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New Methods Based on the Calculation of Specific Decimal Fractions for Decomposing an Integer into a Product of Prime Factors

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Abstract

This article presents for the first time two methods for decomposing integers in products of prime factors which are based on the calculation of decimal fractions. Its originality lies in the fact that the divisors used are decimals and not prime divisors and in addition the decimal part is manipulated in such a way that two decimal digits are fixed and the others are variable. In the first method, the divisors are of type 2^n and which have a very interesting particularity which is that they always have two same digits at the end of their decimal parts (25 or 75). And it is this particularity which is exploited to develop these methods. The other method introduces a new notion that of the decomposition key which is a product of prime factors used to decompose all numbers having the same number of digits. It is similar to the first method because it also uses decimal fractions for the calculation and the denominator is the square root. This article paves the way for new applications in computer science.

Keywords: Method, Factorization, Integers, Prime Numbers, Decimal Fraction, Decomposition

1. Introduction

In mathematics, the fundamental theorem of arithmetic, also called the unique factorization theorem and prime factorization theorem, states that every integer greater than 1 can be represented uniquely as a product of prime numbers, up to the order of the factors (https:// en.wikipedia.org/wiki/Fundamental_theorem_of_arithmetic). However, the problem is to be able to transform any number into a product of prime factors since a theorem must always be correct whatever the number. The solution that has always been imposed is the Euclidean division in series by prime factors called trial division [1]. Trial division is the most laborious but easiest to understand of the integer factorization algorithms. The essential idea behind trial division tests is to see if an integer n, the integer to be factored, can be divided by each prime number in turn that is less than the square root of n. It is for this reason that it has become necessary to develop new algorithms for decomposing natural numbers by taking advantage of the high computing speed of computers. Nowadays, these algorithms are capable of decomposing numbers containing several digits but are all limited by the value of the number to be factored, the larger it is, the more time and energy it requires [2, 3]. Among numbers, biprime numbers with two prime factors relatively closer in value are the most difficult to decompose. Hence their interest for cryptology. The problem of determining whether a given integer is prime is one of the better known and most easily understood problems of pure mathematics. This problem has caught the interest of mathematicians again and again for centuries. However, it was not until the 20th century that questions about primality testing and factoring were recognized as problems of practical importance, and a central part of applied mathematics. The advent of cryptographic systems that use large primes, such as RSA, was the main driving force for the development of fast and reliable methods for primality testing [4].

The defining property of a prime number p is that it is a positive integer $p \geq 2$ that is only divisible by 1 and p. Equivalently, p is prime if and only if p is a positive integer $p \geq 2$ that is not divisible by any integer m such that $2 \leq m < p$. A positive integer $n \geq 2$ which is not prime is called composite. A primality test is a mathematical procedure for determining whether a given number is a prime number (i.e. it has no divisor other than by 1 or itself). How to know if a number is a prime? To determine if a number is prime, it must now pass a primality test which confirms its status as a prime number. There exist several primeness tests to know if a number is a prime number, the oldest is the Sieve of Eratosthenes, and the most common are Miller–Rabin and Lucas-Lehmer tests (https:// www.dcode.fr/primality-test). In number theory, the general number field sieve (GNFS) is the most efficient classical algorithm known for factoring integers larger than 10^{100} (Wikipedia).

There are broadly two categories of number primality tests: A deterministic primality test provides a certain answer about the primality of a number (it guarantees whether the number is prime or not). In contrast, a probabilistic test provides a probable answer, with a controlled margin of error, which means it can tell whether a number is probably prime or probably composite. The Sieve of Eratosthenes is a simple and ancient algorithm used to find the prime numbers up to any given limit [5, 3]. It is one of the most efficient ways to find small prime numbers. For a given upper limit ?n the algorithm works by iteratively marking the multiples of primes as composite, starting from 2. Once all multiples of 2 have been marked composite, the muliples of next prime, ie 3 are marked composite. This process continues until p≤n where p is a prime number (https://brilliant.org/wiki/sieve-of-eratosthenes).

2. The New Proposals in this Article

Let us suppose you have a biprime number Bn = p x q such that p < q that you want to decompose and thus find its two prime factors p and q. You do not want to use a decomposition algorithm that automatically gives you the result but you want to proceed differently by doing your own calculation. First, let's agree on some rules. If you multiply Bn by another prime factor denoted r or a multiple of prime numbers s et t denoted M = s x t and you decompose it with your algorithm; you have three possible outcomes: r x p x q; s x t x p x q if your algorithm is able to decompose it, and r x Bn or s x t x Bn if it is not able to do it. In the first case, you have not really decomposed the number Bn by your own calculation because you have just decomposed one of its multiples. In the second case, the algorithm reaches its limit and returns it to you undecomposed. However, a solution is available to you, which is to look for any numbers that have a common factor with Bn. Let us denote these numbers M' = s' x t' x p and M'' = s'' x t'' x q (p or q are the common prime factors between Bn and M' or M''). These numbers M' and M'' can be less than or greater than Bn, that does not matter. In case they are lesser than Bn, we would say that they are its submultiples and in the other that they are its supermultiples.

If you mark multiple by multiple of a prime number in an Erastothenes sieve, and if you choose a multiple in the middle, the submultiples will be those which precede it and the supermultiples those which follow it. A number to be decomposed therefore has a common factor with an infinite number of numbers which are its submultiples or supermultiples. This is the central idea of this article and it is also a truth of Erastothenes' sieve which has always gone unnoticed and which has not been exploited to its fair value to develop a factorization method. This article will then exploit this fact.

In fact, looking for a common factor between M' or M'' and Bn amounts to going up or down the Erasthotenes'sieve, but if you want to do it manually by searching in this sieve for submultiples or supermultiples and knowing that Bn could be a very large number, you will quickly realize that the task is very difficult and very long or impossible to carry out. How then can you do it? This article gives you answers and alternative methods.

Now you understand that the Erasthtotenes'sieve is a fixed structure like a ladder that can only be ascended or descended step by step. You will therefore have to proceed differently and therefore you must carry out a precise and safe calculation. The question is the following: how can we find a submultiple or supermultiple of Bn which has one prime factor in common with it that we will use to divide it while being sure and certain that it is indeed one of its divisors? In fact, finding a submultiple or supermultiple of a number is another way of breaking it down in the strict sense of mathematics. But be careful, the number sought must only have one factor p or q in common with Bn, never both because in the latter case and as said above we only multiply Bn by an integer and then the algorithm gives it back to us in the form of a product of prime factors, this is not decomposition. Decomposing a number means finding another number which has a single prime factor in common with it or finding an integer or decimal or even irrational divisor which gives a quotient which is a multiple of one of its prime factors. These are the main ideas of this article. The conventional idea is that a divisor is always an integer, this article ignores this classic concept and uses decimal (or irrrational) divisors instead.

New calculation methods are thus necessary to which this article is all dedicated. As a result, this article will propose new methods for decomposing natural numbers. These methods use the calculation of decimal fractions between a denominator and a numerator in a continuous manner using calculators available on the web. An available decomposition algorithm is always necessary but will be used differently, only to find the common factor between Bn and one of its sub- or super-multiples and thus decompose it. This article is undoubtedly the first which uses classic mathematics and which decomposes a number by looking for submultiples which have a common factor with it.

All these new tests are deterministic because we only have two cases: 1) if the number is composite, you will at one time or another have an integer quotient between the numerator and the denominator; 2) if the number is prime, the calculation of fractions will run in an incessant loop giving only decimal or irrational numbers until the end. Examples of calculations will be given and the methods explained. Note that the calculation is mainly the calculation of fractions between a numerator and a denominator with a decimal part limited to few digits. The most important thing is to have a calculator capable of doing this calculation in series and continuously.

