ISSN: 2639-7455

Research article

Earth & Environmental Science Research & Reviews

Refounding the Concepts of Climate Feedback and Radiative Forcing Using a Model of Minimal Complexity

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Submitted: 2024, Mar 27; **Accepted**: 2024, Apr 24; **Published**: 2024, May 10

Citation: de Larminat, P. (2024). Refounding the Concepts of Climate Feedback and Radiative Forcing Using a Model of Minimal Complexity. *Earth Envi Scie Res & Rev, 7*(2), 01-14.

Summary
Radiative forcing and climate feedback are metrics whose function is to assess and compare the behavior of the earth's climate system in response to disturbance factors. Decades after their introduction, their definitions are still unclear and assessments imprecise. This paper sets out to refound these concepts and their assessment methodology. It is based on a climate model of minimal complexity, and reduces the problem to that of its parametric estimation. In addition to redefining and assessing the above metrics, the identification of the model on the basis of millennial data raises the question of the respective contributions of human and natural factors to current global warming.

Keywords: Radiative Forcings; Climate Feedback; Reduced Complexity; Methodology; Identification

1. Introduction

Signals and systems theory has developed mainly since the 1960s, to meet the needs of signal processing and control engineering. It is now taught everywhere at undergraduate level and has widely influenced other disciplines where the behavioral aspects of phenomena are primordial (economy, biology, agronomy, ecology, etc.). Conversely, in the IPCC assessment reports, climate scientists hardly ever refer to the basic concepts of systems theory, such as transfer functions, state-space representations, diagram algebra, model reduction, balanced realizations, algebraic loops, identification of dynamic systems and so on.

This unawareness perhaps explains why efforts in reduced-complexity climate models have not resulted in the Minimum-Complexity Model (MCM) we present here. Once developed, however, the MCM is proving to be a necessary reference in modeling the climate system behavior, firstly because of its simplicity and effectiveness, and secondly for its ability to refund the concepts of radiative forcing and climate feedback. Irrespective of the systemic premises which sustain its genesis, the MCM equations derive directly from the fundamental features of terrestrial thermics, and its understanding does not call upon any notions other than elementary physics.

Terrestrial thermal variations result from the radiative balance at the TOA (top of the atmosphere), i.e. the difference between the incident solar flux - corrected for albedo - and the infrared flux emitted by the Earth. Each of these fluxes is measured in hundreds of $W\ m^{-2}$. In a warming (or cooling) period, variations in this balance do not exceed one Wm^{-2}. Its evaluation is therefore highly sensitive to errors, even small, on each of the two components. Irrespective of the above spectral distribution, its functional decomposition admits two components: *radiative forcings*, caused by external climatic imbalance factors (human and natural), and a *climate feedback* component in response to induced temperature variations.

Over the decades, methodologies have been developed to evaluate these metrics, based on radiative balances simulated by GCMs (General Circulation Models), varying the imbalance factors according to *ad hoc* scenarios, set arbitrarily, most often abrupt variations, and under particular prescriptions or conditions - for example, at fixed sea surface temperature [1]. The results are problematic. In a very comprehensive review of radiative forcings (11 co-authors, 900 references), Ramaswamy, V. et al. (2019) [2] indeed emphasize "*the chronic uncertainty in the value of climate feedback, which persists even now*". They denounce a ''*blurring of the lines between forcing and feedback*". They conclude: *"One of the great challenges of Earth system science is to continue to maintain the relatively simple essence of the radiative forcing concept in a form similar to that originally conceived, while improving the quantification of forcing*. Ultimately, they go so far as to question the viability of the radiative forcing concept, through a *"grand challenge related to the viability of this concept; and its use for the quantification of climate change"*. Knowing that the concept was conceived precisely for these quantification purposes, the mere mention of such a challenge is perplexing.

Meeting these challenges seems possible, provided we revisit the approach to the problem, in order to restore its rigor and coherence. This requires us to start from new foundations, freeing ourselves from certain aspects of previous conceptions - to which we will refer in due course.

The proposed approach is therefore based on the development of a model of minimal complexity (MCM), designed to incorporate among its variables and coefficients the radiative forcings and climate feedback to be determined. The related methodological approach consists in estimating the MCM parameters by *identification*, i.e. on the basis of input-output data from the climate system, and then obtaining assessments of the climate feedback and radiative forcings by simulating the model obtained.

There are two ways of obtaining these data. Either they result from GCM simulations, fed by external, historical or freely-defined imbalance factors (steps, ramps, etc.), but without imposing any constraints or special prescriptions, particularly in terms of surface temperature. Or we collect past climate observations, measured by instruments for recent periods or reconstructed by proxies for the distant past. In both cases, the metrics are ultimately deduced from the parameters or simulation of the MCM model, and not from GCM simulations (reserved for obtaining the necessary input-output data), nor from direct measurements internal to the climate system. In the first case, that of using data from GCM models, assessments are only as good as the models themselves, whatever the methodology adopted. In the second case - that of using real climate data - everything depends on the richness of the information and the validity of the observations. In this study, the applications relate exclusively to this second option.

The article is organized as follows. The Minimum Complexity Model (MCM) is introduced in Section 2. It differs from all previous models, in particular those developed in the context of the Reduced Complexity Model projects [3]: it fills a gap between, on the one hand, existing models of order 1, which turn out to be incomplete for lack of an output equation, and, on the other hand, models of higher degrees. The MCM presents itself as a behavioral model, and its step responses are shown to be able of approximating those derived from GCMs. Section 3 details the specific identification method adopted, based on multivariate linear regression. It is in the line of our previous works (de Larminat, P. 2016, 2023) [4, 5], but applied to the MCM model. Application to millennial input-output data shows a surprising ability to reproduce and, above all, to *project* global mean surface temperatures. Section 4 refunds the concept of radiative forcing by showing that the MCM state equation directly decomposes the balance at the top of the atmosphere into a radiative forcing and a climate feedback. Conversely, climate feedback, as defined by the IPCC, is a *pseudo-reaction*, due to the inclusion of a significant and unquantified fraction of radiative forcing. The result is the confusion of concepts mentioned above, so that the forcings assessed by the IPCC bear little relation to the reality they are supposed to quantify. Paradoxically, despite their design flaws, pseudo-forcings nevertheless remain usable metrics for comparing the respective intensities of climate imbalance factors. However, it is important to interpret them correctly, and to bear in mind that the associated assessment methodologies only imperfectly match the formulation given. Whatever the definition adopted, consistent radiative forcings can be evaluated through the MCM coefficients, estimated by the methodology developed in section 3.

