

# Proving Goldbach's Strong Conjecture by Analyzing Gaps Between Prime Numbers and their Digits

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## Abstract

The main idea of this article lies in the fact that Goldbach's strong conjecture is associated with the progression of natural integers from 0 to infinity, which results in precise gaps between prime numbers. The gap of 6 is the most regular between primes  $6x + 1$  on the one hand and primes  $6x - 1$  on the other. In this article, using the equations  $3x \pm 5$  and analyzing the 6-based gaps between primes while determining the initial conditions that make a prime appear after or before an integer, this article argues for the truth of Goldbach's strong conjecture. Two new concepts are introduced for the first time: Goldbach's gap and Goldbach's transposition. By analyzing its key digits (units and tens), a prime number itself can lead to the conversion of an even number into two primes. A new algorithm is deduced from these results, enabling us to locate prime numbers located at equal distance from any integer, even or odd, prime or composite. This constitutes a decisive proof of Goldbach's strong conjecture, since it means that any even number can be converted into the sum of two prime numbers.

**Keywords:** Goldbach Conjecture, Even, Primes Gaps, Transposition, Algorithm, Conversion, Prime digits, Equidistance

## 1. Introduction

One of the best-known unsolved problems in number theory is Goldbach's conjecture, which appeared in a correspondence between Christian Goldbach and Leonhard Euler in 1742 [1]. The Goldbach's strong conjecture states that every even number larger than 2 can be expressed as the sum of two primes. As the Goldbach's conjecture lies in the eld of number theory and its very core is prime numbers, the distribution of such numbers may be an integral part of any attempted solution to the conjecture. The prime number theorem gives an asymptotic form for the prime counting function, which counts the number of primes less than some integer  $n$ . The set of numbers primes  $<$  an integer  $n$  are denoted  $\pi(n)$  while the asymptotic law for the distribution of prime numbers asserts that  $\pi(n) = n/\ln(n)$  [2].

As the proof of Goldbach conjecture is still out of reach, the conjecture has been extensively verified computationally, with the most recent efforts pushing the boundaries of numerical verification to unprecedented levels [3,4,5]. Despite this, the formal proof of the Goldbach Conjecture remains elusive. Apart from empirical verification, countless reports of research provide

pure mathematical verification of Goldbach's strong conjecture by different logical propositions [6,7,8]. Websites such <https://www.dcode.fr/conjecture-goldbach> allow conversion of evens in sums of two primes to a given limit.

This article is a continuation of efforts to understand the mathematical rules governing Goldbach's strong conjecture. It is essentially based on the analysis of the gaps between primes and the proposal of a new algorithm for the conversion of any even number  $> 4$  into the sum of two primes.

It introduces two new concepts for the understanding of Goldbach's strong conjecture including the Goldbach's gap and Goldbach's transposition. The article also shows that the digits of prime numbers and the primes numbers themselves can be used to prove Goldbach's strong conjecture by using elementary rules of calculation. In addition, it provides a logical and infinitely reproducible line of reasoning that argues for the truth of Goldbach's strong conjecture.

## 1.1 Logical Propositions on Goldbach Strong Conjecture and Gaps Between Primes

### 1a. Meaning of Goldbach's Transposition

*Lemma a.* « If we calculate the Goldbach gaps for a given integer, then the same gaps will give prime numbers with another integer of the same kind (for even or odd multiple of 3 or not). This is what is called here the Goldbach transposition ».

Tables 1A+1B show examples of the Goldbach transposition (a concept used in this article). If  $E$  is any even  $> 4$  and  $p$  and  $p'$  two equidistant primes such that  $p < E/2$  and  $p' > E/2$  then  $E/2 - p = p' - E/2$ . Note that  $E/2$  is any integer  $n$  located between the two equidistant primes  $p$  and  $p'$  such that  $p - n$  or  $n - p' \leftrightarrow 2 \times E/2 = E = 2n = p + p'$ . The Goldbach gap is any value of  $t = E/2 - p = p' - E/2$ . Do not confuse the Goldbach gap with the classical gap between two consecutive prime numbers, it represents the gap between two prime numbers  $p$  and  $p'$  equidistant at  $E/2$  whose sum is  $E$ . Remember that equidistant primes are located at equal distance from an integer before and after. They are essential for Goldbach's strong conjecture to be true. The question is, how far apart are they from an integer? Tables 1 and 2 show that their location depends on the number  $n$ : is it even or odd?  $3n$  or not?

$E/2$  represents any integer  $> 2$  and is either even or odd. To obtain a prime number from an even  $E/2$ , we add odd values of  $t$ , i.e. either values corresponding to primes in ascending order, or odd values of  $3n$  ( $O3n$ ). For an odd  $E/2$ , we obviously add  $2n$  values

in ascending order. Table 1A+1B shows the case of an even  $E/2$  to which we add values  $t = \text{primes}$  or  $t = 3n$  ( $O3n$ ). We can see that an even number  $3n$  always gives equidistant prime numbers when  $t$  values of prime numbers are added to it, whereas a non- $3n$  even number needs  $t = O3n$  (odd  $3n$ ) values to give prime numbers. **This is the Goldbach transposition.** Let us recall that Goldbach's transposition is symmetrical and that an addition is accompanied by a subtraction of the same quantity. For two equidistant primes result from the subtraction and addition of the same value to or from  $E/2$ .

Indeed, if the  $t$  gaps between  $E/2$  and the equidistant primes have prime values in increasing order, all  $3n$  numbers give equidistant primes and therefore verify the strong Goldbach conjecture (Tables 1A+1B). On the other hand, for numbers that are not multiples of 3 (or non- $3n$ ), gaps  $t$  that have  $3n$  values are needed to obtain equidistant primes. This fact is true for all numbers that can be tested, even those with up to 500 equidistant primes or more. *The Goldbach transposition means that even numbers of the same type ( $3n$  or not) give equidistant primes with the same spacings  $t$ .* This means that the strong Goldbach conjecture is a function of the spacings between  $E/2$  and  $p$  or  $p'$  for every even number at infinity. Given that all primes numbers are  $6x \pm 1$  and  $6n$  is the most frequent gap between primes that follow each other, Goldbach's transposition is occurring to infinity so that the same distribution of primes takes place with the same kind of numbers.

	80				72			80				100		
<b>t = p</b>	- ←	<b>40</b>	→ +	- ←	<b>36</b>	→ +	<b>t = O3n</b>	- ←	<b>40</b>	→ +	- ←	<b>50</b>	→ +	
3	<b>37</b>		<b>43</b>	<b>33</b>		39	3	<b>37</b>		<b>43</b>	<b>47</b>		<b>53</b>	
7	33		47	<b>29</b>		43	9	31		49	<b>41</b>		<b>59</b>	
11	29		51	25		47	21	<b>19</b>		<b>61</b>	<b>29</b>		<b>71</b>	
13	27		53	23		49	27	<b>13</b>		<b>67</b>	23		77	
17	23		57	<b>19</b>		53	33	7		<b>73</b>	<b>17</b>		<b>83</b>	
19	21		59	17		55	39	1		79	<b>11</b>		<b>89</b>	
23	17		63	<b>13</b>		59								
29	11		69	7		65								
31	9		71	5		77								
37	3		77											

**Table 1A:** T-values of gaps between  $E/2$  and equidistant primes. If  $E$  is any even  $> 4$  and  $p$  and  $p'$  two equidistant primes such that  $p < E/2$  and  $p' > E/2$  then  $E/2 - p = p' - E/2$  with  $p = E/2 - t$  and  $p' = E/2 + t$ . We add to an even integer either  $t = \text{prime}$  ( $p$ ) or  $t = \text{odd}3n$  ( $O3n$ ) to get a prime number. Equidistant primes are in bold-italic. Note that the sum of two equidistant primes =  $2 \times E/2$  ( $2 \times 40$  ;  $2 \times 36$  ;  $2 \times 50$ ). Note  $E = 80$  and  $E/2 = 40$  does not give equidistant primes with  $t = \text{primes}$  except with 3 but rather it generates them with  $t = O3n$ . if  $t$ -gaps = odd  $3n$  values, the non- $3n$  numbers 80 and 100 do give equidistant primes.

	188			240			188			
<b>t = p</b>	- ←	<b>94</b>	→ +	- ←	<b>120</b>	→ +	<b>t = O3N</b>	- ←	<b>94</b>	→ +
3	91		97	117		123	3	91		97
7	87		101	<b>113</b>		<b>127</b>	9	85		103
11	83		105	<b>109</b>		<b>131</b>	15	<b>79</b>		<b>109</b>
13	81		107	107		133	21	73		115
17	77		111	<b>103</b>		<b>137</b>	27	67		121
19	75		113	<b>101</b>		<b>139</b>	33	<b>61</b>		<b>127</b>
23	71		117	<b>97</b>		<b>143</b>	39	55		133
29	65		123	91		149	45	49		139
31	63		125	<b>89</b>		<b>151</b>	51	43		145
37	57		131	<b>83</b>		<b>157</b>	57	<b>37</b>		<b>151</b>
41	53		135	79		161	63	<b>31</b>		<b>157</b>
43	51		137	77		163	69	25		163
47	47		141	<b>73</b>		<b>167</b>	75	19		169
51	43		145	69		171	81	13		175
53	41		147	<b>67</b>		<b>173</b>	87	7		<b>181</b>
59	35		153	<b>61</b>		<b>179</b>	93			

**Table 1B: Comparison between a 3n number (E = 240) and a non-3 (E = 188). We can clearly see that the 3n only give equidistant prime numbers if the t gaps have prime number values while the non-3n give them with t which have odd 3n values. Equidistant primes are in bold italic.**

The Table 2 shows the different ways to obtain Goldbach gaps and equidistant primes depending on the type of the number. Primes < E (E any even > 4) within  $\pi(E)$  will always give at least one couple of equidistant primes when added to or subtracted of an even E that is 3n; odd 3n numbers will give them as well with non-3n numbers; while 2n are required to obtain them with odd primes or composite numbers.