It is important to note now that the decomposition of a number is all the more difficult as the number is large with large prime factors and that if the known algorithms give a rapid answer it is only thanks to the power of computers. This simplicity is deceptive and breaking down a number could be an extremely difficult or an unachievable task if done manually. But the most important thing is to find an orderly and specific calculation method which leads to the result in a reproducible and programmable way. The calculation must also be done to take the shortest and least expensive path to decompose the number posed. These elements were taken into consideration in the design of the methods described in this article, which will now be described one after the other with illustrative examples.

3. Materials and Methods

This article is based on calculation in the first place which is obviously deduced from an equation which will be described and demonstrated. To calculate all decimal fractions a calculator was used available on the web (https://calculatrices.app/calculatrice-de-grands-nombres). Note that the latter calculates a decimal fraction but not continuously (We must write the decimal denominator). For very large numbers, the calculator (https://www.123calculus.com/) was also used. To factor the numbers, two sites were used: https://www.dcode.fr/decomposition-nombres-premiers; and https://calculis.net/premier. The prime numbers were obtained from the site http://compoasso.free.fr/primelistweb/page/prime/liste_online.php.

4. Results: New Methods of Integer Decomposition

For a method to be valid and even conceivable, it must be based on an invariable and infinitely reproducible fact. Otherwise, the method will generate so many variations and it will be random and impractical. Mathematics reject exceptions and only works with axioms and theorems, but given the great complexity of prime numbers and the decomposition of natural numbers into prime factors, the development of algorithms and probabilistic methods were considered because they take advantage of the high speed of computer calculation and make it possible to verify the authenticity of the theorem of the decomposition of natural numbers at a very high level. In order to avoid probabilistic methods, we must therefore start with sure principles leaving no shadow of doubt.

4.1 Methods Based on the Series of Divisions by 2^n

4.1.1. Principles

As a reminder, this study will be dedicated to biprime numbers because we know that they are the hardest to decompose, especially when their two prime factors are very large halfway between 0 and the square root (SR). But what will be described is valid for any odd number whatever its number of prime factors. If you take an odd number > 1 and divide it by 2 you will have the remainder 1. Any odd number divided by 2 will therefore give a decimal extension 0.5. But if you divide it again by 2 (i.e. 4), you will only have two possible decimal parts 0.25 or 0.75. This is because the remainders of Euclidean divisions of an odd number by 4 is either 1 or 3 and in the first case it is 0.25 and the other it is 0.75. If you continue your divisions by 2n (8, 16, 32, 64, 128, 256, and so on), you will always have a decimal extension which ends either with 25 or 75 (Table 1A-C).

	1/4	1/8	1/16	1/32
91	0.75	0.375	0.6875	0.84375
323	0.75	0.375	0.1875	0.09375
3397	0.25	0.625	0.3125	0.15625
10873	0.25	0.125	0.5625	0.78125
520187	0.75	0.375	0.6875	0.84375
1297603	0.75	0.375	0.1875	0.09375
5321531	0.75	0.375	0.6875	0.84375
20777459	0.75	0.375	0.1875	0.59375
152771243	0.75	0.375	0.6875	0,.34375

Tables 1: Division by 2^n Always Generate 25 or 75 as Digits in the Decimal Parts of the Quotients.
Table 1A

	1/64	1/128	1/256	1/512
91	0.421875	0.7109375	0.35546875	0.177734375
323	0.046875	0.5234375	0.26171875	0.630859375
3397	0.078125	0.5390625	0.26953125	0.634765625
10873	0.890625	0.9453125	0.47265625	0.236328125
520187	0.921875	0.9609375	0.98046875	0.990234375
1297603	0.046875	0.5234375	0.76171875	0.380859375
5321531	0.921875	0.4609375	0.23046875	0.615234375
20777459	0.796875	0.8984375	0.94921875	0.974609375
152771243	0.671875	0.3359375	0.66796875	0.333984375

Table 1B

	1/1024	1/2048	1/4096	1/8192
91	0.0888671875	0.04443359375	0.022216796875	0.0111083984375
323	0.3154296875	0.15771484375	0.078857421875	0.0394287109375
3397	0.3173828125	0.65869140625	0.829345703125	0.4146728515625
10873	0.6181640625	0.30908203125	0.654541015625	0.3272705078125
520187	0.9951171875	0.99755859375	0.998779296875	0.998779296875
1297603	0.1904296875	0.59521484375	0.797607421875	0.3988037109375
5321531	0.8076171875	0.40380859375	0.201904296875	0.6009521484375
20777459	0.4873046875	0.24365234375	0.621826171875	0.3109130859375
152771243	0.6669921875	0.33349609375	0.666748046875	0.8333740234375

Table 1C

These are therefore decimal numbers but whose decimal part is all the longer as n of 2^n tends towards infinity. When 2^n is too high, we can say that the number obtained is between a decimal and irrational number since the digits after the decimal point are unpredictable and non-repeating, except that they always end with 25 or 75. Several examples of numbers of different values that have an increasing number of digits are shown in the following tables. We see that this fact remains true whatever the number of digits of the numbers tested (Table 2A-C). However, the most important fact to note is that for a given 2^n, we have the same number of digits before 25 or 75 in the resulting decimal part. For example, for 1/8, we always have one digit before 25 or 75 (for example 0.375), for 1/16 we have 2 digits (for example 0.6875), for 1/32 we have 3 digits (for example 0.84375) and so on. The decimal part before 25 or 75 increases by one digit when going from 2^n to 2^{n+1}. The number of digits after the decimal separator = n being the exponent of 2^n.

	1/4	1/8	1/16	1/32
4877863751	0.75	0.875	0.4375	0.21875
14048615549	0.25	0.625	0.8125	0.90625
329650448509	0.25	0.625	0.8125	0.90625
1657851033857	0.25	0.125	0.0625	0.03125
59708282945131	0.75	0.375	0.6875	0.34375
472250023232509	0.25	0.625	0.8125	0.90625
772405727553181	0.25	0.625	0.8125	0.90625
5422745703328963	0.75	0.375	0.1875	0.09375
69605379628229681	0.25	0.125	0.0625	0.53125

Tables 2: Division by 2^n still Generate 25 or 75 as Digits in the Decimal Parts of Quotient Obtained with Numbers having Increasing Digits.
Table 2A

	1/64	1/128	1/256	1/512
4877863751	0.109375	0.5546875	0.27734375	0.638671875
14048615549	0.953125	0.9765625	0.48828125	0.244140625
329650448509	0.953125	0.9765625	0.48828125	0.244140625
1657851033857	0.015625	0.0078125	0.00390625	0.501953125
59708282945131	0.671875	0.8359375	0.41796875	0.208984375
472250023232509	0.953125	0.9765625	0.98828125	0.994140625
772405727553181	0.453125	0.2265625	0.61328125	0.306640625
5422745703328963	0.046875	0.5234375	0.76171875	0.380859375
69605379628229681	0.765625	0.3828125	0.19140625	0.095703125

Table 2B

	1/1024	1/2048	1/4096	1/8192
4877863751	0.8193359375	0.40966796875	0.704833984375	0.3524169921875
14048615549	0.1220703125	0.56103515625	0.780517578125	0.8902587890625
329650448509	0.1220703125	0.06103515625	0.530517578125	0.2652587890625
1657851033857	0.2509765625	0.12548828125	0.562744140625	0.7813720703125
59708282945131	0.2509765625	0.12548828125	0.562744140625	0.7813720703125
472250023232509	0.9970703125	0.49853515625	0.249267578125	0.1246337890625
772405727553181	0.6533203125	0.82666015625	0.413330078125	0.2066650390625
5422745703328963	0.1904296875	0.59521484375	0.797607421875	0.3988037109375
69605379628229681	0.0478515625	0.52392578125	0.261962890625	0.1309814453125

Table 2C

In tables 3A-B, there are two examples of 30- and 34-digit numbers produced by three prime factors of different values. Dividing them by 2^n in ascending order generates decimal parts which have the same kinds of decimal parts although with different digits before 25 (or 75). This further confirms that integers divided by 2^n always generate decimal parts ending with 25 or 75 and having the same number of digits, which depend on the magnitude of n in 2^n.