This study is not limited to abstract thinking. It is implemented through identification and simulation programs, fed by databases made available online by the major official climate organizations. Section 5 presents the historical data used for this identification. Section 6 provides an initial assessment of the model and climate metrics, based on the temperature reconstructions proposed in the latest IPCC report (2021). The results are consistent with the principle of anthropogenic global warming (AGW), confirming the effectiveness of the proposed methodology. However, section 7 then highlights the sensitivity of the assessments to climate data, in particular to the reconstruction of pre-industrial temperatures. Specifically, assuming that the climate fluctuation between the medieval warm period and the Little Ice Age would have reached high levels, both parametric assessments and simulations attribute a predominant contribution to current warming to solar activity. These results raise questions, to which the conclusions reported in section 8 are still far from providing definitive answers.

Finally, Appendix A1 provides details of the identification methodology, and Appendix A2 shows how the MCM structure is derived from the two-layer models, by means of order reduction techniques. This approach also leads to detect of a major shortcoming in the usual expression of these models.

2. Design of a minimal complexity climate model (MCM)

By its very nature, a simple model can only concern a limited number of variables. Nevertheless, simplicity does not prevent global behaviors from being described, at least as effectively as the most complex models. The MCM restricts these variables to the global surface temperature T_S (t) (in °C), and to the amount of heat Q_O(t) accumulated by the climate system (in $W\,yr\,m^{-2}$).

The first step when determining a model is to specify the system's input, state and output variables. The state vector $X(t)$ of a system represents its memory, in the sense that its future state depends only on its present state and present and future inputs. For a large class of systems, this principle is expressed by a system of first-order differential equations, or state equations: $dX / dt = f(X, U_i)$. For the basics in system theory, see any undergraduate course (e.g. Boyd, S. 1993) [6]. Note, however, that the simplicity of the final equations dispenses with any prerequisites.

The model's causal inputs U_i(t) are the imposed disturbances, or disequilibrium factors (greenhouse gas concentration, solar irradiance, aerosols, etc.). They are expressed in their own units. For example, U_{CO2} is in log(*ppm*), U_{sol} in $W\ m^{-2}$, Uvolc in AOD, etc.).

For a model of minimum complexity, we propose to reduce the climate state vector to the amount of heat Q_O (t) accumulated by the climate system. Most of this is the ocean heat content (hence the index *O*). It also includes atmospheric heat, the heat of the continental surfaces and the (negative) latent heat of the cryosphere.

The problem is that the global surface temperature T_S – which is of primary interest to us – cannot be deduced directly from the state Q_O(t), nor from some equivalent oceanic temperature T_O. This is why all the reduced complexity models developed to date are required to increase the climatic state vector with – at least – the atmospheric heat content. By failing to address this problem more properly, the climate community has so far missed out on the truly minimal complexity climate model. This is due to a collective unawareness of the state formalism, and in particular of the fact that state equations are necessarily followed by so-called output equations, which in this case translates into the first order equation pair:

$$dQ_O \,/\, dt = f(Q_O,\, U_i)$$

$$T_S \;=\; h(Q_O,\, U_i)$$

NB. All variables considered are relative deviations from a pre-industrial equilibrium level, assumed to be achieved when climate disturbances are identically zero.

Classically, the linear time-invariant (LTI) system resulting from linearization of the state equations by Taylor series expansion limited to first order is set, hence the basic formulation of the MCM model:

$$dQ_O \,/\, dt = f_O\, Q_O + \sum f_i\, U_i \tag{1}$$

$$T_S \;=\; h_O\, Q_O + \sum h_i\, U_i \tag{2}$$

The parameters f_O, f_i, h_O, h_i are the respective partial derivatives of the functions *f* and *h* with respect to the variables Q_O and U_i. In order yo determine these functions; it would be unrealistic to attempt to aggregate the myriad equations governing the finite elements general circulation models (GCM). Another approach is to obtain them directly by adjustment, so as to reproduce as closely as possible the behavior actually observed in the climate – or failing that, the behavior simulated by GCMs, daringly assumed to be representative of the real climate.

The fundamental characteristic of the MCM is that the disturbance factors U_i have a direct effect on the temperature T_S, enabling the first-order model to reproduce the behavior of the climate system with surprising accuracy.

NB. For numerical integration, we use the Euler approximation: $dQ_O \,/\, dt \sim [Q_O\,(t + \Delta t) - Q_O\,(t)] \,/\, \Delta t$. Equation 1 is then iteratively simulated from an initial state Q_O (0) and input series U_i(t). The time *t* is in whole years (Δt = 1 *year*, *t* = 1: year *one* of our era).

A first illustration of MCM's ability to approximate the behavior of the Earth's climate system is provided by Figure 1. It shows (shaded) the temperature T_S simulated by GCM in response to an abrupt change in any perturbation U_i, for example a doubling of CO2 concentration. Dozens of similar responses have been obtained as part of the CMIPs (climate model intercomparison projects) initiated by the IPCC. They differ according to the perturbation factors selected and the models simulated, but they all have the same general characteristics: a rapid initial rise (a few months or years), followed by a much longer-term stabilization.
In all cases, the MCM coefficients allow these characteristics to be approximated from the MCM parameters f_O, f_i, h_i, h_O, as shown in the solid line plot in Figure 1. The dominant time constant τ is approximated by $\tau = -1 \,/\, f_O$, the short-term sensitivity by h_i and the long-term sensitivity by $k_i = h_i + g_i$, where $g_i = -h_O f_O^{-1} f_i$.

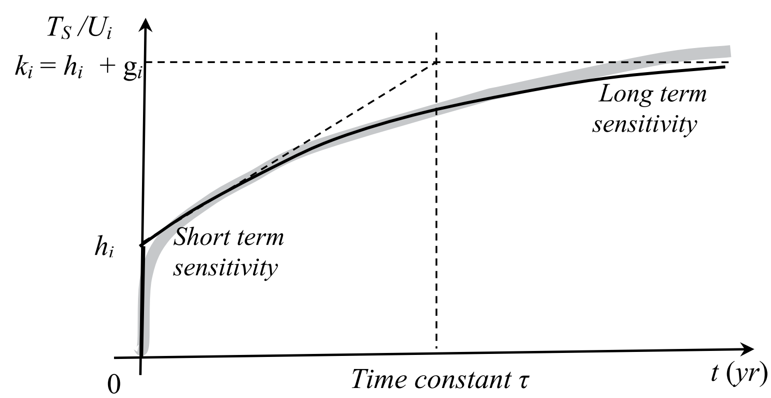

Figure 1. ***Step responses*****:** ▬ ***complex model*****:** ▬▬ ***MCM***

Since the real climate system is non-linear, some deviations result from the LTI approximation. Above all, a first-order model can only define a single time constant. In this case, it ignores the rapid transients implied by the large amount of atmospheric time constants. It results the short-term adjustments due to interactions between troposphere, stratosphere and sea surface upper layer. These deviations are less perceptible when the disturbances are slowly variable, i.e. practically all of them, with the exception of volcanic factors.