These rules apply to all integers to infinity and are therefore responsible for what is called here the Goldbach transposition. Goldbach gaps are therefore produced in the same ways even if

in an irregular and non-linear way. We can see that the larger the number, the more equidistant prime numbers it will generate. If we assume an infinite number, then there will be an infinity of possible gaps with prime numbers < E/2 and it only takes a few prime numbers >E/2 for Goldbach gaps to appear. **Table 2** shows that each integer has its own configuration of equidistant primes as a kind of specific trace or pattern. If we move by one unit, the distribution of equidistant primes varies. For example, **35** - 6 = 29 and **35** + 6 = 41 while **36** - 5 = 31 and **36** + 5 = 41. This shows that the Goldbach's gaps vary infinitely with each integer.

<b>t</b>	Way 1 : adding or subtracting p from $\pi(E)$ to or of E/2				<b>t</b>	Way 2 : adding or subtracting O3n to or of E/2			<b>t</b>	Way 3 : adding or subtracting 2n to or of E/2			
<b>p</b>	<b>48 - p</b>	<b>48 + p</b>	<b>76 - p</b>	<b>76 + p</b>	<b>O3n</b>	<b>76 - 3n</b>	<b>76 + 3n</b>	<b>2n</b>	<b>45 - 2n</b>	<b>45 + 2n</b>	<b>39 - 2n</b>	<b>39 + 2n</b>	
3	45	51	<b>73</b>	<b>79</b>	3	<b>73</b>	<b>79</b>	2	<b>43</b>	<b>47</b>	<b>37</b>	<b>41</b>	
5	<b>43</b>	<b>53</b>	71	81	9	67	85	4	41	49	35	43	
7	41	55	69	83	15	61	91	6	39	51	33	45	
11	<b>37</b>	<b>59</b>	65	87	21	55	97	8	37	53	<b>31</b>	<b>47</b>	
13	35	61	63	89	27	49	103	10	35	55	29	49	
17	31	65	59	93	33	<b>43</b>	<b>109</b>	12	33	57	27	51	
19	<b>29</b>	<b>67</b>	57	95	39	37	115	14	<b>31</b>	<b>59</b>	25	53	
23	25	71	53	99	45	31	121	16	<b>29</b>	<b>61</b>	23	55	
29	19	77	47	105	51	25	127	18	27	63	21	57	
31	<b>17</b>	<b>79</b>	45	107	57	19	133	20	25	65	<b>19</b>	<b>59</b>	

37	<b>13</b>	<b>73</b>	39	113	63	<b>13</b>	<b>139</b>	22	<b>23</b>	<b>67</b>	<b>17</b>	<b>61</b>
41	7	<b>89</b>	35	117	69	7	145	24	21	69	15	63
43	5	91	33	119	75			26	<b>19</b>	<b>71</b>	13	65
47			29	123	81			28	<b>17</b>	<b>73</b>	<b>11</b>	<b>67</b>
53			23	129	87			30	15	75	9	69
59			17	135				32	13	77	7	<b>71</b>
61			15	137				34	<b>11</b>	<b>79</b>	5	73
67			9	143				36	9	81	3	75
71			5	147				38	7	<b>83</b>		
73			3	<b>149</b>				40	5	85		
79								42	3	87		
89								44				
97								46				
101								48				

**Table 2: The transposition ways to get Goldbach's gaps and equidistant primes relatively to an integer n of different types. Either using p of  $\pi(n)$  to subtract p from n or using odd 3n numbers with evens non-3n and 2n with odd numbers of all kinds. This shows that Goldbach's gaps are natural gaps between primes but always occur at least one for any integer. The data shown can be reproduced for whatever number > 4. Be E an even, and p and p' ( $p' > p$ ) two equidistant primes at E/2 then  $t = E/2 - p = p' - E/2$ , the table shows t values to get equidistant primes from different types of numbers. O3n means odds numbers 3n. Equidistant primes are highlighted.**

**1b. A Reasoned Example of Proposition b (lemma b).**

- *Lemma b. « E is any even > 4 et E/2 is any integer > 2. Let us consider  $\pi(E)$  and let us denote  $P_1, P_2, \dots, P_n$  any prime of  $\pi(E) < E/2$  and  $Q_1, Q_2, \dots, Q_m$  any prime of  $\pi(E) > E/2$ . Then there is at least one value of P and one value of Q such that  $E - P = Q$  and  $E - Q = P$  which is a Goldbach's gap. This proposition must be verified for Goldbach's strong conjecture to be true.».*

Here we see (Table 3) that if we subtract a prime number  $< E/2$  from E we get a prime number  $> E/2$  in some cases. This means  $E/2 - t$  and  $E/2 + t$  are primes or  $p + 2t = q$ . This means that a minus gap that generates a prime number can generate it too if it is added. This antisymmetry is common in natural numbers. All prime numbers have an integer  $n = E/2$  in the middle separated from them by the same distance. For any integer n, there exists at least one Goldbach's gap t such that  $n - t$  and  $n + t$  are primes. In Table 3 two numbers are shown. For example,  $11 + 109 = 120$  and therefore  $60 - 11 = 109 - 60$ . And so **11 = 60 - 49** and **109 = 60 + 49**. The difference 49 allows us to obtain two prime numbers

equidistant from 60 which are 11 and 109. Otherwise,  $11 + 98$  gives 109 and 98:  $2 = 49$  and thus  $11 + 49 = 60$  which is therefore the integer in the middle between 11 and 109. Other differences in Table 3 generate other equidistant prime numbers according to the same rule. All these gaps are called here Goldbach's gaps. These rules are true to infinity.

Let us take another example such  $31 + 157 = 188$ . The integer in the middle is  $(31 + 157)/2 = 94$  and thus  $31 + (63 \times 2) = 157$  and so **31 = 94 - 63** and **157 = 94 + 63**. Again  $E/2 - t$  and  $E/2 + t$  must be primes for the Goldbach's conjecture to hold true and that what happens in the set of integers. In fact, there is always any integer n between any two primes p and p' such that  $n - t = p$  and  $n + t = p'$ .

To find Goldbach's gaps it is needed to determine  $\pi(E)$  (E any even > 4) containing primes  $p < E$ . Then determine the gaps t such that  $E/2 - t$  and  $E/2 + t$  are primes and equidistant to E/2. This also represents a method that can be programmed to verify the strong Goldbach conjecture and generate all possible sums of equidistant prime numbers.

p	120 - p	p	188 - p
3	117	7	<b>181</b>
7	<b>113</b>	11	177
<b>11</b>	<b>109</b>	13	175
<b>13</b>	<b>107</b>	17	171
17	<b>103</b>	19	169
<b>19</b>	<b>101</b>	23	165
23	<b>97</b>	29	159
29	91	<b>31</b>	<b>157</b>

<b>31</b>	<b>89</b>	<b>37</b>	<b>151</b>
<b>37</b>	<b>83</b>	41	147
<b>41</b>	<b>79</b>	43	145
43	77	47	141
<b>47</b>	<b>73</b>	53	135
<b>53</b>	<b>67</b>	59	129
<b>59</b>	<b>61</b>	<b>61</b>	<b>127</b>
<b>61</b>	<b>59</b>	67	121
<b>67</b>	<b>43</b>	71	117
71	49	73	115
<b>73</b>	<b>47</b>	<b>79</b>	<b>109</b>
<b>79</b>	<b>41</b>	83	105
<b>89</b>	<b>31</b>	89	99
<b>97</b>	<b>23</b>	97	91
<b>101</b>	<b>19</b>	101	87
<b>103</b>	<b>17</b>	103	85
<b>107</b>	<b>13</b>	107	81
<b>109</b>	<b>11</b>	<b>109</b>	<b>79</b>
<b>113</b>	<b>7</b>	<b>113</b>	<b>71</b>
		<b>127</b>	<b>61</b>
		133	55
		<b>137</b>	<b>47</b>
		139	49
		149	39
		<b>151</b>	<b>37</b>
		<b>157</b>	<b>31</b>
		163	25
		167	21
		173	15
		179	9
		<b>181</b>	<b>7</b>

**Table 3: Goldbach's gaps. Table show  $\pi(120)$  (first column) and  $\pi(188)$  (third column). Any prime of  $\pi(120)$  or  $\pi(188)$  is denoted  $p$ . We see that  $120 - p$  or  $188 - p$  give equidistant prime numbers  $> E/2$  with Goldbach's gaps. These gaps are the distance between  $p$  and  $120/2 = 60$  or  $188/2 = 94$ . This shows that for any even  $E > 4$  there exists a Goldbach's gap denoted  $t$  such that  $E/2 - t$  and  $E/2 + t$  are primes. Equidistant primes are highlighted in bold.**

Let us suppose an even number denoted  $E$  and determine  $\pi(E)$ . Then  $\pi(E)$  will contain the prime numbers which are all  $< E$  (prime number theorem). Let us take  $E/2$  and then we will have the prime numbers of  $\pi(E)$  which are  $< E/2$  and those greater than  $E/2$ . We will then subdivide  $\pi(E)$  into  $P_1, P_2, P_3, \dots, P_N$  which are  $< E/2$  on the one hand, and on the other hand  $Q_1, Q_2, Q_3, \dots, Q_m$  which are  $> E/2$ . Then  $E/2 - P_1 = t_1; E/2 - P_2 = t_2; E/2 - P_3 = t_3; \dots; E/2 - P_n = t_n$  are calculated. Then there is a least one value of  $t$  ( $t_1$  to  $t_n$ ) such that  $E/2 + t = Q_1$  or  $Q_2 \dots$  or  $Q_m$  (Table 4). Since  $Q - P = 2n$  and given  $Q > P$  then  $(Q + P)/2 = E/2$  which means that there is always an integer  $E/2$  at equal distance from  $P$  and  $Q$ . If  $(Q + P)/2 = E/2$  then  $Q + P = E$  and  $Q$  and  $P$  are equidistant relative to  $E/2$ . This proposition postulates that Goldbach gaps before  $E/2$  repeat at least once after  $E/2$  because prime numbers are formed in

a symmetrical manner from 0 to infinity and from infinity to 0. For example, if  $n - t$  is prime then it is likely that  $n + t$  is also prime.

Similarly and symmetrically, the gaps  $Q_1 - E/2 = z_1; Q_2 - E/2 = z_2; Q_3 - E/2 = z_3; \dots; Q_m - E/2 = z_m$  are opposite gaps. And so there exists at least one value of  $z$  ( $z_1$  to  $z_m$ ) such that  $E/2 - z = P_1$  or  $P_2$  or  $P_3$  or  $P_n$  (Table 4). Any even number  $E > 4$  has both types of sets of Goldbach gaps on either side of  $E/2$ .