	185592806269448167697065108937 = 91969 × 16588907 × 121647139715480539
¼	46398201567362041924266277234.25
1/8	23199100783681020962133138617.125
1/16	11599550391840510481066569308.5625
1/32	5799775195920255240533284654.28125
1/64	2899887597960127620266642327.140625
1/128	1449943798980063810133321163.5703125
1/256	724971899490031905066660581.78515625
1/512	362485949745015952533330290.892578125
1/1024	181242974872507976266665145.4462890625
1/2048	90621487436253988133332572.72314453125
1/4096	45310743718126994066666286.361572265625
1/8192	22655371859063497033333143.1807861328125
	5643479997656899909896887654211013 = 17 × 113 × 293778240377766783440754 1725253
/4	1410869999414224977474221913552753.25
/8	705434999707112488737110956776376.625
/16	352717499853556244368555478388188.3125
/32	176358749926778122184277739194094.15625
/64	88179374963389061092138869597047.078125
/128	44089687481694530546069434798523.5390625
/256	22044843740847265273034717399261.76953125
/512	11022421870423632636517358699630.884765625
/1024	5511210935211816318258679349815.4423828125
/2048	2755605467605908159129339674907.72119140625
/4096	1377802733802954079564669837453.860595703125
/8192	688901366901477039782334918726.9302978515625

Tables 3: Division by 2^n still Generate 25 (or 75) as Digits in the Decimal Parts of Quotients Obtained with two Larger Numbers. Table 3A & Table 3B

4.2. First Method based upon Division by 2^n

Let a biprime number Bn = p x q therefore Bn/4 = p/4 x q or p x q/4. Let p < q and let us focus on the smallest factor p. So Bn/4 : p/4 = q and it follows that Bn : p/4 = 4 x q. We can generalize to any 2 to the (power) n by setting the global equation: Bn : $p/2^n$ = 2^n x q. Let's first look at the case of 2^2 (n = 2), 2^3 (n = 3), 2^4 (n = 4), and 2^5 (n = 5). This is therefore the first method of decomposing natural numbers into prime factors in this article as described in Table 4. A calculator of decimal fractions is needed for the whole study. Put the number to decompose Bn as the numerator and the denominators as indicated. Then changes the denominators as indicated and lets the decimal quotients scroll until the desired quotient is obtained, i.e. an integer. The method requires a special calculator or a computer program where 25 or 75 digits are fixed in the decimal part. We only vary the values framed by the two vertical arrows including N for 2^2 fraction or X, XX and XXX for the others. This is also true for the whole study where it is indicated.

	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
Numerator /	Bn /	Bn /	Bn /	Bn /
Denominator	↑N↑.75	N.↑X↑75	N.↑XX↑75	N.↑XXX↑75
		Or		
Denominator	↑N↑.25	N.↑X↑25	N.↑XX↑25	N.↑XXX↑25
The quotient must be an integer =	2^2 x q	2^3 x q	2^4 x q	2^5 x q

Table 4: First Method for the decomposition of integer into prime factors (Bn = p x q). Note that two digits 75 or 25 of the decimal parts are fixed and only X, XX or XXX varies starting with 0, 00 or 000. N varies only in the case of 2^2 fraction. The digits that must change are framed by two vertical arrows in the whole study.

This first method based on the equation Bn : $p/2^n$ = 2^n x q then gives us submultiples of the biprime number Bn = p x q including 4q, 8q, 16q and 32q. This is a new method described for the first time in this paper which allows to factorize integers into products of prime factors. The only particularity is to be able to vary the denominator by fixing the two digits 75 or 25 of the decimal part while increasing X, XX or XXX starting from 0, 00, or 000. Here are the instructions or the calculation program in the box below to apply this method for any number to decompose. As soon as we have an integer as a result, we divide it by 2^n, and we have the prime factor of Bn.

a) First, put the decimal fraction with Bn as the numerator and as denominators the decimals shown below.

b) Put ↑N↑.75 or ↑N↑.25 as denominators then increase N to divide Bn.

c) Put N.↑X↑75 or N.↑X↑25 as denominators with $0 \leq X \leq 9$. Change X to divide Bn. N will increase from 1 to +∞.

d) Put N.↑XX↑75 or N.↑XX↑25 as denominators with $00 \leq XX \leq 99$. Change XX to divide Bn. N will increase from 1 to +∞.

e) Put N.↑XXX↑75 or N.↑XXX↑25 as denominators with $000 \leq XXX \leq 999$. Change XXX to divide Bn. N will increase from 1 to +∞.

f) Factorize Bn once you get an integer as a result of the decimal fractions

To explain the method well, biprime numbers Bn = p x q whose prime factors we know are used and then we apply the method to fall back on their prime factors. We will give examples of numbers with an increasing number of digits (Tables 5A-D). All integers which are the quotients of the fractions in the tables are in the form of 2^n x q corresponding to the box in which they are found.

Bn = p x q (q > p)	22 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
t = $p/2^n$	7/4 = 1.75	7/8 = 0.875	7/16 = 0.4375	7/32 = 0.21875
Q = 91/t	52	104	208	416
t = $q/2^n$	13/4 = 3.25	13/8 = 1.625	13/16 = 0.8125	13/32 = 0.40625
Q = 91/t	↑N↑.25	N.↑X↑25	N.↑XX↑25	N.↑XXX↑25
	28	56	112	224
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2. We will then find that 91 = 7 x 13.				

Tables 5: Examples of the Application of the first Method based on series of Divisions by 2^n ($n \leq 5$) to decompose randomly chosen biprime numbers (Bn). Quotients (Q) are highighted.

Table 5A: The number 91 to decompose (SR = 9.5393920); 91 = p x q with p = 7 and q = 13.

Bn = p x q (q > p)	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
t = p/2^n	11383/4 = 2845.75	11383/8 = 1422.875	11383/16 = 711.4375	11383/32 = 355.71875
Q = 152771243/t	53684	107368	214736	429472
t = q/2^n	13421/4 = 3355.25	13421/8 = 1677.625	13421/16 = 838.8125	13421 /32 = 419.40625
Q = 152771243/t	45532	91064	182128	364256
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2. We will then find that 152771243= 11383 x 13421				

Table 5B: The number 152771243 to decompose (SR = 12360.0664642); p = 11383 and q = 13421

Bn = p x q (q > p)	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
t = p/2^n	6247/4 = 1561.75	6247/8 = 780.875	6247/16 = 390.4375	6247/32 = 195.21875
Q = 4877863751/t	3123332	6246664	12493328	24986656
t = q/2^n	780833/4 = 195208.25	780833/8 = 97604.125	780833/16 = 48802.0625	780833/32 = 24401.03125
Q = 4877863751/t	24988	5649976	99952	199904
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2. We will then find that 4877863751 = 6247 x 780833				

Table 5C: The number 4877863751 to decompose (SR = 69841.7049548); p = 6247 and q = 780833.