On the other hand, the heat circulation in the ocean layers results in a continuum of large time constants, the longest of which approach or exceed a millennium. For the model to approximate responses on a humanly relevant time scale (the century), the model must present a time constant τ of the same order, hence the longer-term deviations shown in Figure 1. Once again, these deviations are of little consequence if the disturbances do not involve strong variations on a multi-secular scale. In this respect, it should be noted that the TCR (transient climate response) metric was introduced by the climate community to avoid overemphasizing very long-term behavior.

Finally, we note that the three features τ, h_i , k_i shown in Figure 1 are not sufficient to trace back to the four parameters f_O, h_O, f_i, h_O. Adjusting the simulations of Q_O to the observations removes the indeterminacy, as shown in the next section dedicated to identifying the MCM.

3. MCM identification

It is known that the behavior of an LTI system is entirely determined by its set of step responses. We can therefore presuppose that the MCM is able to reproduce the behavior of the climate system in response to any other disturbance signals. This point raises the problem of designing a methodology for determining model parameters, without reference to unrealistic step responses. One solution is provided by the theory of *dynamic systems identification, i.e.* estimation of the parameters of a behavioral model from input and output data series [7, 8, 9, 10, 11, 12]. A variety of methods are achieved statistical optimality (e.g. ARMAX, B&J, PEM, etc.) or simplify implementation (e.g. ARX, OLS).

The climate community makes extensive use of statistical estimation, particularly for the *"detection and attribution"* [13, 14], but without ever referring to the prior identification theory. In the present case, the relative paucity of data leads us to prefer the most robust method, in this case parameter fitting according to the least squares criterion of output error (OE).

Model 1-2 is first transformed through the state variable change $T_O = h_O Q_O$. In the limit, if all the heat Q_O were entirely contained in a homogeneous ocean, T_O would be its temperature and $C_O = 1 / h_O$ its heat capacity. Regardless of this interpretation, T_O defines a state variable, not measured, and C_O is a parameter to be identified. The equivalent state equation and the equations of outputs T_S and Q_O can then be written as follows:

$$dT_O / dt = -\tau^{-1}(T_O - \sum_i g_i U_i) \tag{3}$$

$$T_S = T_O + \sum_i h_i U_i \tag{4}$$

$$Q_O = C_O T_O \tag{5}$$

The model parameters to be identified are now:

$$\tau = -1 / f_O, \quad g_i = -f_i \ h_O / f_O \ , \ h_i \, , \ C_O = 1 / h_O \tag{6}$$

Let's take another look at the time constant $\tau = -1 / f_O$. It aggregates the multitude of atmospheric and oceanic time constants. Depending on the spectrum of U_i inputs, ones or others may predominate in the responses, and have repercussions on all the identified parameters. Furthermore, Figure 1 shows the possibility of over-parameterization: on limited time intervals, various combinations τ, h_i, g_i are likely to achieve about equivalent approximations. Under these conditions, it is preferable to set τ at a value fixed a priori, for example the round value 100 *yr*, consistent with the projection horizon of interest to decision-makers. In addition, this stipulation facilitates the identification process by reducing it to simple multivariable linear regressions.

This identification methodology is detailed in Appendix A1, and details on the input data are given in section 5. Note these data include oceanic indexes of internal variability. Then, Figure 2 illustrates the MCM's ability to reproduce real climate behavior.

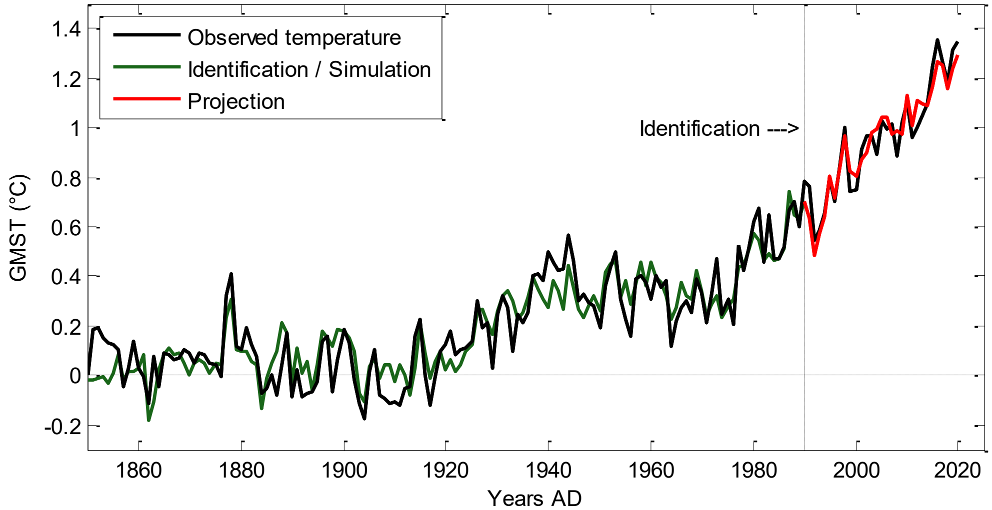

Figure 2. ***MCM identification and projection.***

The important point is that, from 1990 onwards, the simulations (red) are *projections*, i.e. the MCM parameters are identified from the output data stopped at the indicated date. After this date, the projections continue the simulation on the basis of the forcing factors that have actually occurred (human activity, solar activity, etc.), but they owe nothing to knowledge of the surface temperatures observed subsequently. The longer the identification period, the better are the projections. Actually, we find that projections deteriorate significantly if identification is stopped before that variations in forcing factors – particularly human ones – have provided sufficient information on the behavior of the climate system. It is also pointed out that the identification data are not limited to the historical period (1850-now), but covers also the previous millennium (850-1850). In conclusion, Figures 1 and 2, together with Figures 3 and 5 in the following sections, demonstrate the astonishing ability of the MCM to reproduce the behavior of the climate system – limited to global surface temperature – but with a quality equal to or better than that of much more complex.

We conclude that the MCM is capable of defining climate metrics that are not limited to the radiative forcings of primary interest to us here.

4. Refounding

An essential property of the MCM is that the radiative balance at TOA appears directly in the equations. We know that telluric flux through the Earth's crust is negligible, and that the climate system has no significant internal source of energy. According to the principle of energy conservation, the accumulated heat quantity Q_O results from the summation of the radiative balance $R(t)$ at TOA, i.e. $Q_O(t) = \int_0^t R(\tau)d\tau$. Reciprocally, the radiative balance satisfies the equation $R = dQ_O / dt$. Equation 1 leads to:

$$R = -\lambda T_O + \sum f_i U_i \tag{7}$$

$$\text{where} \quad T_O = h_O Q_O \quad \text{and} \quad \lambda = -f_O / h_O \tag{8}$$

The MCM, initially considered as a behavior model, therefore ranks in the class of energy balance models (EBM), long known in the literature [15, 16, 17, 18, 19, 20, 21, 22].

In equation 7, the first term $-\lambda T_O$ clearly defines a climate feedback flux, and the second $\sum f_i U_i$ is a sum of radiative forcings. The question therefore seems answered: these metrics are defined through the coefficients of an MCM, and their evaluation is a matter for identification techniques.