Equidistant primes are highlighted in italic and there are 10 Goldbach's gaps. The table show that there is at least one value of  $t$  such that  $E/2 + t$  and  $E/2 - t$  numbers are both primes which represents an initial condition essential for Goldbach's strong conjecture to be true. The Table shows that there are 10 Goldbach's

gaps and 10 couples of  $E/2 + t$  and  $E/2 - t$  prime numbers obtained with the number  $E = 120$  ( $E/2 = 60$ ). This will happen with any

even  $E > 4$  of same type ( $3n$ ), only the values and positions of primes changes between 0 and  $E/2$  and between  $E/2$  and  $E$ .

t	60 - t	60 + t	Goldbach gap
3	57	63	
5	55	65	
7	53	67	1
11	49	71	
13	47	73	2
17	43	77	
19	41	79	3
23	37	83	4
29	31	89	5
31	29	91	
37	23	97	6
41	19	101	7
43	17	103	8
47	13	107	9
53	7	113	10
59	1	119	

**Table 4: Goldbach's gaps with  $E/2 + t$  and  $E/2 - t$  numbers with  $E = 120$  ( $3n$ ) and  $E/2 = 60$  and  $t =$  prime value in an increasing order. This can be reproduced for any number but it must be considered whether the number is even or odd,  $3n$  or not (see above). If it is odd, then it is necessary to subtract and add  $2n$  numbers in ascending order.**

### 1c. Specific Case of an Even Number Denoted E Which Tends to Infini

If  $E$  tends to infinity, the Goldbach' gaps tend to infinity.  $\pi(E)$  will contain an infinity of prime numbers  $P < E/2$  and  $Q > E/2$ . And so for a prime number  $Q > E/2$ , there must exist a Goldbach gap such that  $Q - E/2 = E/2 - P$ . Even if there is a long empty gap of prime numbers after  $E/2$  and the number of prime numbers  $Q > E/2$  are rarer and more dispersed, there will still exist a Goldbach gap such that  $Q - E/2 = E/2 - P$  because all possible Goldbach gaps exist before  $E/2$  for a number  $E$  that tends to infinity. For example, there are infinitely many gaps between 3 and all other primes ( $7 - 3$ ;  $11 - 3$ ;  $17 - 3$ ;...to infinity) and this is true for every prime number and therefore all possible Goldbach gaps exist at infinity to the point that any prime number  $Q > E/2$  will find a number  $P < E/2$  which is symmetric to it such that  $Q - E/2 = E/2 - P$ . Goldbach's conjecture is true because there are infinitely many possible gaps between known or possibly known prime numbers at infinity and any prime  $E/2 + t$  whatever its value and  $> E/2$  would find a symmetric prime  $E/2 - t < E/2$ . This rule of symmetry can be more easily seen with twin prime numbers to infinity.

Let us suppose an even number  $E$  such that  $E/2$  tends to infinity, so  $[E/2 \rightarrow E]$  is in infinity. On the other hand, from 0 to  $E/2$  we have all the prime numbers that we could discover by the means available to date or which are known at the maximum limit. So if

we take any prime number  $p$  and any prime number  $q$  that we know such that  $q > p$ , then  $q - p$  can take any possible value  $2n$  and all possible  $2n$  gaps between any two prime numbers (not just twins but distant from each other by any gap) exist from 0 to  $E/2$ . We know well today that prime numbers are infinite and probably at a much higher density than we imagine. Let us then suppose that between  $E/2$  and  $E$  we have two prime numbers among others  $s$  and  $t$  such that  $s - t = 2n'$ . Then, there must exist two other prime numbers  $u$  and  $v < E/2$  such that  $u - v = 2n'$ .

Therefore  $s - t = u - v$  and  $s + v = t + u$  and therefore if  $u$  and  $t$  are equidistant at  $E/2$  then  $s$  and  $v$  are also equidistant. Since  $s$  and  $t > E/2$  and  $u$  and  $v < E/2$  with  $u$  and  $t$ , and  $s$  and  $v$ , equidistant from  $E/2$ , then  $2 \times E/2 = s + v = t + u$  and therefore  $E = s + v = t + u$ . Thus Goldbach's conjecture can be verified at infinity knowing that any even number that we know or can imagine will be the sum of two prime numbers. Indeed for a number which tends to infinity, all possible gaps  $2n$  between any two primes are limitless.

Let us remember that if  $P1 \leftarrow E/2 \rightarrow P2$  means  $P1$  and  $P2$  are equidistant to  $E/2$  then  $E/2 - t = P1$  and  $E/2 + t = P2$  ( $t$  any integer  $< E/2$ ). Therefore  $E = E/2 + E/2 = (E/2 - t) + (E/2 + t) = P1 + P2$ . Any time there are two equidistant primes relatively to  $E/2$  the strong Goldbach conjecture is verified and holds true. The  $t$  is the Goldbach's gap and is also limitless as the number tends to infinity which means all primes at any  $t$  gap are possible. Note here that

two initial conditions are necessary  $s - t = u - v$  and  $s, v$  on one hand and  $t, u$  on the other are equidistant to  $E/2$ . This means that all equidistant primes to  $E/2$  are related to each other by  $2n$  gaps and related to  $E/2$  by  $t$  Goldbach's gaps. The two gaps overlap in all possible ratios. If  $E$  and  $E/2$  extend to infinity, and even if the density of primes  $> E/2$  is lower or there are long sequences devoid of primes, the distances between primes close to  $E/2$  and  $0$  would be as infinite as the distances between these distant primes and  $E/2$ . Moreover, given their higher density, more possible gaps occur

before  $E/2$ . It is therefore quite possible that these distant primes are equidistant to primes  $< E/2$ .

To illustrate this with an example, we limit ourselves to a small number like 100 knowing that what is shown applies to infinity (Table 5A+B). These tables show the large number of possibilities for a small number and a fortiori for a number which extends to infinity.

$v \rightarrow$	3	5	7	11	13	17	19	23	29	31	37	41	43	47
$u \downarrow$														
47	44	42	40	36	34	30	28	24	18	16	10	6	4	
43	40	38	36	32	30	26	24	20	14	12	6	2		
41	38	36	34	30	28	24	22	18	12	10	4			
37	34	32	30	26	24	20	18	14	8	6				
31	28	26	24	20	18	14	12	8	2					
29	26	24	22	18	16	12	10	6						
23	20	18	16	12	10	6	4							
19	16	14	12	8	6	2								
17	14	12	10	6	4									
13	10	8	6	2										
11	8	6	4											
7	4	2												

**Table 5A :**  $E$  is any even and be  $E = 100$ . Let us calculate  $\pi(100)$  and calculate  $u - v$  such that  $u$  is any prime  $> v$  in primes  $\pi(100) < E/2 = 50$ .

$t \rightarrow$	53	59	61	67	71	73	79	83	89	97
$s \downarrow$										
97	44	38	36	30	26	24	18	14	8	
89	36	30	28	22	18	16	10	6		
83	30	24	22	16	12	10	4			
79	26	20	18	12	8	6				
73	20	14	12	6	2					
71	18	12	10	4						
67	14	8	6							
61	8	2								
59	6									

**Table 5B :** Let us calculate  $\pi(100)$  and calculate  $s - t$  such that  $s$  is any prime  $> t$  in primes  $> E/2 = 50$ .

Let determine any  $u - v = s - t$  such that  $u + t$  or  $s + v = 100$ . Examples :

$$\begin{array}{ll}
 47 - 41 = 59 - 53 & \rightarrow 47 + 53 = 59 + 41 = 100 \\
 47 - 3 = 97 - 53 & \rightarrow 47 + 53 = 97 + 3 = 100 \\
 47 - 17 = 83 - 53 & \rightarrow 47 + 53 = 17 + 83 = 100 \\
 41 - 17 = 83 - 59 & \rightarrow 41 + 59 = 17 + 83 = 100 \\
 41 - 11 = 89 - 59 & \rightarrow 41 + 59 = 89 + 11 = 100 \\
 17 - 3 = 97 - 83 & \rightarrow 17 + 83 = 97 + 3 = 100
 \end{array}$$

## 2. Demonstrate and Prove the Correctness of the Goldbach's Strong Conjecture by a Table Using $3x + 5$ and $3x - 5$ Equations While Following the Remainders of Euclidean Divisions

### 2a. Goldbach's Strong Conjecture and the Gaps that Separate Prime Numbers Using $3x \pm 5$ Equations

Prime numbers and their multiples except multiples of 2 and 3 are all  $6x \pm 1$  [9]. Here the equations  $3x \pm 5$  reconstruct all prime numbers and their multiples in their natural order (Table 6A+B). However, using Euclidean division, one can know why a number is prime or not by examining the remainders of the Euclidean division of  $3x$  by the prime numbers below it. This is possible with the equations  $3x \pm 5$ . Let's take an example, the number 35 has 5 as a factor and therefore cannot be prime because  $35 = 3 \times 10 + 5$ . But if we take 77, we have  $77 = 3 \times 24 + 5$  and in fact  $77 - 5 = 72$  and  $72 : 11 = 6$  and the remainder (denoted  $r$  in the table) = 6 and therefore if we add 5, we will have a new factor  $11 = 5 + 6$  or  $5 = 11 - 6$  and therefore 77 is not prime. In general for the equation  $3x + 5$  if  $5 = \text{prime factor} - \text{remainder}$  (the factor is denoted  $q$  in the table) then the number obtained is not prime. This could be used as a factorization method by examining each time the remainder of the division of the number  $3x$  by the prime factors  $q$  which are less than its square root, if  $5 = q - r$  then the number obtained with the equation  $3x + 5$  is composite. *Here are the steps, take the number  $3x + 5$  and subtract 5, you get  $3x$ , then divide it by primes  $q < \text{its square root}$ . See the remainders and determine if  $5 = q - r$ . If it is the case, then the number is composite. You get the prime factor by adding 5 to the remainder. This method can work well to decompose a number in product of prime factors. If  $5 \neq q - r$ , then the number is prime.* Factoring an integer into a product of prime factors is still a topic of primary importance in mathematics [10].

Let us take another example,  $119 = 3 \times 38 + 5$ . We have  $3 \times 38 = 114$  and  $114 : 17 = 6$  and  $r = 12$ . Therefore  $5 = 17 - 12$  and thus 119 is composite. By contrast, if the number is prime,  $5 \neq q - r$  in all Euclidean divisions.