B^n = p x q (q > p)	t = 2^2 Fraction	t = 2^3 Fraction	t = 2^4 Fraction	t = 2^5 Fraction
t = p/2^n	854299/4 = 213574.75	854299/8 = 106787.375	854299/16 = 53393.6875	854299/32 = 26696.84375
Q = 69605379628229681/t	325906408076	651812816152	1303625632304	2607251264608
t = q/2+	81476602019/4 = 20369150504.75	81476602019/8 = 10184575252.375	81476602019/16 = 5092287626.1875	81476602019/32 = 2546143813.09375
Q = 69605379628229681/t	3417196	6834392	13668784	27337568
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2. We will then find that 69605379628229681 = 854299 x 81476602019				

Table 5D: The number 69605379628229681 to decompose (SR = 263828314.6825406); p = 854299 and q = 81476602019

The method can be extended to all powers of 2 to infinity. As above, these results are summarized in the tables below: tables 6A-C for 2^n such n ranges from 6 to 13; and tables 7A-B for n ranging from 14 to 17.

Bn = p x q (q > p)	2^6 Fraction	2^7 Fraction	2^8 Fraction	2^9 Fraction
t = p/2^n	6247/64 = 97.609375	6247/128 = 48.8046875	6247/256 = 24.40234375	6247/512 = 12.201171875
Q = 4877863751/t	49973312	99946624	199893248	399786496
t = q/2^n	780833/64 = 12200.515625	780833/128 = 6100.2578125	780833/256 = 3050.12890625	780833/512 = 1525.064453125
Q = 4877863751/t	399808	799616	1599232	3198464
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2. We will then find that 4877863751 = 6247 x 780833				

Table 6B: The number 69605379628229681 to decompose (SR = 263828314.6825406); p = 854299 and q = 81476602019

Bn = p x q (q > p)	2^{10} Fraction	2^{11} Fraction	2^{12} Fraction	2^{13} Fraction
t = p/2n	6247/1024 = 6.1005859375	6247/2048 = 3.05029296875	6247/4096 = 1.525146484375	6247/8192 = 0.7625732421875
Q = 4877863751/t	799572992	1599145984	3198291968	6396583936
t = $q/2^n$	780833/ 1024 = 762.5322265625	780833/ 2048 = 381.26611328125	780833/ 4096 = 190.633056640625	780833/ 8192 = 95.3165283203125
Q = 4877863751/t	6396928	12793856	25587712	51175424
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2. We will then find that 4877863751 = 6247 x 780833				

Table 6C: The number 4877863751 to decompose (SR = 69841.7049548); p = 6247 and q = 780833

Bn = p x q (q > p)	2^{14} Fraction	2^{15} Fraction	2^{16} Fraction	2^{17} Fraction
t = $p/2^n$	6247/16384 = 0.38128662109375	6247/32768 = 0.190643310546875	6247/65536 = 0.0953216552734375	6247/131072 = 0.04766082763671875
Q = 4877863751/t	12793167872	25586335744	51172671488	102345342976
t = $q/2^n$	780833/ 16384 = 47.65826416015625	780833/ 32768 = 23.829132080078125	780833/ 65536 = 11.914566040039062 5	780833/ 131072 = 5.95728302001953125
Q = 4877863751/t	102350848	204701696	409403392	818806784
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2. We will then find that 4877863751 = 6247 x 780833				

Tables 7: Examples of the Application of the first Method based on series of Divisions by 2^n ($14 < n \leq 17$) to Decompose Randomly Chosen Biprime Numbers (Bn). Quotients (Q) are highighted.
Table 7A: The number 4877863751 to decompose (SR = 69841.7049548); p = 6247 and q = 780833.

Bn = p x q (q > p)	2^{14} Fraction	2^{15} Fraction	2^{16} Fraction	2^{17} Fraction
t = $p/2^n$	549319/16384 = 33.52777099609375	549319/32768 = 16.763885498046875	549319/65536 = 8.3819427490234375	549319/131072 = 4.19097137451171875
Q=5422745703328963/t	161738926931968	323477853863936	646955707727872	1293911415455744
t = q/2n	9871760677/ 16384 = 602524.45538330078 125	9871760677/ 32768 = 301262.22769165039 0625	9871760677/ 65536 = 150631.11384582519 53125	9871760677/ 131072 = 75315.5569229125976 5625
8.3819427490234375	549319/131072 =	204701696	409403392	818806784
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2. We will then find that 5422745703328963 = 549319 x 9871760677				

Table 7B: The number 5422745703328963 to decompose (SR = 73639294.5602343); p = 549319 and q = 9871760677

Note that the more 2^n increases the more the decimal fraction obtained tends towards 0 which shows that this method can be very quick to decompose a natural number into products of prime factors. The most important thing is to have a continuous decimal fraction calculator where the denominator can be a decimal fraction. It is also possible to create a computer program or an algorithm by following the instructions indicated in Table 4 which directly gives the quotient Q and the prime factor of the number to be decomposed. Here are the instructions or the calculation program of the method in the box below (n from 4 to 7 for example). Make up the decimal fraction and put Bn as the numerator. Then follow what is below.
=a) As denominators, Put N.↑XXXX↑75 or N.↑XXXX↑25 with 0000 ≤ XXXX≤ 9999. Change XXXX to divide BN. N will increase from 1 to +∞.
b) As denominators, Put N.↑XXXXX↑75 or N.↑XXXXX↑25 with 00000 ≤ XXXXX≤ 99999. Change XXXXX to divide BN. N will increase from 1 to +∞.
c) As denominators, Put N.↑XXXXXX↑75 or N.↑XXXXXX↑25 with 000000 ≤ XXXXXX≤ 999999. Change XXXXXX to divide BN. N will increase from 1 to +∞.

d) As denominators, Put N.↑XXXXXXX↑75 or N.↑XXXXXXX↑25 with 0000000 ≤ XXXXXXX≤ 9999999. Change XXXXXXX to divide BN. N will increase from 1 to +∞.

Here are the instructions or the calculation program of the method in the box below (n from 10 to 13) (see Table 6C). Make up the decimal fraction and put Bn as the numerator. Then follow what is below.

a) As denominators, Put N.↑XXXXXXXXXX↑75 or N.↑XXXXXXXXXX↑25 with 0000000000 ≤XXXXXXXXXX≤ 9999999999. Change XXXXXXXXXX to divide Bn. N will increase from 1 to +∞.
b) As denominators, Put N.↑XXXXXXXXXXX↑75 or N.↑XXXXXXXXXXX↑25 with 00000000000 ≤XXXXXXXXXXX≤ 99999999999. Change XXXXXXXXXXX to divide Bn. N will increase from 1 to +∞.
c) As denominators, Put N.↑XXXXXXXXXXXX↑75 or N.↑XXXXXXXXXXXX↑25 with 000000000000 ≤ XXXXXXXXXXXX≤ 999999999999. Change XXXXXXXXXXXX to divide Bn. N will increase from 1 to +∞.
d) As denominators, Put N.↑XXXXXXXXXXXXX↑75 or N.↑XXXXXXXXXXXXX↑25 with 0000000000000 ≤ XXXXXXXXXXXXX≤ 9999999999999. Change XXXXXXXXXXXXX to divide Bn. N will increase from 1 to +∞.

We can continue for increasing values of n of the powers of 2^n to infinity but this study is limited to what has been shown (n = 17).