However, there are other ways of breaking down the radiative balance. From equation 4 of the MCM, we have $T_O = T_S - \sum h_i U_i$. Hence, substituting in (7):

$$R = -\lambda T_S + \sum \kappa_i U_i \tag{9}$$

$$\text{where} \quad T_S = T_O + \sum h_i U_i \quad \text{and} \quad \kappa_i = f_i + \lambda h_i \tag{10}$$

This gives us two different expressions 7-8 and 9-10 that are supposed to define the same concepts. If we persist in maintaining $-\lambda T_S$ as the expression of climate feedback, then the radiative forcing $F = \sum \kappa_i U_i$ results from the difference $F = R - (-\lambda T_S)$, in accordance with the IPCC definition: "*ERF is determined by the change in the net downward radiative flux [R] at the TOA, after the system has adjusted to the perturbation but excluding the radiative response* $[-\lambda T_S]$ *to changes in surface temperature*" (AR6, 2021) [23].

Relationships 7-8 and 9-10 are both admissible. They are based on the same MCM, recognized above as representative of the fundamental behaviors of the climate system. The difference lies in the definitions of climate feedback. In 7, it is the response to a variation in the state temperature T_O. In 9, it is "*the radiative response to changes in surface temperature*". In both cases, radiative forcing and climate feedback result from the identified coefficients of the MCM, but through different expressions 8 and 9.

At this stage, there's nothing to prevent us from adopting the IPCC definition. However, it is flawed. In fact, the forcing factors U_i have a direct impact on the climate feedback $-\lambda T_S$ via the terms $h_i U_{ii}$ in T_S. This results in the blurring in the lines between the concepts of reaction and forcing, denounced above by Ramaswamy, V. et al.2019) [2].

The climatic community's difficulty in perceiving this design flaw stems from the fact that it is only familiar with EBMs of orders greater than or equal to 2 (cf. the two-layer models in appendix A2). In these models, surface temperature necessarily appears as a component of the state vector. In such models, only variations dT_S / dt are a direct function of U_i, the temperature T_S being not. Hence the illusion that a climate feedback mediated by T_S would be independent of forcing factors. On the other hand, this dependence reveals in the short term, *"after the system has adjusted to the perturbation",* as stated in the IPCC definition. An essential property of the MCM is that it makes this dependence explicit and quantifiable through the $h_i U_i$ terms of the output equation (2).

In the light of these considerations, it would be logical to prefer climate feedback via the thermal state (7-8) to the IPCC formulation. However, there are other contingencies to consider. Since the emergence of the concept, climate feedback has been determined –regrettably – n the basis of surface temperature. Switching to a different definition, however justified, hampers the continuity of works and assessments. For example, we would have to rule out the expression of equilibrium surface temperature in terms of the ratio of *radiative forcing to climate feedback coefficient.*

We thus retain the formulation 9-10 – that of the IPCC – but rejecting the associated terminology. IPCC metrics are neither climate feedbacks nor radiative forcings in the strict sense, since they are not given by the appropriate formulation 7-8. They are *pseudo-radiative forcings*. The fact that their values do not correspond to the correct values remains secondary, as long as their function is limited to comparing either the intensities of the disturbances caused by the imbalance factors, or their assessment methodologies.

Furthermore, it is unfortunate to define radiative forcing as the result of contingent and possibly evolving methodologies, and not on the basis of *a priori* premises. Indeed, the IPCC's formulation above is not an *a priori* definition, but an *a posteriori one*, based on a methodology: "*ERF is determined by* [...]". The methods for obtaining the simulated variables R and T_S are subject to complex prescriptions detailed, for example, in AR5, (2013, Fig. 8.1, panel d) [24]: *"full atmospheric model simulations allowing atmospheric and land temperature to adjust while prescribed ocean conditions are fixed (SSTs and sea ice)"*. This can be seen as an attempt to bring climate feedback closer to its ideal definition based on total heat quantity, at the risk of distorting GCMs, which are supposed to represent the earth's climate system.

More generally, defining parameters or metrics on the basis of their assessment methodologies leads to systemic aberrations. For example, Hansen, J. et al. (2005) [1] introduce a climate feedback coefficient dependent on forcing mode, and for Geoffroy, O. et al. (2013-b) [25], as a time-dependent one. Models corrupted in this way are no more than simulation algorithms rigged to correct the behavior of poorly designed models, when they are not erroneous (see Appendix A2).

The MCM approach represents a genuine paradigm shift, which is why we are talking about refounding. The metrics to be determined are defined directly as parametric combinations *of* a well-designed LTI model of truly minimal complexity. Despite this minimality, the $[f_O, h_O, f_i, h_i]$ parameters are overabundant in number compared to the two metrics to be evaluated – climate feedback λ and radiative forcing f_i. Nevertheless, the set of MCM parameters constitutes an in indissociable whole for their assessment. The pair $[\lambda, f_i]$ alone is not sufficient to account for the fact that transient responses to disturbances depend not only on their intensity – quantified by radiative forcings – but also on their nature. Indeed, the ratio of initial and final sensitivities (see figure 1) differs according to the modes of forcing. On the contrary, the omission of surface forcings in two-layer models (see appendix A2) wrongly implies a proportionality of transient responses. Thanks to the identification of the four parameters $[f_O, h_O, f_i, h_i]$, the evaluations of λ and f_i do not suffer from the same flaw. Moreover, the additional metrics thus introduced are of their own interest, particularly those in equation, 4 where the parameter h_i quantifies the adjustment to perturbations, and where the state temperature T_O is associated with the total amount of heat Q_O via a heat capacity C_O.

Finally, through the MCM approach, the scope of the methodology is strictly limited to that of identifying the model from the input data output, and these data are not subject to any methodological constraints. The data may therefore be observation series of actual climate. They can also be data simulated by GCM, according to realistic scenarios, other than steps, and without imposing 'prescriptions' likely to distort the metrics they are intended to evaluate.

5. Input-output climate data

The generation and processing of simulation data using complex models is not part of the work presented here. It is limited to the use of data from climate observations since the beginning of our era.

Climate scientists have accumulated a considerable volume of quantified information on the factors of climatic perturbations that have occurred over the millennia. Much of this corpus is in the form of presumed radiative forcings $\mathscr{F}_i$, possibly obtained by applying existing methodologies (and constituting pseudo-forcings at best). One way of exploiting these information without direct reference to the underlying U_i factors is to treat the presumed forcings $\mathscr{F}_i$ as if they were themselves external perturbation factors, and to apply to them the coefficients κ_i (eq. 10) obtained by the MCM methodology, i.e. $F_i = \kappa_i \mathscr{F}_i$. In other words, we assume that the F_i forcings have the same profile and differ from the $\mathscr{F}_i$ series only by a multiplicative factor κ_i to be determined. Ideally, we expect to obtain $\kappa_i = 1$. Depending on whether the estimated κ_i are less than or greater than 1, we conclude that $\mathscr{F}_i$ are respectively overestimated or underestimated compared with F_i.