On the other hand, for  $3x - 5$  equation, if  $5 = r$  of the Euclidean divisions then the number is composite.

For the rest, let us denote any prime number **P** and any composite number **C**. The prime numbers obtained by  $3x + 5$  are the **P+** and those of the equation  $3x - 5$  are **P-**. Similarly, we have the **C+** and **C-**. Note that multiples of 3 are excluded from the tables. We see that there are gaps of  $6n$  between the **P+** primes on the one hand and between the **P-** on the other hand. Each line break = gap of 6 by going down or up. If we go up, we have gaps of  $-6n$  and if we go down we have  $+6n$ . On the other hand, we have variable gaps of  $2n$  between the **P+** and the **P-** primes. We have the same pattern of gaps between the **C+** and the **C-**. Let us note that  $3x + 5$  corresponds to  $6x - 1$  primes and  $3x - 5$  to the  $6x + 1$  equation. In fact, there are two types of primes of which the former are  $6x - 1$  and the latter  $6x + 1$ . Not only primes, but all their multiples except those of 2 and 3 can be written as  $6x + 1$  or  $6x - 1$ .

Since all composite or prime numbers can be written as the equation  $3x \pm 5$ , we can then develop a method for their factorization by

applying the rules  $5 = q - r$  or  $5 = r$  for the equations  $3x + 5$  and  $3x - 5$ , respectively. We can clearly see that the integers, during their progression to infinity, give prime or composite numbers, depending on the remainders of their Euclidean divisions by the prime numbers that are less than their square roots. Conversely and by the same process, a number **P** or **C** comes from a natural integer **P** or **C** which precedes it.

*This progression of integers in tables-6 by  $3x \pm 5$  equations can be used to demonstrate and prove the Goldbach's strong conjecture.* In fact, it appears clear that this conjecture is a result of the progression of natural numbers into primes or composite numbers. The proof of this conjecture lies in this progression itself.

Here are the key arguments to prove Goldbach's strong conjecture:

- Any odd number **P** or **C** is preceded and followed by prime numbers at regular intervals of  $6n$ . Therefore  $P' (+6n) \text{---} P \text{---} P'' (-6n) \leftrightarrow 2 \times P = 2n = P' + P''$ .  $P''' (+6n) \text{---} C \text{---} P'''' (-6n) \leftrightarrow 2 \times C = 2n = P''' + P''''$ . Take any number **P** or **C** in the tables-6 using the equation  $3x + 5$  or  $3x - 5$ , go up and down till you get the two equidistant primes then Goldbach's strong conjecture is demonstrated this way.

For example  $7(+6 \times 5) \text{---} 37 \text{---} 67 (-6 \times 5) \leftrightarrow 2 \times 37 = 7 + 67 = 74$ .  
 $71(+6 \times 1) \text{---} 77 \text{---} 83 (-6 \times 1) \leftrightarrow 2 \times 77 = 71 + 83 = 154$ .

This table shows that since all numbers **P** and **C** are equidistant from prime numbers preceding and following them, even numbers are sums of two prime numbers because if a number  $n$  is equidistant from two prime numbers **P1** and **P2** then  $2n = P1 + P2$ . Goldbach's strong conjecture can thus be demonstrated by  $6n$  gaps.

- However, we are missing the even numbers in the tables-6 (multiples of 3 or not) and the odd multiples of 3. We just need to convert them first to **P** or **C** of the table and then move in gaps of 6. *The numbers **P** and **C** in tables-6 are converters because they are used to transform any even  $> 4$  into the sum of two prime numbers.*

- Note that we can also use the gaps between the prime numbers  $3x + 5$  and  $3x - 5$  but they are  $= 2n$  and are variable and just require more attention and calculation but the table demonstrates Goldbach's strong conjecture in all directions and cases.

- For example :  
 $23(+6) \text{---} 29 (+1) \text{---} 30 \text{---} 31(-1) \text{---} 37(-6) \leftrightarrow 2 \times 30 = 60 = 31 + 29 = 23 + 37$

- $37(+6 \times 5) \text{---} 67 (+2) \text{---} 69 \text{---} 71(-2) \text{---} 101(-6 \times 5) \leftrightarrow 2 \times 69 = 138 = 67 + 71 = 37 + 101$ .

- $37(+6 \times 5) \text{---} 67 (+3) \text{---} 70 \text{---} 73(-3) \text{---} 103(-6 \times 5) \leftrightarrow 2 \times 70 = 140 = 67 + 73 = 37 + 103$ .

- It is important to note that if we start with a non- $3n$  even number like 56 or 88, we should not convert it to a  $3n$  number because we will no longer be able to find prime numbers with steps of 6.

Example if we do  $56 - 2 = 54$ , we cannot have prime numbers below or above 54. Rather  $56 - 3 = 53$  or even  $56 - 1 = 55$  because  $55 - 6 = 49 - 6 = 43$  or  $55 + 6 = 61 + 6 = 67$  and so on.



• According to this table here is the demonstration of Goldbach's strong conjecture: Be E any even  $>4$ , C is composite and P is prime.

$$E = C_1 + C_2 = (C_1 + 6n) + (C_2 - 6n) = P_1 + P_2. E = C_1 + P = (C_1 + 6n) + (P - 6n) = P_3 + P_4.$$

$$E = P' + P'' = (P' + 6n) + (P'' - 6n) = P_5 + P_6$$

• Because there is always a  $2n$  gap between any two prime  $p$  and  $q$  such that  $q > p$ , then there is always an integer in the middle. Be  $q > p$  and  $q = p + 2t \leftrightarrow \exists n$  such that  $p + t = n$  and  $q - t = n \leftrightarrow 2n = p + q$ . All prime numbers taken to infinity will generate all possible evens  $2n$  and therefore  $2n$  would always be the sum of two primes.

• Conversely, be  $2n \leftrightarrow n$  is in the middle of two primes  $p$  and  $q \leftrightarrow 2n = p + q$ .

•  $2n \leftrightarrow 2n/2 = n \leftrightarrow n = 3x + 5$  or  $n = 3x - 5 \leftrightarrow n$  is P or C  $\leftrightarrow n - 6n$  and  $n + 6n \leftrightarrow p + q$ .

• There are four possible equalities or Goldbach equations to

complete to convert any even number into the sum of two prime numbers P and P' using tables-6 and  $3x \pm 5$  to infinity :

$$P' (+6n) \text{---} P \text{---} P'' (-6n) \text{ (Even} = 2n = 2 \times P = P + P') \text{.}$$

$$P' (+6n) \text{---} C \text{---} P'' (-6n) \text{ (Even} = 2n = 2 \times C = P + P') \text{.}$$

$$P' (+6n) \text{---} C \text{ or } P \text{---} 2n + x \text{---} 2n \text{---} 2n - x \text{---} C \text{ or } P \text{---} P'' (-6n) \text{ (Even} = 2n = 2 \times 2n = P + P') \text{.}$$

$$P' (+6n) \text{---} C \text{ or } P \text{---} 3n + x \text{---} O3n \text{---} 3n - x \text{---} C \text{ or } P \text{---} P'' (-6n) \text{ (Even} = 2n = 2 \times O3n = P + P') \text{.}$$

Note  $2n$  might be  $3n$  or not while  $O3n$  is odd  $3n$  in the last equation. Note that  $x$  is used to preconvert the even or  $O3n$  into the converters P or C before searching for the P by the jumps of  $6n$  (see examples above). Or we can use any gap between primes to complete these equations but  $6n$  is the most regular.

### 6A

$x = 2n$	$3x + 5$	P or C ( $5 = q - r$ )	$x = 2n$	$3x - 5$	P or C ( $r = 5$ )
2	11	P	4	7	P
4	17	P	6	13	P
6	23	P	8	19	P
8	29	P	10	↓25↑	$q = 5$
10	↓35↑	$q = 5$	12	31	P
12	41	P	14	37	P
14	47	P	16	43	P
16	53	P	18	↓49↑	$r = 5$
18	59	P	20	↓55↑	$q = 5$
20	↓65↑	$q = 5$	22	61	P
22	71	P	24	67	P
24	↓77↑	$5 = 11 - 6$ $5 = 11 - 6$	26	73	P
26	83	P	28	79	P
28	89	P	30	↓85↑	$q = 5$
30	↓95↑	$q = 5$	32	↓91↑	$r = 5$
32	101	P	34	97	P
34	107	P	36	103	P
36	113	P	38	109	P
38	↓119↑	$5 = 17 - 2$ $5 = 7 - 2$	40	↓115↑	$q = 5$
40	↓125↑	$q = 5$	42	↓121↑	$r = 5$
42	131	P	44	127	P
44	137	P	46	133	$r = 5$
46	↓143↑	$5 = 11 - 6$ $5 = 13 - 8$	48	139	P
48	149	P	50	↓145↑	$q = 5$
50	↓155↑	$q = 5$	52	151	P
52	↓161↑	$5 = 7 - 2$ $5 = 23 - 18$	54	157	P

54	<b>167</b>	P	56	<b>163</b>	P
56	<b>173</b>	P	58	↓ <b>169</b> ↑	<b>r = 5</b>
58	<b>179</b>	P	60	↓ <b>175</b> ↑	q = 5
60	↓ <b>185</b> ↑	q = 5	62	<b>181</b>	P
62	<b>191</b>	P	64	↓ <b>187</b> ↑	<b>r = 5</b>
64	<b>197</b>	P	66	<b>193</b>	P
66	↓ <b>203</b> ↑	5 = 29 - 24 5 = 11 - 6	68	<b>199</b>	P
68	↓ <b>209</b> ↑	5 = 19 - 14 5 = 11 - 6	70	↑ <b>205</b> ↑	q = 5
70	↓ <b>215</b> ↑	P	72	<b>211</b>	P