Conclusion: This method is used to find submultiples of the number to be decomposed or, in other words, to go up against the grain of Erastothenes'sieve. The goal is to find by specific calculations a submultiple which has a prime factor of the number to be decomposed. This is achieved by using powers of 2 to divide the number to be decomposed. The number is finally factorized according to the equation initially posed Bn : $p/2^n = 2^n$ x q. The instructions given show how to apply the method for numbers with unknown prime factors.

4.3. Variant of the First Method Described by Standardizing the Denominators by Multiplication with Powers of 10

Since the method uses decimal divisors it is possible to modify it and make it uniform, i.e. operating in the same way whatever the decimal length of the divisors. Below are the instructions to follow and this time we will just use a few divisors (up to 2^5) without covering them all because the method will always apply in the same way. Make up the decimal fraction and put Bn as the numerator. Then follow what is below. Here are the instruction to follow to apply this new method. Change N for ¼ and change NX, NXX, or NXXX for 1/8, 1/16 and 1/32.

a) Divide BN by N.25 and N.75 → N.25 or N.75 (1/4)
b) If the divisor has one digit in the decimal part before 25 or 75 digits such N.X25 ou N.X75 multiply by 10 →
c) ↑NX↑.25 or ↑NX↑.75 as denominators (1/8)
d) If the divisor has two digits in the decimal part before 25 or 75 digits such N.XX25 ou N.XX75 multiply by 100 →
e) ↑NXX↑,25 or ↑NXX↑.75 as denominators (1/16)
f) If the divisor has three digits in the decimal part before 25 or 75 digits such N.XXX25 ou N.XXX75 multiply by 1000 → ↑NXXX↑.25 or ↑NXXX↑.75 as denominators (1/32)

Therefore, all denominators of the decimal fractions have the same form N.75 or N.25. It is as if you divided Bn by the decimals N.75 and N.25 except that this time you will recover at the same time the quotients 1/8 and 1/16 and 1/32 during the same one calculation because these will be decimals which we will make integers by multiplying them by 10^m. To explain it differently, you only divide Bn by 4 but this time you do not limit yourself to taking only the integer quotients but also any decimals with 1, 2 or 3 digits after the decimal point which you multiply by 10^m in order to be able to decompose them and find the q factor of Bn. You could recover all the decimal quotients having one digit, two, three, four, five and by increasing n in 2n (or the expnonent n) and so on to find all the submultples of the number to be decomposed. This method does not offer a single solution for factoring a number as in Trial division, but an infinity.

Representative examples are shown in Tables 8A-D. Indeed, with this method three quotients obtained are decimals with one, two and three digits after the decimal separator following the initial equation Bn : $p/2^n = 2^n$ x q and so its two terms are multiplied by 10^m ($1 \leq m \leq 3$ for 1/8, 1/16 and 1/32) to have integer quotients such (Bn : $p/2^n$) x 10^m = (2^n x q) x 10^m. We therefore multiply the obtained quotient by 10^m so as to have a natural number as a quotient Q' = p x 2^n whose decomposition will give the factor of the number Bn.

Bn = p x q (q > p)	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
t = p/2^n	7/4 = 1.75	7/8 = 0.875	7/16 = 0.4375	7/32 = 0.21875
N.75	1.75	8.75	43.75	218.75
Q = 91/t	52	10.4	2.08	0.416
Q' (integer) = Q x 10^m	52	104	208	416
t = q/2^n	13/4 = 3.25	13/8 = 1.625	13/16 = 0.8125	13/32 = 0.40625
N.25	3.25	16.25	81.25	406.25
Q = 91/t	28	5.6	1.12	0.224
Q' (integer) = Q x 10^m	28	56	112	224
Decompose the integers (or the quotients Q') obtained and divide Bn by their prime factors > 2. We will then find that 91 = 7 x 13				

Tables 8: Examples of the application of the first method with variations aimed at standardizing the calculation. Note 1 ≤ m ≤ 3 in 10^m.

Table 8A: The Number 91 to Decompose (SR = 9.5393920)

Bn = p x q (q > p)	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
t = p/2^n	11383/4 = 2845.75	11383/8 = 1422.875	11383/16 = 711.4375	11383/32 = 355.71875
N.75	2845.75	14228.75	71143.75	355718.75
Q = 152771243/t	53684	10736.8	2147.36	429.472
Q' (integer) = Q x 10^m	53684	107368	214736	429472
t = q/2^n	13421/4 = 3355.25	13421/8 = 1677.625	13421/16 = 838.8125	13421/32 = 419.40625
N.25	3355.25	16776.25	83881.25	419406.25
Q = 152771243/t	45532	9106.4	1821.28	364.256
Q' (integer) = Q x 10^m	45532	91064	182128	364256
Decompose the integers (or the quotients Q') obtained and divide Bn by their prime factors > 2. We will then find that 152771243= 11383 x 13421				

Table 8B: The number 152771243 to decompose (SR = 12360.0664642)

Bn = p x q (q > p)	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
t = p/2^n	6247/4 = 1561.75	6247/8 = 780.875	6247/16 = 390.4375	6247/32 = 195.21875
N.75	1561.75	7808.75	39043.75	195218.75
Q = 4877863751/t	3123332	624666.4	124933.28	24986.656
Q' (integer) = Q x 10^m	3123332	6246664	12493328	24986656
t = q/2^n	780833/4 = 195208.25	780833/8 =	13421/16 = 838.8125	13421/32 = 419.40625
97604.125	780833/16 = 48802.0625	780833/32 =	83881.25	419406.25
24401.03125	45532	9106.4	1821.28	364.256
N.25	195208.25	976041.25	4880206.25	24401031.25
Q = 4877863751/t	24988	4997.6	999.52	199.904
Q' (integer) = Q x 10^m	24988	5649976	99952	199904
Decompose the integers (or the quotients Q') obtained and divide Bn by their prime factors > 2. We will then find that 4877863751 = 6247 x 780833				

Table 8C: The number 4877863751 to decompose (SR = 69841.7049548)

Bn = p x q (q > p)	t = 2^2 Fraction	t = 2^3 Fraction	t = 2^4 Fraction	t = 2^5 Fraction
t = p/2^n	854299/4 = 213574.75	854299/8 = 106787.375	854299/16 = 53393.6875	854299/32 = 26696.84375
N.75	213574.75	1067873.75	5339368.75	26696843.75
Q=69605379628229681/t	325906408076	65181281615.2	13036256323.04	2607251264.608
Q' (integer) = Q x 10^m	325906408076	651812816152	1303625632304	2607251264608
t = q/2^n	81476602019/4 = 20369150504.75	81476602019/8 = 10184575252.375	81476602019/16 = 5092287626.1875	81476602019/32 = 2546143813.09375
N.75	20369150504.75	101845752523.75	509228762618.75	2546143813093.75
Q=69605379628229681/t	3417196	683439.2	136687.84	27337.568
Q' (integer) = Q x 10^m	3417196	6834392	13668784	27337568
Decompose the integers (or the quotients Q') obtained and divide Bn by their prime factors > 2. We will then find that 69605379628229681 = 854299 x 81476602019				

Table 8D: The number 69605379628229681 to decompose (SR = 263828314.6825406)

The other option is to completely remove the decimal point and continue the same divisions process. So we must then multiply Bn by a power of 10 to avoid having decimals as a result. We choose a power of 10 (10^m) high enough ($m \geq 5$ for 2^5 and much more when increasing n of 2^n) to be divisible by all the divisors obtained and give an integer number. Even if 10m is too high, this has no impact because the most important thing is to recover the prime factor which divides Bn. It follows that the quotient is also a multiple of 5 and 2 and of the prime factor which divides Bn. Make up the decimal fraction and put Bn as the numerator. Then follow what is below. The instructions are summarized below and representative examples are shown in Tables 9A-C.

a) Calculate B_n' = Bn x 10^m ($m \geq 5$ for up to 2^5)
b) Divide B_n' by changing N only and without changing 75 or 25 digits such that ↑N↑75 or ↑N↑25 with N increasing from 1 to to + ∞. Note that the decimal separator is removed.
c) When you get an integer as the quotient, Decompose it and get prime factors of B_n'. You also get prime factors by dividing the denominators that give integer quotients by 2 x 5n.