The presumed radiative forcings most often presented in the literature are those of the IPCC AR5 report (2013) [26]. Data are available eg. in *Climate Explorer* (*World Climate Organization*). The total increase in presumed anthropogenic forcings $\mathscr{F}_{anth}$ since the pre-industrial period is around 3 $W\ m^{-2}$ in 2020. The components of this total (GHGs, land use, industrial aerosols, etc.) all show monotonic trends (positive or negative). The result is a high degree of mutual correlation, so that it is not possible to discriminate between the associated parameters hi and gi through the combined effects on surface temperature. The application of the MCM methodology to real climate observations is therefore necessarily limited to the assessment of global anthropogenic forcing.

The main natural forcings are volcanic and solar activities. The former are reflected in negative peaks caused by volcanic eruptions. Solar forcing is assumed to be very weak (not exceeding 0.3 $W\ m^{-2}$). It is reconstructed by the NRLSI2 solar model [27]. This series begin only in 1610, and is extended by the 10Be cosmogenic series of Delaygue, G. et al., (2011) [28], aligned and calibrated on the solar model.

The $\mathscr{F}_{anth}$, $\mathscr{F}_{sol}$, $\mathscr{F}_{volc}$ forcings are centered on their pre-industrial average (850-1850). We add to these the ENSO and AMO oceanic indices. These indices, which are representative of so-called *internal* climate variability, are not strictly speaking external imbalance factors. Nevertheless, they can be treated as such, considering that they do not cause forcing at the TOA, but only exchanges across the surface, i.e. forcing at the BOA (see Appendix A2). We therefore assign them the constraint $k_i = 0$.

Identification requires data on output variables, primarily global surface temperature. In section 6, this is made up of historical measurements (HadCRUT5) for the historical period and of a proxy based reconstruction, resulting from a consortium initiated by the *PAst climate chanGES* network (PAGES2k, 2019) [29]. It shows a pre-industrial climate anomaly (PCA), which combines the Medieval Warm Period (MWP) and the Little Ice Age (LIA). The excursion of this PCA is small, around - 0.2°C. Section 7 will test the effects of a more significant PCA.

The measurement of the heat quantity Q_O is limited to the ocean heat content (OHC) (Chen, L. et al. 2017) [30]. It is shown in Figures 3-c and 5-c. It has no impact on the temperature simulations, but only on the evaluation of the climate metrics.

6. Results in line with the principle of anthropogenic global warming

Figure 3 shows simulations from the model identified from the above data (Section 5). It completes Figure 2 with the uncertainty ranges and contributions of the forcing factors. To take full advantage of the data, the identification process has been extended to 2020. The contribution of natural factors (sun, volcanism, and internal variability) is very much in the minority, although significantly greater than in the comparable figure SPM1-b (IPCC, 2021) [31]. Internal variability (thin blue lines) has contributed around two-tenths of a degree to warming since 1970.

Panel (b) shows the PAGES2k pre-industrial reconstruction and its reproduction by the identified model, resulting from the only contributions of natural factors (solar and volcanic) alone.

Panel (c) refers to oceanic heat content. The black line shows the observed heat content, limited to the recent period. Simulation of Q_O shows a drop at the beginning of the 19th century, due to the combined effects of the Dalton solar minimum and the volcanic activity (Tambora, 1815). The Q_O information is used to determine the ocean heat capacity C_O, (indicated in equivalent ocean depth) and finally the metrics λ and κ_i by the relations (9-10).

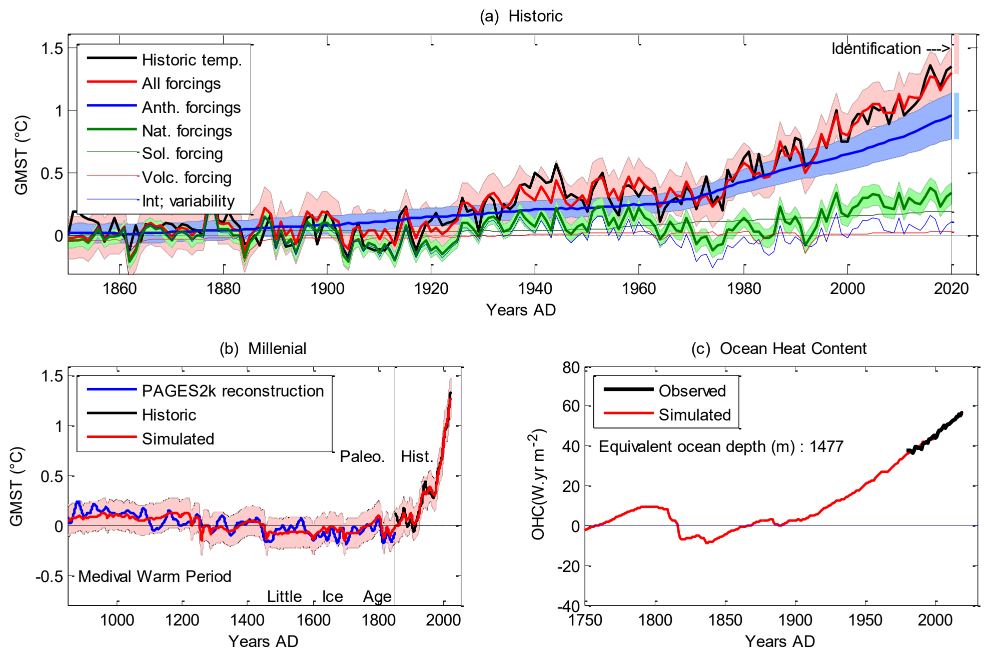

Figure 3: ***Results from PAGES2k (Weak PCA)***

Table 1 shows the metrics evaluated using PAGES2k data. The uncertainty ranges shown are 90% confidence intervals. They result exclusively from the statistical treatment of deviations between observed and simulated outputs, as observed in frame b. There is no room for subjective probabilities, either directly or indirectly, for example through *a priori* confidence ranges on observations. Appendix A1 states that these ranges are conditional on the assumption that the observations are representative of the uncertainty affecting all possible observations (ergodic assumption). As a result, the conditional ranges shown are narrower than would be the case for unconditional ranges.