**6B**

<b>x = 2n</b>	<b>3x + 5</b>	<b>P or C (5 = q - r)</b>	<b>x = 2n</b>	<b>3x - 5</b>	<b>P or C (r = 5)</b>
72	↓ <b>221</b> ↑	5 = 13 - 8 5 = 17 - 12	74	↓ <b>217</b> ↑	<b>r = 5</b>
74	<b>227</b>	P	76	<b>223</b>	P
76	<b>233</b>	P	78	<b>229</b>	P
78	<b>239</b>	P	80	↓ <b>235</b> ↑	q = 5
80	↓ <b>245</b> ↑	q = 5	82	<b>241</b>	P
82	<b>251</b>	P	84	↓ <b>247</b> ↑	<b>r = 5</b>
84	<b>257</b>	P	86	↓ <b>253</b> ↑	<b>r = 5</b>
86	<b>263</b>	P	88	↓ <b>259</b> ↑	<b>r = 5</b>
88	<b>269</b>	P	90	↓ <b>265</b> ↑	q = 5
90	<b>275</b>	q = 5	92	271	P
92	<b>281</b>	P	94	277	P
94	↓ <b>287</b> ↑	5 = 41 - 36 5 = 7 - 2	96	<b>283</b>	P
96	<b>293</b>		98	↓ <b>289</b> ↑	<b>r = 5</b>
98	↓ <b>299</b> ↑	5 = 23 - 18 5 = 13 - 8	100	↓ <b>295</b> ↑	q = 5
100	↓ <b>305</b> ↑	q = 5	102	↓ <b>301</b> ↑	<b>r = 5</b>
102	<b>311</b>	P	104	<b>307</b>	P
104	<b>317</b>	P	106	<b>313</b>	P
106	↓ <b>323</b> ↑	5 = 17 - 12 5 = 19 - 14	108	↓ <b>319</b> ↑	<b>r = 5</b>
108	↓ <b>329</b> ↑	5 = 7 - 2 5 = 47 - 42	110	↓ <b>325</b> ↑	q = 5
110	↓ <b>335</b> ↑	q = 5	112	<b>331</b>	P
112	↓ <b>341</b> ↑	5 = 31 - 26 5 = 11 - 6	114	<b>337</b>	P
114	<b>347</b>	P	116	↓ <b>343</b> ↑	<b>r = 5</b>
116	<b>353</b>	P	118	<b>349</b>	P
<b>118</b>	<b>359</b>	P	120	↓ <b>355</b> ↑	q = 5
120	↓ <b>365</b> ↑	q = 5	122	↓ <b>361</b> ↑	<b>r = 5</b>
122	↓ <b>371</b> ↑	5 = 7 - 2 5 = 53 - 48	124	<b>367</b>	P
124	↓ <b>377</b> ↑	5 = 13 - 8 5 = 29 - 24	126	<b>373</b>	P

126	383	P	138	379	P
138	389	P	140	↓385↑	q = 5
140	↓395↑	q = 5	142	↓391↑	r = 5
142	401	P	144	397	P
144	↓407↑	5 = 11 - 3 5 = 37 - 32	146	↓403↑	r = 5
146	↓413↑	5 = 7 - 2 5 = 59 - 54	148	409	P
148	419	P	150	↓415↑	r = 5
150	↓425↑	q = 5	152	421	P

**Table 6A+B :** The equations  $3x \pm 5$  reconstruct all the prime numbers and odd multiples of prime numbers except those of 2 and 3. Depending on the remainders (r) of the Euclidean division of a number  $3x$  by the primes  $<$  its square root denoted  $q$ , the number is prime (P) or composite (C). The table shows that all-natural numbers (primes, odd multiples of primes, evens and multiples of 3) can be in the middle of two primes by making  $6n$  jumps in two opposite directions (up down  $\downarrow N \uparrow$ ) or left-right by  $2n$  gaps. In the table P is prime and C is composite while  $r$  is the remainder of Euclidean division of numbers  $3x$  in  $3x + 5$  or  $3x - 5$  by  $q$  which is any prime less than their square roots. If  $5 = q - r$  in the case of  $3x + 5$  or  $5 = r$  in the case of  $3x - 5$ , the number is not prime (C).

**2b. A new algorithm-I to Convert an Even In Sum of two Primes**

Be E an even. Calculate  $E/2$ .  
 If  $E/2$  is P or C  
 Follow table-6 lines (to infinity). Perform conversion  
 $P'( +6n) \rightarrow P$  or  $C \rightarrow P''(-6n)$   
 if P or C =  $P' + 6n$  and  $P'' = P$  or  $C + 6n$ ;  $E = 2x P$  or  $2x 2C = P' + P''$ .  
 If  $E/2$  is even (2n) non-3n  
 Covert it to P or C. Calculate  $E/2 - 1$  and  $E/2 + 1$   $E/2 - 1$  and  $E/2 + 1$  should be  $\neq$  odd  $3n$  ( $O3n$ )  
 If  $O3n$  calculate  $O3n - 2$  and  $O3n + 2$  If the result is C or P proceed as above  
 $P'( +6n) \rightarrow P$  or  $C \rightarrow 2n \rightarrow P$  or  $C \rightarrow P''(-6n)$  if  $2n = P' + 6n$  and  $P'' = 2n + 6n$   
 $E = 2x 2n = P' + P''$ .  
 If  $E/2$  is even  $3n$  Covert it to P or C  $E/2 - 1$  and  $E/2 + 1$   
 If the result is C or P proceed as above  $P'( +6n) \rightarrow P$  or  $C \rightarrow 2n \rightarrow P$  or  $C \rightarrow P''(-6n)$  if  $2n = P' + 6n$  and  $P'' = 2n + 6n$   
 $E = 2x 2n = P' + P''$ .

If  $E/2$  is odd  $3n$   
 calculate  $E/2 - 2$  and  $E/2 + 2$   
 If the result is C or P proceed as above  $P'( +6n) \rightarrow P$  or  $C \rightarrow O3n \rightarrow P$  or  $C \rightarrow P''(-6n)$  if  $O3n = P' + 6n$  ad  $P'' = O3n + 6n$   
 $E = 2x O3n = P' + P''$ .

Note the method works with other gaps =  $2n$  or any gap  $N$  ( $N$  any integer  $> 0$ ) From  $3x + 5$  to  $3x - 5$  primes or vice versa  
 $P'( +N) \rightarrow P$  or  $C \rightarrow P''(-N)$   $E = 2x P$  or  $C = P' + P''$ .

**2c. Corresponding Examples of the Algorithm-I**

$E = 170$   $E/2 = 85$   
 $E/2$  is C  
 $67(+18) \rightarrow 85 \rightarrow 103(-18)$

$170 = 2x 85 = 67 + 103$ .

$E = 194$  and  $E/2 = 97$

$E/2$  is P  
 $67(+30) \rightarrow 97 \rightarrow 127(-30)$   
 $194 = 2x 97 = 67 + 127$ .

$E = 320$   $E/2 = 160$

$E/2$  is even non-3n  
 $160 - 1 = 159$  and  $160 + 1 = 161$   
 $159$  is odd  $3n$  to discard  
 $160 - 3 = 157$  and  $160 + 3 = 163$   
 $157 \rightarrow 160 \rightarrow 163$  or  
 $139 (+6x3) \rightarrow 157 \rightarrow 160 \rightarrow 163 \rightarrow 181(-6x3)$  or  
 $139 (+21) \rightarrow 160 \rightarrow 181(-21)$   
 $320 = 2x 160 = 157 + 163$   
 $320 = 2x 160 = 139 + 181$   
 $E = 660$  and  $E/2 = 330$   
 $E/2$  is even  $3n$   
 $330 - 1 = 329$  and  $330 + 1 = 331$   
 $329 \rightarrow 330 \rightarrow 331$   
 $311(+18) \rightarrow 329 \rightarrow 330 \rightarrow 331 \rightarrow 349(-18)$  or  $313(+17) \rightarrow 330 \rightarrow 347(-17)$   
 $660 = 2x 330 = 311 + 349$  or  $660 = 2x 330 = 313 + 347$

$E = 3006$  and  $E/2 = 1503$

$E/2$  is  $O3n$   
 $1503 - 2 = 1501$  and  $1503 + 2 = 1505$   
 $1501 \rightarrow 1503 \rightarrow 1505$   
 $1483(+18) \rightarrow 1501 \rightarrow 1503 \rightarrow 1505 \rightarrow 1523(-18)$   
 $3006 = 2x 1503 = 1483 + 1523$

**3. Key Digits of primes numbers and equidistance to verify Goldbach's Strong Conjecture**

**3a. The Goal Behind the Use of Prime Digits**

Between two primes there is an integer in the middle the double of which is the even equal to the sum of the two primes.

$$q > p \rightarrow q - p = 2n \rightarrow p \leftarrow E/2 \rightarrow q \rightarrow 2 \times E/2 = 2n = p + q.$$

$$p = E/2 - t \text{ and } q = E/2 + t \text{ and } p + 2t = q.$$

The integer in the middle is the mean =  $(p + q)/2$ . The unit digit of this average value always depends on the units or other key digits of the prime numbers  $p$  and  $q$ . Here we demonstrate that starting from these digits, we can convert an even into the sum of two prime numbers.

First, formulas are given following this article that are based on the location of the digits of prime numbers. Secondly, how to apply them to convert an even number into a sum of two prime numbers. Note that these formulas do not cover all the rules that we could know but only a part and the purpose of this paper is mainly to show that they exist. Here examples of calculation are given for illustration purposes.

### 3b. Unit Digits in Case $q - p = 6$ or $q - p = 10$ or $q - p = 42$ and therefore $t = 3$ or $t = 5$ or $t = 21$ .

There are very common gaps between prime numbers that can be deduced from their unit digits in case of all primes (see below). There is always a  $2n$  gap between two primes and an integer in the middle between them. Hence Omnipresence of the equidistance between primes in all-natural numbers.