7	7/4	7/8	7/16	7/32
Divisors N75	175	0875	04375	021875
Q' = 9 100 000/N75	52000	10400	2080	416
13	13/4	13/8	13/16	13/32
Divisors N25	325	1625	08125	040625
Q ' = 9 100 000/N25	28000	5600	1 120	224
Decompose the integers (or the quotients Q') obtained and divide Bn by their prime factors > 5. We will then find that 91 = 7 x 13				

Tables 9: Examples of the application of the first method with other variations aimed at standardizing the calculation. This involves eliminating the decimal point. Note that we multiply Bn by 10^5 in all cases.
Table 9A: The number 91 to decompose (SR = 9.5393920). Note that we multiply Bn = 91 by 105.

11383	f/4	f/8	f/16	f/32
Divisor N75	284575	1422875	7114375	35571875
Q = 15277124300000 /N75	53684000	10736800	2147360	429472
13421	f/4	f/8	f/16	f/32
Divisor N25	335525	1677625	8388125	41940625
Q' = 15277124300000 /N25	45532000	9106400	1821280	364256
Decompose the integers (or the quotients Q') obtained and divide Bn by their prime factors > 5. We will then find that 152771243 = 11383 x 13421				

Table 9B: The number 152771243 to decompose (SR = 12360.0664642). Note that we multiply Bn =152771243 by 10^5.

854299	854299/4 = 213574.75	854299/8 = 106787.375	854299/16 = 53393.6875	854299/32 = 26696.84375
Divisor N75	21357475	106787375	533936875	2669684375
Q' = 6960537962822968100 000/N75	32590640807600000	6518128161520000	1303625632304000	260725126460800
81476602019	81476602019/4 = 20369150504.75	81476602019/8 = 10184575252.375	81476602019/16 = 5092287626.187	81476602019/32 = 2546143813.09375
Divisor N75	2036915050475	10184575252375	50922876261875	254614381309375
Q' = 6960537962822968100 000/N75	3417196000	683439200	136687840	683439200
Decompose the integers (or the quotients Q') obtained and divide Bn by their prime factors > 5. We will then find that 69605379628229681 = 854299 x 81476602019				

Table 9C: The number 69605379628229681 to decompose (SR = 263828314.6825406). Note that we multiply Bn = 69605379628229681by 10^5.

Overall conclusion on the methods based on divisions by 2^n

First this method is set with a calculator of decimal fractions that works continuously. We put Bn as the numerator and the denominators are defined in the tables and in the instructions cited above. These first methods also require specific calculators where it is necessary to fix two decimal digits after the decimal point (or without the decimal separator) which are 75 or 25 and only vary those which are before (framed by arrows) as if 25 and 75 are separated from the rest of the number which continues to grow as if they were not present. The other option is that you can have several calculators for each fraction based on 2^n because as soon as an integer quotient appears, all the calculations on all the calculators combined automatically stop. Here the approach differs from that of serial divisions because the calculation consists of determining decimal fraction after decimal fraction by increasing the values of the divisors as indicated in the instructions. The calculation is therefore continuous but the path is shorter than that of Euclidean divisions in series and the number of operations can be drastically reduced in these new methods because in the end we are only counting, i.e. increasing the value of the divisor until you find the one which gives a quotient in the form of an integer. This method is robust, and applicable to any number, even those with two digits like 91. It can help in the decomposition of very large numbers by calculating the quotients with several fractions whose denominator is the desired factor divided by powers of 2. It therefore applies without limits.

4.4. Can we use other Decimal Divisors?

If we were able to use the 2^n divisors it is because they form a repeating pattern and therefore predictable, however this is not the case for all other numbers. We can always calculate submultiples or supermultiples of the number Bn = p x q, that is to say multiples of p or q.

Let r = p/n. Then Bn : r = pq : r = pq : p/n = pq x n/p = nq. However, very few numbers give recognizable fractions and among them are 2, 3, 6 in addition to powers of 10.
16

Bn = 5601385979857 (p x q)	F:2	Bn/f:2
99793 (p)	49896.5	112260098 = 2 × 56130049
56130049 (q)	28065024.5	199586 = 2 × 99793
Bn = 5601385979857 (p x q)	F:3**	Bn/f:3
99793 (p)	33264.333333	168390147 = 3 x 56130049
56130049 (q)	18710016.33333	299379 = 3 × 99793
Bn = 5601385979857 (p x q)	F: 6**	Bn/f:6
99793 (p)	16632.16666666666666666667	336780293 = 2 x 3 x 56130049
56130049 (q)	9355008.16666666666666666667	598758= 2 x 3 × 99793
Bn = 5601385979857 (p x q)	F:7	Bn/f:7
99793 (p)	14256.14285714285714285714	392910343 = 7 x 56130049
56130049 (q)	8018578.42857142857142857143	698551 = 7 x 99793

Table 10: Decimal divisors that can work in a decompisition method. The table shows 2, 3, 6 as good divisors but not 7 that vary from a prime number to another. The number Bn = 5601385979857 is used as an exemple but the data are true for all composite odd numbers. F means prime factor either p or q of the chosen numbers Bn.

** An odd number divided by 3 can also give a decimal extension = 0.666..While divided by 6 can also give another decimal extension of 0.83333...This is true for all odd numbers.

Table 10 shows us the other decimal extensions obtained with other denominators. All prime numbers divided by 2 give a decimal extension 0.5 for example. Odd numbers divided by 3 give two extensions either 0,3333...or 0.66666... The table shows a prime number p divided by 3 giving a decimal extension of 0.33333. But prime number like 597566339 divided by 3 gives instead an extension of 0.66666...Prime numbers divided by 6 either gives an extension of 0.1666666... like in the table or 0.8333333 with another prime numbers like 597566339. Altough prime numbers divided by 11 give decimal fractions with a repeating pattern, it varies from one number to another. For example 99793/11 = 9072.0909090909... while another prime number 597566273/11 = 54324206.6363636363... Therefore, we cannot make a method with 11 because of variable results. Prime numbers divided by 7 give several decimal extensions either 0.428571428571...; 0,1428571428571428571 4...; 0.28571428571428571... or 0.71428571428571428571.... Even if we can tinker with 7, this will complicate the method and therefore 7 is not a good candidate nor is 11 nor 6^n like 36. We can only develop a calculation method with 2, 3 or 6. We have seen the case of 2^n but this does not work with the powers of 3 or 6. Here are the instructions in the case where one wants to decompose a number using decimal extensions of 2, 3 and 6.
In a similar way this method is based on the calculation of decimal fractions:

a) Put Bn to decompose into numerator (Bn = p x q such that q > p)
b) Put the denominator by fixing the decimal part and only change N. Therefore, ↑N↑.5 (we will detect the submultiple 2 x q), ↑N↑.3333 or ↑N↑.6666 (we will detect 3 x q), ↑N↑.1666... or ↑N↑.83333... (we will detect 6 x q). The decimal part in bold must remain unchanged despite N changes and therefore you must have a calculator capable of carrying out this type of specific calculation as metioned above with 2n.
c) If the quotient is an integer or very close** then decompose Bn by its larger prime factor.