	PAGES2k (Weak PCA)	**IPCC**
$\kappa_{anth} = F_{anth} / \mathscr{F}_{anth}$	0.601 [0.493 to 0.709]	1
$\kappa_{sol} = F_{sol} / \mathscr{F}_{sol}$	3.69 [2.82 to 4.56]	1
$\lambda\ (W\,m^{-2}\ °C\,)^{-1}$	1.41 [1.15 to 1.68]	1.16 [0.51 *to* 1.81]
ECS (°C)	1.59 [1.31 to 1.88]	3 [2 to 5]
TCR (°*C*)	1.19 [0.982 to 1.4]	1.8 [1.2 to 2.4]

Table 1. ***Metrics from Weak PCA***

The metrics presented in Table 1 only very approximately match the IPCC assessments. According to the estimate $F_{anth} = \kappa_{anth}$, $\mathscr{F}_{anth}$ the total anthropogenic radiative forcing assumed by the IPCC (2013) would be overestimated in the ratio $1 / \kappa_{anth} = 1.66$, and the corresponding uncertainty range does not include unity. Nevertheless, given the possible underestimation of the ranges, the ratio κ_{anth} remains admissible.

Conversly, the coefficient $\kappa_{sol} = 3.9$ makes the solar forcing $\mathscr{F}_{sol}$ incompatible with F_{sol}. Remember, however, that variations in are $\mathscr{F}_{sol}$ in a ratio of 10 to those in $\mathscr{F}_{anth}$, which explains the low relative contribution of solar forcing to warming in Figure 3a.

More successfully, the estimated range of the climate feedback coefficient λ is narrower and is fully included in that of the IPCC (2021).

Finally, the equilibrium climate sensitivity to CO_2 doubling2 (ECS) was calculated under the following assumptions: firstly, that the evaluated global sensitivity k_{anth} is identically distributed over all components of the anthropogenic factor $\mathscr{F}_{anth}$, and secondly, that the assumed forcing to CO_2 doubling2 is 3.75 $W\,m^{-2}$ (IPCC, 2021). The climate sensitivity obtained (ECS = 1.57°C) is well below the lower limit of the range accepted by the IPCC (2021).

There are many possible explanations for the disagreements that have arisen. The most immediate is that the principle of identifying the climate system suffers, on the one hand, from the low precision of paleoclimatic (and even historical) data, and, on the other, from a lack of excitation by forcing factors. Climate history and proxies being what they are, no spectacular progress can be expected in the coming decades. The fact remains that the results confirm the AGW principle, and not on the basis of speculative models not validated by observations, but on the basis of the observations themselves. The crucial question is therefore that of the validity of these data.

7. Strong PCA assumption

The weak excursion of the PAGES2k pre-industrial climate anomaly is difficult to reconcile with historical evidence (Le Roy Ladurie, E., 1967)[32]. It is also at odds with a number of reconstructions listed by the IPCC in AR5, Figs. 5.7 to 5.9 and table 5.A.6 (IPCC, 2013), compiled (*Pangaea* database) by Masson-Delmotte, V. et al. (2013)[24]. Therefore, in this section, we test the sensitivity of the results to a higher PCA. For the visibility of the results, the average of the four AR5 reconstructions with the highest PCA is used (Figure 4, red plot): [33, 34, 35, 36]. For comparison, the figure also shows the PAGES2k reconstruction (blue plot) selected in section 6.

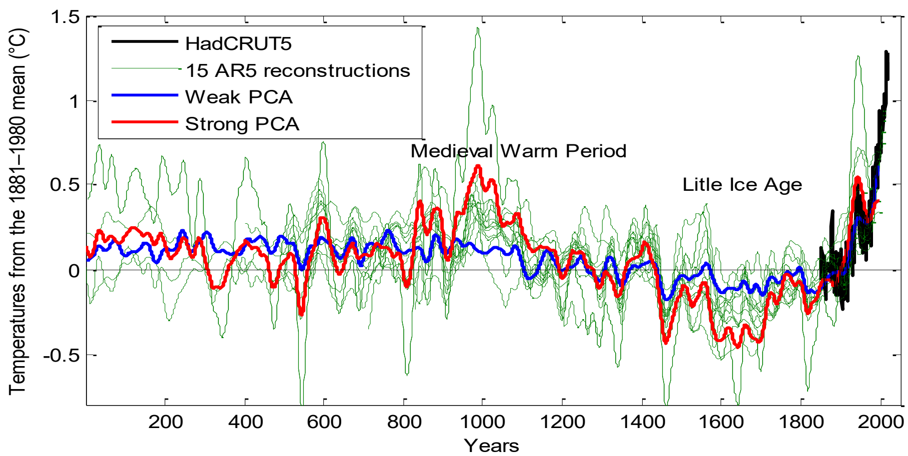

Figure 4: ***Weak and strong preindustrial climate anomalies, respectively from PAGES (IPCC,2021) and AR5 (IPCC,2013)).***

Figure 5 shows the simulations resulting from the MCM identified from the high PCA paleoclimate data.

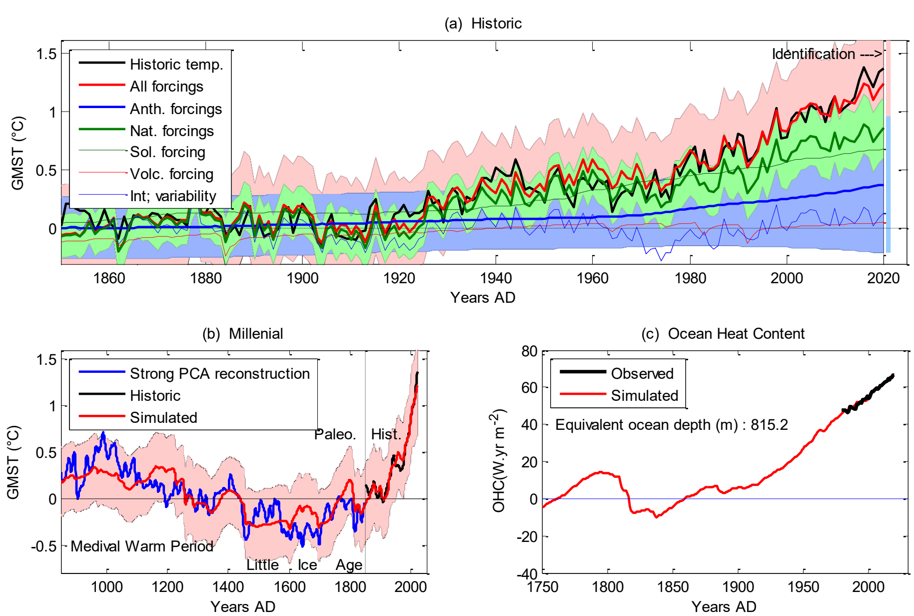

Figure 5: ***Results from strong PCA***

The simulations in frame (a) are now at odds with the AGW principle: natural contributions to warming (in green) are greater than anthropogenic contributions. These simulations are nevertheless consistent with the climate series selected. Before the advent of human forcing, the adjustment of the pre-industrial simulation (frame b) in fact requires a high solar contribution, which then contributes strongly to recent warming, resulting in a correlative reduction in the estimate of human activity.