Here gaps of 3, 10 and 21 are used. The gap used is indicated in parentheses (see below). In case of gap = 6 and  $t = 3$ , the difference between the unit digits is 6 example 11 and 17 (7 - 1). However if the unit of the larger prime is < that of the smaller prime we have to add 10 to see the gap. For example 17 and 23, we do not do 3

- 7 but  $13 - 7 = 6$ . See in those having three digits we can see the difference of 6 between the last two digits example 103 and 109 ( $9 - 3 = 6$ ) or 131 and 137 ( $7 - 1 = 6$ ). Here are other examples not shown. The primes numbers 1187 and 1193 such that  $193 - 187 = 6$  or 1117 and 1123 such that  $123 - 117 = 6$ . In case of gap = 10 the larger and smaller primes have the same unit digit. In case of gap = 42 =  $2 \times 21$ , the unit digit of the larger is = that of the smaller + 2. For example 11 and 53 ( $3 = 1 + 2$ ). Again we add 10 if the unit digit of the larger prime < that of the smaller example 19 and 61 such that 1 of 61 + 10 - 9 = 2. All this show that digits of primes are the result of the gaps that are in between them. In addition, the digits of the even that are sum of two primes are also the result of those gaps. In case of gap = 6,  $t = 3$ , we see that the difference between the unit digit of the average value (the integer in the middle) and that of the smaller prime = 3. Again we add 10 in case it is smaller example in case 5-8-11 we have  $8 - 5 = 3$  or 11-14-17 we have  $4 - 1 = 3$  but in case of 37-40-43 we do  $10$  (0 of 40) - 7 (of 37) = 3. We see that the same difference can be deduced from the unit digit of the larger prime and that of the integer in between example 11-14-17 ( $7 - 4 = 3$ ); 47-50-53 ( $3 - 0 = 3$ ).

5	→ 8	→ 11 (+ 6)	3	→ 8	→ 13 (+10)
7	→ 10	→ 13	7	→ 12	→ 17
11	→ 14	→ 17	13	→ 18	→ 23
13	→ 16	→ 19	19	→ 24	→ 29
17	→ 20	→ 23	31	→ 36	→ 41
23	→ 26	→ 29	37	→ 42	→ 47
31	→ 34	→ 37	43	→ 48	→ 53
37	→ 40	→ 43	61	→ 66	→ 71
41	→ 44	→ 47	73	→ 78	→ 83
47	→ 50	→ 53	79	→ 84	→ 89
53	→ 56	→ 59	97	→ 102	→ 107
61	→ 64	→ 67	127	→ 132	→ 137
			139	→ 144	→ 149
67	→ 70	→ 73	11	→ 32	→ 53 (+ 42)
73	→ 76	→ 79	17	→ 38	→ 59
97	→ 100	→ 103	19	→ 40	→ 61
103	→ 106	→ 109	29	→ 50	→ 71
107	→ 110	→ 113	31	→ 52	→ 73
131	→ 134	→ 137	37	→ 58	→ 79

### 3c. Algorithm-II for the Conversion of an even Number into the sum of two Prime Numbers by their units' digits.

Even numbers have 0, 2, 4, 6, and 8 as unit digits and so if an even number is the sum of two prime numbers, the unit digits of the latter count to deduce that of the even number of which they are the

sum. The partitions of 0, 2, 4, 6, and 8 are therefore determinant in Goldbach's conjecture excluding the number 5 because we know that there is no prime number ending with this digit. For example evens ending with 8 are sums of two primes having units digits such 1 and 7 or 9 twice and those with 4 are sums of primes with 1 and 3 or 7 twice while 6 is either sum of two primes having both 3

as units' digits or 7 and 9.

This counts for converting an even number into the sum of two primes. Let  $E$  be an even number and calculate  $\pi(E)$ . We separate the primes  $< E/2$  and those  $> E/2$ . Knowing that the even number is the sum of a prime  $< E/2$  and another  $> E/2$ , we will then sort the prime numbers of  $\pi(E) < E/2$  and  $> E/2$  according to their unit digits. Note that this process is symmetrical for example if we take prime numbers  $< E/2$  ending with a digit like 1 and those  $> E/2$  ending with a digit like 7, we must also do the inverse or the reciprocal i.e. those  $< E/2$  having 7 as the unit digit and those  $> E/2$  having 1 as the unit digit. *In all cases and each time we convert an even  $> 4$  into a sum of two prime numbers, we apply these rules whether we realize it or not. This article seeks, however, to state them.*

This selection of primes by their unit digits therefore results in a method or a **new algorithm-II** that is simple to execute in computer science and which eliminates all other useless prime numbers. For example, an even number ending in 4 will not be affected by all the prime numbers ending in 9 since there are no prime numbers ending in 5. This is also the case for evens with unit digit = 8 with primes ending in 3. This exclusion accelerates the process of converting an even number into the sum of two prime numbers. There is also the total number of digits. For example, an even number of 4 digits will be the sum of a prime number of

2 digits and another of not less than 3 digits. A three-digit prime number depending on its value relative to the even number of 4 digits to be converted will add either another three-digit prime or a 2-digit prime or even primes of one digit. The two selections can be superimposed, those based on the unit digit and those based on the total number of digits, which will further speed up the process of converting an even number into the sum of two prime numbers. In this article, we restrict ourselves to safer and simpler rules that go in one direction only. For example, eliminate prime numbers with 9 as the unit digit as soon as we convert an even number ending in 4 or those with 3 with even numbers ending in 8. The algorithm-II is more robust with only one parameter which is the unit digits.

Here we give a single example of  $E = 580$  and  $E/2 = 290$  by using unit digit only and the rule applies in the same way in the case of other unit digits and for any even number. Indeed, a prime number has 1, 3, 7 and 9 as unit digits. Therefore, an even number that ends in 0 is either the sum of two prime numbers having 3 and 7 as unit digits or 9 and 1. We will then look for them before  $E/2$  and after  $E/2$  and identify those that are equidistant. We can do it in a reciprocal manner, that is to say, look for those having 1 before  $E/2$  and 9 after  $E/2$  and vice versa (same for 3 and 7). **Only those prime numbers that satisfy the partition rule of the unit digit of the even number count to convert it into a sum of two prime numbers** (Tables 7A+7B).

**Table 7A**

$< E/2 = 290$	$> E/2 = 290$	P	P'	$< E/2 = 290$	$> E/2 = 290$	P	P'
11	269	11	569	19	311	59	521
31	349	71	509	29	331	89	491
41	359	101	479	59	401	149	431
61	379	131	449	79	421	179	401
71	389	191	389	89	431	269	311
101	409			109	461		
131	419			139	491		
151	439			149	521		
181	449			179	541		
191	479			199	571		
211	499			229			
241	509			239			
251	569			269			
271							
281							

**Table 7B**

$< E/2 = 290$	$> E/2 = 290$	P	P'	$< E/2 = 290$	$> E/2 = 290$	P	P'
7	293	17	563	3	307	3	577
17	313	137	443	13	317	23	557
37	353	197	383	23	337	113	467
47	373	227	353	43	347	233	347
67	383			53	367	263	317
97	433			73	397		
107	443			83	457		

127	463			103	467		
137	503			113	487		
157	523			163	557		
167	563			173	577		
197				193			
227				223			
257				233			
277				263			
				283			

**Table 7 : Algorithm-II application. Conversion of an even (E = 580 and E/2 = 290) following the partition in sum of 0, let there be prime numbers with digits 1 and 9 (table 7A) or 3 and 7 (Table 7B). The equidistant prime numbers P and P' make a sum = 2 x E/2 = 2 x 290 = 580. Note that this method based on digits operates in the two directions (1,9) and (9,1) (table 7A) or (3,7) and (7,3) in table 7B.**

**3d. Digits in case XX and X0X or XX and X00X.**

The remainder of this article focuses on the relationship that exists between numbers which have in common digits placed in key positions. When applied to prime numbers, we find the average or integer that is equally distant from two prime numbers, therefore confirming again that prime equidistance is omnipresent among integers. Finally, these rules will be used to convert any even into prime numbers with some examples.

- Let note digits by X except the key digit. Example XX and X0X. Here 0 is a key digit and so  $X0X - XX = 90$ .  $11$  and  $101 \rightarrow 101 - 11 = 90 \rightarrow 11 + 45 = 56$  and  $101 + 45 = 146 \rightarrow 56 + 146 = 202 = 101 \times 2$ .

Hence  $11 \leftarrow 56 \rightarrow 101$  so that  $56 - 11 = 101 - 56 = 45$ .  
 $17$  and  $107 \rightarrow 107 - 17 = 90 \rightarrow 17 + 45 = 62$  and  $107 + 45 = 152 \rightarrow 62 + 152 = 214 = 107 \times 2$ .

Hence  $17 \leftarrow 62 \rightarrow 107$  so that  $62 - 17 = 107 - 62 = 45$ .  
 •  $X00X - XX = 990$ .

$13$  and  $1003 \rightarrow 1003 - 13$  and  $\rightarrow 1000 - 10 = 990$ . Then,  $990 : 2 = 495 \rightarrow 13 + 495 = 508$  and  $1003 + 495 = 1498 \rightarrow 508 + 1498 = 2006 = 2 \times 1003$ . Hence  $13 \leftarrow 508 \rightarrow 1003$ .

$X000n...X - XX = 9999n \ 0$ . Example  $1000003 - 13 = 999990$ .

**3e. Digits in case XXY and XX.**

$XXY - XX = XX (10 - 1) + Y$ .

Examples :

101	103	107	109	113	127	131	137	139	149
151	157	163	167	173	179	181	191	193	197
199	211	223	227	229	233	239	241	251	257
263	269	271	277	281	283	293	307	311	313
317	331	337	347	349	353	359	367	373	379
383	389	397							

Note this is not limited to numbers with three digits but to infinity as show below with numbers of 9 digits.

$139 - 13$ . Given that  $(139) - 13 = (13 \times 10 + 9) - 13 = 13 \times (10 - 1) + 9 = 13 \times 9 + 9 = 126$ .

$126 : 2 = 63 \rightarrow 13 + 63 = 76$  and  $139 + 63 = 202 \rightarrow 76 + 202 = 278 : 2 = 139$ . Therefore

$13 \leftarrow 76 \rightarrow 139$ .

$173 - 17 = 17 \times 9 + 3 = 156 \rightarrow 156 : 2 = 78$ . Therefore  $17 + 78 = 95$ . And  $95 + 78 = 173$ .

$17 \leftarrow 95 \rightarrow 173$ .

**3f. Digits in case YXX and XX  $\rightarrow$  YXX - XX = Y00 or Yn... XX - XX = Yn...00.**

$661 - 61 = 600$ . Given  $600 : 2 = 300$  we have  $61 + 300 = 361$  and  $361 + 300 = 661$ .

$61 \leftarrow 361 \rightarrow 661 \rightarrow 361 \times 2 = 61 + 661 \rightarrow 722 = 61 + 661$ .

$673 - 73 = 600$ . Given  $600 : 2 = 300$  we have  $73 + 300 = 373$  and  $373 + 300 = 673$ .