** When we divide BN by a decimal number having .3333... or .6666... or similar decimal parts we could obtain a quotient close to an integer having the form N.999999999...XXX or N.00000000... XXX..., in the first case take N + 1 and in the second case N.

By contrast, it is entirely possible to develop a reliable and extended method with 10 and its powers which is as robust and relevant as that which we saw with the 2^n subdivisions.

a) Put the number BN to be decomposed into the numerator of the decimal fraction
b) As a denominator, Put N.↑X↑ ($0 \leq X \leq 9$) to detect 10 x q. Or N.↑XX↑ ($00 \leq XX \leq 99$) to detect 100 x q. N.↑XXX↑ to detect 1000 x q ($000 \leq XXX \leq 999$). N.↑XXX....Xn↑ ($000...0n \leq XXX....Xn \leq 999...9n$) to detect 10^n x q. Change X;
c) XX; XXX;...and then N will increase from 1 to +∞
d) Divide the quotient by 10^n et get the prime factor of Bn.

Here is an exemple Bn = 77633670588622783 = 9865069 x 7869551707. If we divide Bn by a decimal number like 986506.9 (in the form of N.↑X↑) we will get 10 x 7869551707. If we divide it by 98650.69 ((in the form of N.↑XX↑) we will get 10^2 x 7869551707. If we divide it by 9865.069 (in the form of N.↑XXX↑) we will get 10^3 x 7869551707. If we divide it by 9.865069 (in the form of N.↑XXXXXX↑)we will get 10^6 x 7869551707. Finally by a decimal divisor like 0.9865069 (in the form of N.↑XXXXXXX↑) we will get 10^7 x 7869551707. Even with 0.0009865069 (in the form of N.↑XXXXXXXXXX↑) we will get 10^{10} x 7869551707 and so on.

Let us recall to better understand the basis of the method. In reality we have no idea about the prime factor of a number that we want to decompose, we will just put the decimal fraction Bn/N.X or Bn/N. XX, or Bn/N.XXX and so on. We are just going to vary X from 0 to 9; XX from 00 to 99, and XXX from 000 to 999 and so on. N will increase and increase till we get that decimal divisor which gives a quotient that is an integer ending up with one or many zeros as digit units and which is = 10^n x q. Then we can decompose our number Bn by dividing it with q.

4.5. Case of very Large Numbers

The larger the number, the more we can increase the exponent n of the denominator 2^n. If we divide an odd number by 2^n, it will have a decimal part of n digits that end in 25 or 75. Here are four examples of numbers divided by 2^n (n= 20; 40; 60 and 80). The quotients are shown in the table below.
A= 447842627831497390805494736120920236322683279722 3
B= 437167028939242386178065726320960973486317657714 9611
C=296656153839032545132756393640972499493454450571947475882071924710747657058731
D=122389557149739906602533324562904297975123047381083842035962765384958406174755360357788075768515853039701361219

Number	$/2^{20}$	$/2^{40}$	$/2^{60}$	$/2^{80}$
A	4270960119547819049887606965264513362147171.78079891204833984375	111312653780023453330189415697796384663445046260775037993513502932269912541847166754249708516773885 3.706124574472596577834337949752807 6171875	3884415600212213621387133539669.33333335616387630481044501706833216303493827581405639648 4375	3704467392170156117808469.333333333333355106235794840283410137493289027155185522133251652121543884 27734375
B	41691496747898329370314190513702485417014852 30.58854198455810546875	397601096610053342536107926499390463037 6.32487353185842948732897639 27459716796875	25730819717877848150085588200487425906055759473526785622617 8.17492793391384823652873192756374010059516876935958 8623046875	3616160907859496933598238015.287192529627941324662841388533096484382209356844839476252673 1938123703002929687 5
C	28291335472014669907832755436036348294587559754557368839461510153841748 9.1078290939331054 6875	26980720016493482501824145732914303106868324045712822761022100595 3.2332316473284663516096 77076339721679687 5	25730819717877848150085588200487425906055759473526785622617 8.17492793391384823652873192756374010059516876935958 8623046875	245388219050196153164726144795297869740064234481113296.724489378860410608146893051845481456508732409638184890354750677943229675292968 75
D	1167197772500418725991566967547325418448565528279384462390864148507078558134799347265041423576846936263753.2818 0217742919921875	111312653780023453330189415697796384663445046260775037993513502932269912541847166754249708516773885 3.706124574472596577834337949752807 6171875	10615601900102944691676084108142507997841362596585754203177786153056136373696057010102244235 68.47653742420632788905771048959536528855 0965487957000732421875	10123826885321564380336841686384685514298784824929956629922662880951057790466362962820 28.6021885648130647719649210526566406205799947271017913408286403864622116 0888671875

Here is an example where we see that if we divide a number by 2^{20}, we can factor it after a quick number of operations. In the example below where A is the numerator, we must reach the denominator 75307.34239673614501953125 to obtain an integer quotient whose decomposition gives the largest factor of the number to be decomposed (A).

2966561538390325451327563936409724994934544505719474758820719247107476570587 31 (A) = 78965471861 (B) × 37567831464519759027031592252967427562479673861813292594676910406 71 (C)

78965471861(B) / 2^{20} = 75307.34239673614501953125

29665615383903254513275639364097249949345445057194747588207192471074765705873 1 (A)/ 75307.34239673614501953125 = 39392726445740270841528678878247573323754686499324735095755936006626344 96 (D) = 2^{20} × 375678314645197590270315922529674275624796738618132925946769104067 1

4.6. The second Method based on the use of a Decomposition key

4.6.1. Decomposition Key (Kd)

A decomposition key is the product of prime numbers (p_1 x p_2 x p_3 x p_4 x p_5 x... p_n) whose values are in ascending order (p_n... > p_5 > p_4 > p_3 > p_2 > p_1) which helps with decomposition of all numbers having a given number of digits. So there is one key to decomposing two-digit, another for three-digit, another for 4-digit numbers, and so on. The more digits the number has, the more the key will encompass more prime factors and will have a much greater value. Not only is it a product of prime factors, but some are put into power (p^n_1 x p^m_2 x p_3 x p_4 x p_5 x... p_n) depending on the number of digits of the numbers. This key is calculated by analyzing the value of each number having a given number of digits. This article is limited to Kd specific for numbers with two or three digits. Two-digit numbers are denoted NX and three-digit numbers are NXX. But since the decimal numbers such like N.X or N.XX can be converted into fractions between two integers with 10^n as the denominator such that NX/10 or NXX/100, whether the number is NX or N.X or is NXX or NX.X or N.XX has no importance for the calculation that we will develop subsequently. Since the key will considerably inflate the value of the number Bn, it is necessary to have very large number calculators or specialized computer programs. See the box below where Kds used in this article are presented. To put it another way, Kd2 will factor all two-digit numbers NX such as 1≤ NX ≤99 and Kd3 for three-digit numbers NXX such as 100 ≤ NXX ≤ 999.

*Kd2 = 2671979643323542381608979200 = $2^8 \times 3^2 \times 5^2 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47$.