	Strong PCA	**IPCC**
$\kappa_{anth} = F_{anth} / \mathscr{F}_{anth}$	0.13 [0.0899 to 0.17]	1
$\kappa_{sol} = F_{sol} / \mathscr{F}_{sol}$	6.72 [5.7 to 7.75]	1
$\lambda\ (W\ m^{-2}\ °C\ )^{-1}$	0.78 [0.539 to 1.02]	1.16 [0.51 to 1.81]
ECS (°C)	0.626 [-0.215 to 1.47]	3 [2 to 5]
TCR(°C)	0.468 [-0.161 to 1.1]	1.8 [1.2 to 2. 4]

Table 2. ***Metrics from strong PCA***

Despite the widening of uncertainty ranges, the metrics in Table 2 become incompatible with those of the IPCC (with the exception of climate feedback coefficient λ). To explain such discrepancies, remember that $\mathscr{F}_{anth}$, as assessed by the IPCC, from the addition of positive and negative factors, each of which beeing subject to large uncertainty. Their sum is by nature highly uncertain, and the presumed overall uncertainty could also be significantly underestimated. Above all, recall that in the $F_i = k_i\, \mathscr{F}_i$ relationship, the coefficients κ_i must incorporate all possible climatic interactions, excluding only those passing through the total heat quantity Q_O. Reactions due to cloud cover are particularly poorly understood. All in all, there is nothing implausible about the very low impact of human activity on current warming, as assessed here.

For solar activity, the presumed forcing $\mathscr{F}_{sol}$ retained by the IPCC is derived from a solar activity model whose multi-century variations (from the Maunder minimum to the current grand maximum) are among the lowest compared with many other admissible models [37]. Moreover, solar forcing is not just a result of total solar irradiance (TSI). It also depends on its spectral distribution. We should also mention variations in solar magnetism (Courtillot et al., 2007) [38], its shielding effects vis-à-vis cosmic radiation, impacting cloud genesis (Svensmark et al., 2017, 2021) [39, 40], the solar wind generated by solar flares (Landscheidt 2000), etc. See Soon (2015) [41] and Conolly (2021) [42] for comprehensive reviews. Finally, a strong contribution from solar activity to current warming cannot be ruled out.

8. Conclusions

We have proposed here a paradigm shift for understanding the concepts of radiative forcing and climate feedback, and for evaluating them, a methodology based on the principle of identifying a climate model of minimal complexity. This principle could be applied to input-output data generated by GCM, without scenario constraints or other specifications. We have not tested the MCM methodology on such data, due to lack of facilities for GCM simulation. It would certainly give results in line with AGW, knowing that all GCM models currently retained by CMIPs lead to radiative balances where human activity is preponderant [43].

The MCM methodology has been applied here to real climate observation data. If we limit ourselves to the millennial temperature data recommended by the IPCC, the results of the MCM methodology are positive. They do not overlap exactly with the metrics evaluated by the IPCC, but they are close enough to stimulate fruitful prospects. In particular, the range of the climate feedback coefficient confirms that of the IPCC, with a substantial reduction in uncertainty.

On the other hand, we tested a temperature reconstructions in which the Medieval Warm Period and the Little Ice Age reach significant levels. Then the primacy of human activity is called into question and also the solar forcing presumed by the IPCC. To preserve the AGW principle, the climate community must therefore reserve confidence in its speculative models and exclude any plausibility of disturbing observations.

Appendix A1. MCM identification

Identification of parameters g_i and h_i

By transferring the summation $\sum_i g_i$ from the U_i inputs to the, state T_O, equation 3 breaks down into:

$$T_O = \sum_i g_i V_i \quad , \text{ where } \quad dV_i / dt = -\tau^{-1}\left(V_i - U_i\right)$$

The time constant τ being fixed, the signals V_i are determined by simulating equation 9 from the given input series U_i . Equation 4 then takes a form *linear in the parameters* g_i and h_i:

$$T_S = \sum_i (g_i V_i + h_i\, U_i)$$

Let T_S^* be an observed series where T_{pi} is the pre-industrial temperature (unknown) and v is the set of noises and disturbances brought back to the output:

$$T_S^* = T_S + T_{PI} + v$$

We estimate g_i , h_i and T_{PI} by linear regression. As this regression is applied to an output signal, it minimizes an output error (*Output Error method*) and not a simple equation error (*Ordinary Least Squares*), whose careless application to step responses [44, 45].

Observation of the global surface temperature T_S therefore provides estimates of the parameters g_i and h_i , which is already a substantial achievement. This makes it possible to assess the contributions of the various disequilibrium factors to climatic variations, making the focus on radiative forcings almost superfluous.

Initial conditions

Simulations are carried out from year 1 of our era, and outputs are faced to observations from year 850. The model is initialized at zero, knowing that the extinction of the initial error will be acquired after more than eight times the time constant τ.

Identification of heat capacity C_O

A second identification is based on the observed series Q_O^*. Having determined $T_O = \sum_i g_i V_i$ in the expression $Q_O = C_O T_O$, we then estimate C_O by linear regression.

In the case of real climate series, the observation period for the heat quantity Q_O is short (a few decades) compared with the hundreds of years of observations or reconstructions available for T_S. It is nevertheless sufficient, being devoted to estimating the parameter C_O alone.

Solar activity

Most TSI reconstructions agree on the amplitude of the Schwabe cycles (11-year period). However, the amplitude of the low frequencies (from the Maunder minimum (17th century) to the current 'grand maximum') can vary by a ratio of 1 to 10 depending on the solar models adopted: NRL, SATIRE, Shapiro, CHRONOS (Egorova, T. et al., 2018) [37]. This situation leads to decomposing the indicator U_{sol} into a low-frequency component U_{sol1}, obtained by smoothing, and its high-frequency complement U_{sol2}, treated as an independent forcings.

Uncertainty ranges

Equations 7 and 8 can be written in the generic form:

$$y = X\,\theta + v$$

where y is the vector of observations (here $T^*(t)$, of dimension N), where X is the matrix of input data (...U_i , V_i ...) and θ the vector of parameters (... g h_i, i ...,). The estimate minimizing the sum of squares $\|y - X\theta\|^2$ is written $\hat{\theta} = X^+ y$, where $X^+ = (X^T X)^{-1} X^T$ is the pseudo-inverse of X. The estimator $\hat{\theta}$ is statistically optimal (asymptotically unbiased and minimum variance) when the noises and disturbances v reduce to additive white noise on the output. Whether white or not, the variance of the estimation error $\theta - \hat{\theta}$ is given by $V_{\theta\theta} = X^+ V_{vv} X^{+T}$, where $V_{vv} = E(v\,v^T)$ is the variance matrix of v. We approximate $[V_{vv}(i,j)] \sim [\phi_{vv}(i-j)]$ where $\phi_{vv}(\tau)$ is the autocorrelation function of the residuals $\hat{v} = y - X\hat{\theta}$. Hence the estimate $V_{\theta\theta} \sim X^+[\phi_{vv}(i-j)]X^{+T}$. (NB. The calculations are performed without explicitly using the V_{vv} matrix, of dimension NxN).