$73 \leftarrow 373 \rightarrow 673 \rightarrow 373 \times 2 = 73 + 673 \rightarrow 746 = 73 + 673$ .

**3g. Common digits in primes XX...ZZ (ZZ are digits of primes p such that  $3 \leq p \leq 97$ )**

The digits of primes  $\leq 97$  are often at the end of prime numbers of three digits and more to infinity.

85412401	785412409	785412449	785412469	785412479	785412491	785412503	785412517
785412553	785412569	785412571	785412581	785412583	785412601	785412613	785412619
785412697	785412701	785412731	785412737	785412751	785412781	785412791	785412811
785412853	785412877	785412893	785412919	785413021	785413037	785413049	785413081
785413151	785413207	785413217	785413229	785413249	785413273	785413297	785413309
785413423	785413459	785413477	785413483	785413493	785413543	785413553	785413609
785413679	785413691	785413703	785413721	785413751	785413771	785413781	785413793
785413829	785413883	785413901	785413907	785413921	785413927	785413949	785413961
785414083	785414107	785414111	785414159	785414177	785414209	785414213	785414237

In all cases  $XXXn...YY - YY = XXXn... \times 10^2$  (n is the total number of the digits of the number reduced by 1). Example 107 has three digits and so  $n = 2$  and therefore  $107 - 07 = 01 \times 10^2 = 100$ . Given that  $100 : 2 = 50$

we have

$7 + 50 = 57$  and  $57 + 50 = 107$ . Therefore  $7 \leftarrow 57 \rightarrow 107$  and so  $57 \times 2 = 114 = 7 + 107$ . Note here that we use this rule of digits to find the even which makes the sum of the two primes  $XnYY$  and  $YY$ .  $785412613 - 13 = 785412600 : 2 = 392706300$ . And we have  $13 + 392706300 = 392706313$  and  $392706313 + 392706300 = 785412613$ . Therefore,  $13 \leftarrow 392706313 \rightarrow 785412613$ .

If we do not have a prime at  $YY$  such the case of the prime number 785412791, we can still perform the calculation.  $785412791 + 6 \rightarrow 785412797 - 97 = 785412700$  and  $785412700 : 2 = 392706350$  then  $392706350 + 97 = 392706447$  and  $392706447 + 392706350 = 785412797$ . We have  $97 \leftarrow 392706447 \rightarrow 785412797$  and therefore  $(97 + 6) \leftarrow 392706447 \rightarrow (785412797 - 6)$ . Then  $(103) \leftarrow 392706447 \rightarrow (785412791)$ .

### 3h. Convert an even in sum of two primes by starting with close or neighboring prime numbers :

These rules will now be used to put Goldbach's strong conjecture into practice and thus convert an even number into the sum of two prime numbers. Although some specific numbers are used here, the method applies to any even  $> 4$  as described.

The method is to look for two prime numbers around the even to convert into the sum of two prime numbers and put them in the form  $Xn...YY - YY$  and from there convert it. Here is a detailed example, but we will limit ourselves to a single conversion afterwards.

$E = 84$  as an example.

$83 - 3 = 80 : 2 = 40$ , then  $3 + 40 = 43$ , and  $43 + 40 = 83$ .

$3 \leftarrow 43 \rightarrow 83 \leftrightarrow 3(+2) \leftarrow 43(-1) \rightarrow 83(-4) \leftrightarrow (-5) + (-2) = -3$ .

If subtracting  $x$  from the terms of the equation on the right or left is the same as subtracting  $x/2$  of the center. However, the center remains unchanged if subtracting and adding a same quantity from the two terms. In this example, we added 2 to 3 and subtracted 4 from 83 which means subtraction of 2 then we subtract 1 from the

center such that  $3(+2) \leftarrow 43(-1) \rightarrow 83(-4) \leftrightarrow 5 \leftarrow 42 \rightarrow 79$ . We continue by the addition and subtraction of equal  $6n$  gaps on terms on the left and right.

$3(+2) \leftarrow 43(-1) \rightarrow 83(-4) \leftrightarrow 5 \leftarrow 42 \rightarrow 79 \leftrightarrow 11 \leftarrow 42 \rightarrow 73 \leftrightarrow 17 \leftarrow 42 \rightarrow 67 \leftrightarrow$

$23(+24) \leftarrow 42 \rightarrow 61(-24) \leftrightarrow 47 \leftarrow 42 \rightarrow 37 \leftrightarrow 53(+8) \leftarrow 42 \rightarrow 31(-8) \leftrightarrow 61 \leftarrow 42 \rightarrow 23$

$42 \times 2 = 84 = 5 + 79 = 11 + 73 = 17 + 67 = 23 + 61 = 47 + 37 = 53 + 31 = 61 + 23$ .

Let us convert 240 in sum of two primes following the same method. First we put 240 in the form of  $YXX$  and  $XX$  so we can apply one of the rules shown above such like  $240 = 70 + 170$ . The sum chosen must give numbers close to primes such that 67 and 73 for 70 or 173 for 170. The numbers 173 and 73 are in the form  $YXX$  and  $XX$  and can be used in a subtraction like seen above.

$173 - 73 = 100 : 2 = 50$  and so we have  $73 + 50 = 123$  and  $123 + 50 = 173$ . Therefore  $73 \leftarrow 123 \rightarrow 173$  but  $123 \times 2 = 246 = 73 + 173$ . We therefore make  $(73 - 6) \leftarrow 123(-3) \rightarrow 173(-0) \rightarrow (67) \leftarrow 120 \rightarrow 173 \rightarrow 240 = 67 + 173$ . Or  $73 \leftarrow 123(-0) \rightarrow (173 - 6) \rightarrow 240 = 73 + 167$ .

In both  $(73 - 6) \leftarrow 123 \rightarrow 173$  and  $73 \leftarrow 123 \rightarrow (173 - 6)$  it is needed to equilibrate this way :

$(73 - 6) \leftarrow 123 - 3 \rightarrow 173$  and  $73 \leftarrow 123 - 3 \rightarrow (173 - 6)$ .

Let us convert a number like 1268. Then using the same calculation as above let us pose  $1268 = 38 + 1230$ . We take  $1268 + 6 = 1274 = 37 + 1237$ .

Then let take a close prime 1237. Then  $1237 - 37 = 1200 : 2 = 600$ . We have  $37 + 600 = 637$  and  $637 + 600 = 1237$ . We then have  $37 \leftarrow 637 \rightarrow 1237$ . But  $637 \times 2 = 1274 = 37 + 1237 = 1274$  and  $1274 - 1268 = 6$ . Therefore  $(37 - 6) \leftarrow 637(-3) \rightarrow 1237 \leftrightarrow 31 \leftarrow 634 \rightarrow 1237$  and then  $31 + 1237 = 1268$ . Otherwise  $37 \leftarrow 637 \rightarrow (1237 - 6) \rightarrow 37 + 1231 = 1268$ . Or  $37(-18) \leftarrow 637 \rightarrow (1237 + 12) \rightarrow 19 + 1249 = 1268$ . In all these cases we have to subtract 3 from the middle namely  $637 - 3 = 634$ .

1)Note that in both two examples cited above the conversion of 240 and 1268 we have a difference =  $6n$  ( $n = 1$ ) which is the best way to find out new primes and increase the combinations of primes in sums. In the case of 340 we start with  $346 = 73 + 173$

and in the latter we start with  $1274 = 37 + 1237$ .

2)  $N1 \leftarrow M \rightarrow N2$  is an equation or a balance that we can modify by adding and subtracting the same quantity from both sides ( $N1$  and  $N2$ ). If we add and subtract a quantity  $x$  for from  $N1$  and  $N2$ , we must add or subtract  $x/2$  to or of  $M$ .

Let convert **18985474**. We add  $6n$  to it like  $18985474 + 6 = 18985480$ . Then examine the list of prime numbers so that we have  $18985480 = 29 + 18985451$ . We then have  $18985451 - 51 = 18985400 : 2 = 9492700 + 51 = 9492751$  and then  $9492751 + 9492700 = 18985451$ . Therefore  $51 \leftarrow 9492751 \rightarrow 18985451$ .

But  $9492751 \times 2 = 18985502 - \mathbf{18985474} = 28$ . We then have to search in primes list to find out how to add them by standing close to  $18985451$ . Then we have  $51 (-28) \leftarrow 9492751 (-14) \rightarrow 18985451$  and finally  $23 \leftarrow \mathbf{9492737} \rightarrow \mathbf{18985451}$ . Therefore,  $9492737 \times 2 = 18985474 = 23 + 18985451$ .

We will do the same method again but this time by drawing up a table and thus writing the step-by-step instructions to follow to apply this method. Let us see this with a number like **489776**. First  $489776 + 6 = 489782$  which is converted into the sum of two terms, the larger of which is prime such that for example  $489782 = 15 + 489761$ . We then establish the balance equation between the two terms of the addition and the average at the center as shown above.

$489761 - 61 = 489700 : 2 = 244880 + \mathbf{61} = 244941$ , and then  $244941 + 244880 = 489821$ . Then  $\mathbf{61} \leftarrow \mathbf{244941} \rightarrow \mathbf{489821}$  (**not prime**). However,  $244941 \times 2 = 489882$  as expected. We have  $489882 - \mathbf{489776} = 106$ . We will have to remove 106 units from this number to find our initial number **489776** while converting it into sums of two prime numbers.

- n	489821↓	Prime or not	+ n	61↑	Prime or not
<b>106</b>	489715	not	0	<b>61</b>	prime
6	489709	not	6	67	prime
6	489703	not	6	73	prime
6	489697	not	6	79	prime
6	489691	prime	6	85	not
12	<b>489679</b>	<b>prime</b>	12	97	<b>prime</b>
6	<b>489673</b>	<b>prime</b>	6	103	<b>prime</b>
60	<b>489613</b>	<b>prime</b>	60	163	<b>prime</b>
60	<b>489553</b>	<b>prime</b>	60	223	<b>prime</b>

Therefore we get 4 conversion of the number **489776** =  $489679 + 97$  ; **489776** =  $489673 + 103$  ; **489776** =  $489613 + 163$  ; and **489776** =  $489553 + 223$ . This can be continued further.

