \mathbf{E}_2^*\,(t)\cdot\mathbf{J}_1(t)dV \tag{78}$$

$$\langle\mathbf{J}_2|\mathbf{E}_1\rangle = \iiint_{V_2} \mathbf{J}_2^*\,(t)\cdot\mathbf{E}_1(t)dV \tag{79}$$

$$\langle\xi_2|\xi_1\rangle = (\xi_2,\xi_1) = \oiint_{\Gamma} (\mathbf{E}_1\times\mathbf{H}_2^* + \mathbf{E}_2^*\times\mathbf{H}_1)\cdot\hat{n}d\Gamma \tag{80}$$

Considering the formula (55), the mutual energy flow theorem can also be written as,

$$-\langle\xi_2|L\xi_1\rangle = \langle\xi_2|\xi_1\rangle = \langle L\xi_2|\xi_1\rangle \tag{81}$$

or

$$-\langle\xi_2|L_R|\xi_1\rangle = \langle\xi_2|\xi_1\rangle = \langle\xi_2|L_L|\xi_1\rangle \tag{82}$$

L_R Is the operator L acting on the right, L_L Is the operator acting on the left. We know from electromagnetic theory that ξ_2 is the wave emitted by the receiving antenna, ant it is the advanced wave. This advanced wave appears in the bra. It can be understood that the quantity appearing in the bra in quantum mechanics is the advanced wave. It's not because the quantity in the bra has a conjugate sign. It's an advanced wave. This is because we use it to represent the wave emitted by the receiving antenna, which is the advanced wave.

5. Conclusion

The author supports that the advanced wave and the retarded wave are both physical objective existence. The author studies the expression of advanced wave. It is found that because the advanced wave and the retarded wave satisfy the same equations, such as Maxwell's equations and Schrodinger equation, the advanced wave can use the same symbol as the retarded wave. For example, ψ can represent retarded wave or advanced wave. Where ψ (t), t ≥ 0, when ψ (t) is the retarded wave, when t < 0, ψ (t) is the advanced wave. It is not necessary to use ψ^* to represent the advanced wave at all. For a current element in a waveguide, a wave running to the left and a wave running to the right will be generated. The wave running to the left moves from x = −∞, t = −∞ to x = 0, t = 0, and then continues to x = ∞, t = ∞. The current occurs at x = 0 and t = 0. This wave is the advanced wave at t < 0 and is called the retarded wave at t ≥ 0. Therefore, when the wave moving to the left passes through the current element, it changes from advanced wave to retarded wave. Note that Wheeler and Feynman's absorber theory only tells us that the current emits half retarded wave and half advanced wave. This paper tells readers that the retarded wave and the advanced wave are combined to form a leftward wave and a rightward wave. Both waves are generated by the current. The wave in each direction ether left or right is composed of a advanced wave and a retarded wave.

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