The above approximation is based on two assumptions: time invariance and ergodicity of the ν sequence. The validity of the invariance assumption is only a matter of degree. Ergodicity is more problematic. The observed series *y would* have to be representative of the probability space of all conceivable data. In fact, in the climate context, this space is not probabilizable: it makes no sense to assign *a priori* probabilities to reconstructions whose proxies and treatments have been arbitrarily chosen.

This objection does not prevent the IPCC from assigning probability ranges and confidence levels to all its assessments and assertions, while warning that *"the probability of an event is the degree of belief that exists among* [ours] *lead authors and reviewers that the event will occur. When complex systems are the topic, both prior and updated probability distributions usually contain a high degree of (informed) subjectivity"* [46].

Rather than providing highly subjective probabilities (allegedly *informed*), we stick to the $V_{\theta\theta}$ variance calculated above and the associated ranges. These are limited (not unimportantly) to the uncertainties deduced from noise and disturbances as they concretely manifest themselves through temporal variations in the observed signal.

NB. As in IPCC reports, the uncertainty ranges given here are 90% intervals.
N.B. The calculation of the uncertainty in λ takes into account the uncertainties in the estimates of h_i and g_i. It is performed by varying hi and gi using the Monte Carlo method.

Constraints

The paucity of observations means that we need to make the most of the *a priori* knowledge available, taking care not to bias the results, especially long-term sensitivities.

The numerous simulations from abrupt variations in CO_2, carried out within the CMIP, all show a ratio of initial sensitivity to final sensitivity of the order of 2/3. Knowing moreover that CO_2 is predominant in anthropogenic forcings, we introduce the linear constraint $h_{anth} = 0.66\, k_{anth}$, without constraining the final sensitivity k_{anth}.

Similarly, transients in response to volcanic eruptions are improved by freely constraining the parameter h_{volc}, while leaving free the centennial trends determined by the sensitivity K_{volc} . Recall also the constraint $\kappa_i = 0$ on oceanic oscillation indices.

Method. All these constraints translate into a linear relationship:

$$y_c = X_c[...g_i...h_i....]^T$$

By means of an orthogonal transformation derived from the singular value decomposition of X_c, we separate the parameters into a combination determined by y_c, and a part to be estimated, to which we apply the previous generic method.

Appendix A2. From two-layer models to MCM

In section 2, the MCM is set directly in the state space form, where the output equation (2) incorporates an explicit action of the forcing factors on the global mean temperature. We show here that the MCM can be obtained by reducing the complexity of models accepted by the climate community – provided that certain deficiencies have been detected and corrected. Using previous notations (C_O, Q_O, T_O, T_S …), a two-layer model is classically written (eg. Geoffroy, O. et al. 2013) [21], in the form of state equations of dimension 2, where C_S is the heat capacity and Q_S the atmospheric heat content:

$$C_S\, dT_S / dt = dQ_S / dt = R - S \qquad \text{A1}$$

$$C_O\, dT_O / dt = dQ_O / dt = S \qquad \text{A2}$$

The energy balances R at the TOA and S at the BOA (base of the atmosphere) are usually developed as :

$$R = -\lambda T_S + \sum_i F_i \;, \quad \text{where} \quad F_i = f_i U_i \qquad \text{A3}$$

$$S = -\gamma (T_O - T_S) \qquad \text{A4}$$

where γ is a heat transfer coefficient. Defining the state of the system as the vector $X = [T_S \,;\, T_O]$, equations A1 to A4 can be written under matrix form:

$$dX / dt = AX + BU \qquad \text{A5}$$

$$T_S \;=\; CX + DU \,, \qquad \text{A6}$$

In this form, where $C = [1\ 0]$ and $D = 0$, the U disturbances have no direct effect on the surface temperature. However, we know that they do act in the short term, after atmospheric equilibrium has been achieved, Q_O being constant. This finding has far-reaching consequences, as it implies an internal contradiction in the very definition of radiative forcing (AR6, 2021): *"ERF is determined by the change in the net downward radiative flux at the TOA, after the system has adjusted to the perturbation but excluding the radiative response to changes in surface temperature"*. Since the adjustment to the perturbation concerns the surface temperature, this temperature cannot be excluded from the determination of the radiative forcing, defined as a change resulting from the perturbation. This contradiction, which is all the more damaging as it remains unspoken, is however easily resolved by techniques for reducing the order of systems on several time scales, leading here to a first-order system, i.e. an MCM.

Before that, we need to correct a serious deficiency in equation A4. It ignores the existence of a second type of forcing, *surface forcings*, similar to the radiative forcings $F_i = f_i U_i$, with the exception of the point of application (at the BOA). It is clear, for example, that a variation in solar activity U_{sol} not only causes forcing at the TOA $F_{sol} = f_{sol}\, U_{sol}$, but also a forcing at the BOA $G_{sol} = \beta_{sol}\, U_{sol}$ resulting from the fraction of solar flux directly absorbed by the surface. In A4, the S balance (A4) must therefore be increased by surface forcing $G_i = \beta_i\, U_i$:

$$S = -\gamma (T_O - T_S) + \sum_i G_i \qquad \text{A5}$$

It's surprising that the climate community didn't immediately correct such a glaring omission. It concerns not only solar energy forcing, but all the forcings in which it intervenes indirectly: volcanic or industrial aerosols, changes in land use, etc. Note that these surface forcings are distinct from *ocean heat uptake* (Winton, M., et al. 2010), which has a long-term impact on transient responses.

We can now proceed to reduce the dimension of the state. The singular perturbation method [47] simply involves approximating the thermal inertia CS of the atmosphere by zero. Eq. A1 results in equal balances at BOA and TOA: $R = S$. Combined with equations A2, A3 and A5, we find the MCM model. After all calculations, the parameters τ, g_i, h_i can be deduced from those of the two-layer model by the formulas:

$$g_i = f_i / \lambda, \qquad h_i = (f_i - \beta_i) / (\lambda + \gamma),$$
$$\tau = C_O\, (\lambda^{-1} + \gamma^{-1}) \qquad \text{A7}$$

This is where the direct transmission term h_i appears in the MCM output equation (2), a term that quantifies the adjustment to disturbances specified in the IPCC definition and whose presence is the key to the MCM.

An important point is that the omission of surface forcings in two-layer models leads to a $h\,/g_{ii}$ ratio independent of i, unlike CMIP simulations which show different ratios between initial and final sensitivities depending on the forcing modes.

Finally, let's note that oceanic multilayer models can be reduced to a single time constant τ, by state aggregation derived from the method of balanced realizations [48].

The sensitivities g_i and h_i are closely associated, as parameters of the same behavior model. The TOA and BOA forcings that determine them are therefore also two inseparable metrics. Methodologies that do not implicitly or explicitly integrate the assessment of surface forcings with that of radiative forcings are intrinsically unviable [49-52].

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