Let take a number like 890 and then convert it in sum of two primes by manipulation of digits. We have  $890 = 880 + 10 \rightarrow 10$  is either  $7 + 3$  or  $9 + 1$  to get a prime. We then pose the possible combinations either  $883 + 7$  or  $881 + 9$ . Therefore  $880 = 883 + 7$  but in the second case 9 is not prime, but we can transfer units from 881 to 9 and so :  $881 - 4 = 877$  and  $9 + 4 = 13 \rightarrow 890 = 13 + 877$   $881 - 22 = 859$  and  $9 + 22 = 31 \rightarrow 890 = 31 + 859$

Note we cannot use a number with unit digit = 6 to add to 9 because we will get 5 ad thus not prime.  $881 - 853 = 28$  and  $9 + 28 = 37 \rightarrow 890 = 37 + 853$   $881 - 52 = 829$  and  $9 + 52 = 61 \rightarrow 890 = 61 + 829$  and so on.

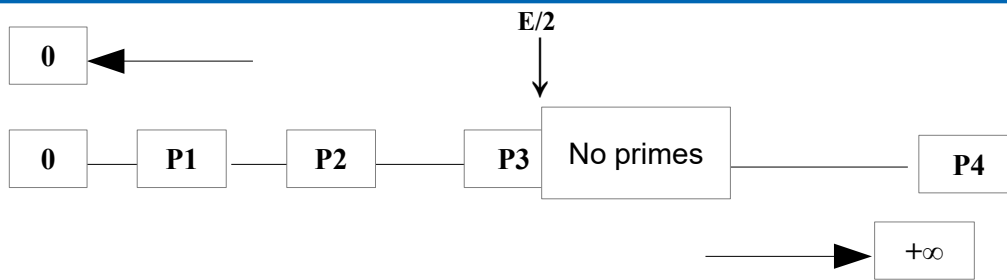
Let us take another example. An even number has a unit digit = 0, 2, 4, 6, 8. The number 78956. Here we start with  $78956 = 78950 + 6$  with  $6 = 1 + 5$  or  $6 = 3 + 3$ . Then we make  $78956 = 78953 + 3$  or  $78956 = 78951 + 5$ . However neither 78953 nor 78951 is prime. We can this time pose  $78956 = 78940 + 16$  with  $16 = 15 + 1$  ;  $13 + 3$  ;  $11 + 5$  ; and  $9 + 7$ . Then  $78956 = 78941 + 15$  (not primes) ;  $78956 = 78943 + 13$  ;  $78956 = 78949 + 7$  but none are primes.

However, all those sums give us chances to find out two primes that sum up. For instance,  $78956 = 78953 + 3 = (78919 + 34) + 3 = 78919 + 37$  (both primes).  $78956 = 78953 + 3 = (78889 + 64) + 3 = 78889 + 67$  (both primes).  $78956 = 78953 + 3 = (78877 + 76) + 3 = 78877 + 79$  (both primes) and so on.

Let E be an even number such that  $E = (A\downarrow) + (1\uparrow)$  or  $E = (A\downarrow) + (7\uparrow)$ . A is an odd number that can be prime or composite but not  $3n$ . So  $E = (A\downarrow - 6n) + (1\uparrow + 6n)$  or  $E = (A\downarrow - 6n) + (7\uparrow + 6n)$  such that  $A - 6n$  and  $1 + 6n$  or  $7 + 6n$  ( $1 \leq n \leq +\infty$ ) will produce other odd numbers that are either composite or prime. Let us assume this time that A is in infinity and therefore  $(A - 6n\downarrow)$  will tend to 0. Conversely,  $(1 + 6n\uparrow)$  or  $(7 + 6n\uparrow)$  will tend to infinity. We will therefore admit that  $A - 6n\downarrow$  or  $1 + 6n\uparrow$  or  $7 + 6n\uparrow$  will produce all the prime numbers that we know or that exist. Whether we start from infinity to 0 or from 0 to infinity, we will see the same prime numbers with the same gaps in opposite directions. Since every prime number occupies a position, we can predict with certainty that at times both  $A - 6n\downarrow$  and  $1 + 6n\uparrow$  or  $7 + 6n\uparrow$  are primes.

During this process, the equidistant prime numbers continue to add up, however large they may be. The gaps devoid of primes are compensated by the infinite inter-prime gaps that exist between the prime numbers taken two by two to infinity (Figure 1).





**Figure 1 :** It shows that while  $P_4 - P_3$  might be relatively large because of the empty sequence of primes, the difference between  $P_3$  and  $P_2$  or  $P_1$  (which are close to 0) would be as large or even larger. Let us call  $P_0$  all primes close to 0. The very higher density of primes  $P_0$  increases the chances that a difference  $P_3 - P_0 = P_4 - P_3$  which is a correct verification of the strong Goldbach conjecture. Because this implies that  $E/2 - P_0 = P_4 - E/2$  knowing that  $E/2$  is very close to  $P_3$ . Hence  $P_0$  and  $P_4$  are equidistant at  $E/2$ .

#### 4. Discussion

The main idea of this article lies in the fact that Goldbach's strong conjecture is associated with the progression of natural integers from 0 to infinity, which results in precise gaps between prime numbers. The gap of 6 is the most regular between primes  $6x + 1$  on the one hand and primes  $6x - 1$  on the other. This progression means that any integer is preceded and followed by a prime number. Since all  $2n$  distances are possible between the primes  $6x + 1$  and  $6x - 1$ , it follows that a natural number is always located at an equal distance from two primes of which one precedes it and the other follows it. This paper shows that prime equidistance is omnipresent among integers.

Two new concepts are introduced in this article for the first time. The first is the Goldbach gap, noted here as  $t$ , which for any even number noted  $E > 4$  means the gap separating its half  $E/2$  from two symmetrically equidistant primes  $p$  and  $p'$  such that  $p' > p$  and such that  $E/2 - t = p$  and  $E/2 + t = p'$ .  $E/2$  represents any natural number  $> 2$  and since  $p - E/2 = -p'$  then  $2 \times E/2 = E = p + p'$ . Without Goldbach's gaps, an even number would not be the sum of two primes. The second concept is the Goldbach transposition, which means that adding a given quantity to a number will result in a prime number, not necessarily in the same position, but somewhere between 0 and  $E$ , or in  $\pi(E)$ . If an integer is an even  $3n$ , we should add to it values of prime numbers in an increasing order like for example ( $E = 120$ ,  $E/2 = 60$ , to get primes we do  $60 + 7$ ;  $60 + 11$ ;  $60 + 13$ ;  $60 + 17$ ;  $60 + 19$ ;  $60 + 23$ ; ...). If it is non- $3n$ , then odd  $3n$  values should be added to it (like for example  $80 + 3$ ;  $80 + 9$ ;  $80 + 15$ ;  $80 + 21$ ;  $80 + 27$ , ...). Even numbers obviously differ in their values, but Goldbach's transposition means that there are common rules governing the appearance of equidistant primes.

Using  $3x + 5$  and  $3x - 5$  equations and by examining the remainders ( $r$ ) of Euclidean divisions of  $3x$  by prime factors  $q < \sqrt{3x}$ , if  $5 = q - r$ , we can understand why a number  $3x + 5$  is prime or composite and in a similar way if  $5 = r$  the  $3x - 5$  number is composite. Both equations show that any integer  $> 4$  is bounded by two equidistant primes. This led to the development of a new algorithm for converting even numbers into the sum of two primes, whatever the even number in question. Given the equality or balance  $X - E/2 = Y$ , we do  $(X + 6n) - E/2 = (Y - 6n)$  until we obtain  $p' - E/2 = p$ , which means  $2 \times E/2 = E = p + p'$ . This article is the first

to offer this conversion algorithm. Others have used the equation  $6x \pm 1$  to prove Goldbach's strong or to locate the equidistant primes around composite numbers [11,12]. However, the present article is original in that it uses a different equation  $3x \pm 5$  by examining the remainders of Euclidean divisions, and furthermore shows that any integer  $> 4$  is surrounded by equidistant primes and not just composites. In addition, this article is of practical interest, as it proposes two algorithms that could be easily programmed and links the key digits of the prime numbers to the gaps that separate them and thus to Goldbach's strong conjecture. Prime numbers digits can also be used to predict gaps between primes, and even to use a near or neighboring prime to convert an even into the sum of two primes. This approach is also original in this article. Using unit digits of primes, the proposed algorithm of even conversion in sum of two primes can be greatly improved by only focusing on primes whose unit digits are suitable to form the even.

All the data in this article argue strongly in favor of the truth of Goldbach's strong conjecture at infinity.

The other idea that prevails in mathematics is the following question: if we start from evens (and not with prime numbers as usually done), how many prime numbers will they be the sum of? However let's be precise, Goldbach did not rule that every even *is only the sum of two prime numbers*, but he just said that every even is the sum of two prime numbers. In mathematics, we sometimes emphasize propositions or conjectures with words like *only* or *if only if* to set the context, but Goldbach used no such emphasis. An even number can therefore be the sum of more than two prime numbers; the larger it is, the more it is the sum of several prime numbers, or even an infinity. An infinite even number is the sum of an infinity of even numbers, each of which is the sum of two prime numbers or even much more. But what Goldbach's conjecture means is that the sum of two prime numbers is the most common and invariable form of all evens. While two evens of different values can be sums of variable numbers of primes, Goldbach's conjecture tries to find a common and invariant and minimalist property, i.e. every even is the sum of TWO prime numbers.

Goldbach wanted an invariable law for all evens which unites them all and any law is defined in minimal conditions otherwise it would be subject to exceptions. Therefore the sum of two prime numbers

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would be this law common and true to all the even numbers (which nevertheless continue to vary by the possible number of prime numbers of which they can be the sum). This is undoubtedly the most decisive point and the deepest meaning of this conjecture. It would rather be wise to know if all the even numbers are the sum of two prime numbers at the same time as they are the sum of several prime numbers. However, this article shows that an integer can only be surrounded by two equidistant prime numbers at a time and therefore Goldbach's conjecture is correct. This article sets up an algorithm that starts with an even to convert in sums of two primes.

What this article shows is that any integer  $(E/2 \text{ or } n) > 2$  is in the middle of two or more equidistant prime numbers whose sum always gives the same even number  $(E \text{ or } 2n) > 4$ . This configuration of numbers is natural, it is in this way that the natural numbers progress unit by unit to infinity. The natural numbers form a single set and therefore equidistant prime numbers will go to infinity.

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