*Kd2' (Kd2 extended) = Kd2 x 53 x 59 = 8355280344672717027291277958400

*Kd2'' (Kd2 extended) = Kd2 x 53 = 16356523386605064689019 3661728000 = $2^8 \times 3^3 \times 5^3 \times 7^4 \times 11^3 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 53$

**Kd3 = 2^9 x 3^6 x 5^4 x 7^3 x 11^2 x 13^2 x 17^2 x 19^2 x 23^2 x 29^2 x 31^2

x 37 x 41 x 43 x 47 x 53 x 59 x 61 x 67 x 71 x 73 x 79 x 83 x 89 x 97 x 101 x 103 x 107 x 109 x 113 x 127 x 131 x 137 x 139 x 149 x 151 x 163 x 167 x 173 x 179 x 181 x 191 x 193 x 197 x 199 x 211 x 223 x 227 x 229 x 233 x 239 x 241 x 251 x 257 x 263 x 269 x 271 x 277 x 281 x 283 x 293 x 307 x 311

* Used in this study. ** Not used but replaced by extended Kd2 including Kd2' and Kd2".

4.7. Principles

Let Bn = p x q. Let's call Sri the integer part of its square root (SR), ignoring the decimal part. We calculate a decimal fraction between the number Bn x Kd to be decomposed and Sri such that Bn x Kd /Sri. Let Q = (Bn x Kd)/Sri. We will from the start decide the calculation such that Q will be in the form of N.X or N.XX. In the first case we use a Kd2 and in the other a Kd3. For this method as for the first ones we need a calculator for decimal fractions in series, that is to say in a continuous and automatic way.

The principle of this method is based on the fact that if we vary the value of the SRi by increasing or reducing it continuously, we will go through decimal values which can be used by the decomposition key Kd to factorize the number Bn. If Bn = p x q such that p < q, at one time or another we will come across a value of SRi that allows Bn decomposition. For instance, Sri = N.X x p; Sri = N.XX x p or Sri = N.XXX x p and so on. If we divide Bn x Kd by these values of SRi, we can obtain a number whose decomposition will give the largest factor of the number Bn (q). Let's see an example with Kd2, that is to say a two-digit number like N.X. For exemple, Q = Bn x Kd2/SRi and suppose that one value we reach is Sri = N.X x p therefore Q = Bn x Kd2 / N.X x p = Bn x Kd2 x 10 / NX x p. Let Kd2a = Kd2 x 10 and Bn = p x q. Hence p x q x Kd2a / NX x p = q x Kd2a / NX. Let NX = p1 x p2 and knowing that Kd2a = p1 x p2 x p3 x p4 x p5 x p6 x...pn x 10 then Q = q x Kd2a' with Kd2a' = Kd2a' = p3 x p4 x p5 x p6 x...pn x 10. Indeed, Kd2 contains all the prime factors which factor any two-digit number, here we have assumed that NX = p1 x p2 to simplify but whatever the prime numbers which are factors of NX, they will be included in Kd2, and therefore by dividing Kd2 by NX, we eliminate the prime factors of NX. Since the chosen value of Sri corresponds to NX x p, by dividing Bn x Kd/Sri, we have Bn x Kd/p x NX and given that BN = p x q, we then eliminate p from the numerator and denominator, and then we finally have the remainder of Kd x q (the largest factor of Bn).

Any Q value that is divisible in this way by Kd2 allows Bn decomposition. This also applies to the largest factor p if we increase SRi value. A certain number of operations is required before obtaining a desired value with a very short decimal part such like N.X or N.XX; N.XXX; or N.XXXX depending on the Kd we want to use. In all cases, we will have the closest one when the progress of the continuous calculation of decimal fractions reaches it. The speed of the method depends on the distance between Sri and the values like Sri x N.X (in the case of Kd2) or Sri x N.XX (in the case of Kd3) (or even Sri x N.XXX in the case of kd4 and so on). Note that these values of Sri could be obtained by reducing and/or increasing Sri and this is why two calculators must be operational in parallel.

4.8. Representative examples of the Calculation

In order to explain more this method, we will give various examples. We will see a first example before establishing the instructions to follow for this method.

Here, a Kd2 adapted and readjusted to the chosen examples is used to save space and make the explanations more plausible and be able to carry out the calculations because if Kd3 or Kd4 are used, the limits of the calculators available on the web or conventional will be exeeded. The Kd3 in the box above gives you an idea of the complex construction of the decomposition keys which must be the product of all the prime factors and powers of prime numbers capable of forming a three-digit number (up to 999) either combined together into products or in powers. Examples of the use of Kd to factor numbers are listed in tables 11.

<table>
<tr><td>SF</td><td>LF</td><td>Bn</td><td>SRi</td><td>SRi/SF</td><td>2.5SF</td><td>2.5SF - SR</td></tr>
<tr><td>144598763437</td><td>895514715619</td><td>129490320518
144110022503</td><td>35984763514
3</td><td>2.4885941386
3370574904</td><td>361496908592
.5</td><td>1649273449.3
131689930371
0662683</td></tr>
<tr><td colspan="7">267197964332354238160897920 0 (Kd2) x 129490320518144110022503 = 345995500431921880816230698761118569805858858937600
345995500431921880816230698761118569805858858937600 : 361496908592.5 = 957118836172255643149703593577954449920 = $2^9 \times 3^2 \times 5 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times$ 895514715619</td></tr>
<tr><td>SF</td><td>LF</td><td>Bn</td><td>SRi</td><td>SRi/SF</td><td>6.5SF</td><td>6.5SF - SR</td></tr>
<tr><td>9471240377</td><td>389714237467</td><td>369107722138
9216605059</td><td>60754236242</td><td>6.4146018708
9495089055</td><td>61563062450.
5</td><td>808826208.5</td></tr>
<tr><td colspan="7">2671979643323542381608979200 (Kd2) x 3691077221389216605059 = 986248319748721086334613933548235352225454577 2800
986248319748721086334613933548235352225454577 2800 : 61563062450.5 = 16020130911156630104892314669050210 5600 = $2^9 \times 3^2 \times 5^2 \times 7^3 \times 11^2 \times 13 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times$ 389714237467</td></tr>
</table>

<table>
<tr><td>SF</td><td>LF</td><td>Bn</td><td>SRi</td><td>SRi/SF</td><td>1.32SF</td><td>SR – 1.32SF</td></tr>
<tr><td>398714257013</td><td>698754303007</td><td>278603302758
072676738091</td><td>527828857451</td><td>1.3238273981
1029692832</td><td>526302819257.16</td><td>1526038193.84</td></tr>
<tr><td colspan="7">2671979643323542381608979200 (Kd2) x 278603302758072676738091 = 744422353532275922257173111915367167756492266707200 744422353532275922257173111915367167756492266707200 : 526302819257.16 = 1414437328272298725648035663718757920000 = $2^8 \times 3 \times 5^4 \times 7^3 \times 11 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times$ 698754303007</td></tr>
<tr><td>SF</td><td>LF</td><td>Bn</td><td>SRi</td><td>SRi/SF</td><td>4608.35 x SF</td><td>(4608.35 x</td></tr>
<tr><td>22823697877
120277</td><td>484703753292
603814487663</td><td>11062732025
056632153858
950364786583
642651</td><td>10517952284
0981894967</td><td>4608.3471402
072668085387</td><td>105179588112
027228512.95</td><td>1526038193.84</td></tr>
<tr><td colspan="7">2671979643323542381608979200 (Kd2) x 53 x 59 = 8355280344672717027291277958400 (Kd'2)
141614921096147746225275897600 x 11062732025056632153858950364786583642651 = 9243222744733708232359218881913436
5189249446133336982453719218443718400 92432227447337082323592188819134365189249446133336982453719218443718400 :
105179588112027228512.95 = 87880385449776740043619050489979257184066310515200 = $2^{10} \times 3^2 \times 5^3 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 41 \times 43 \times 59 \times$ 484703753292603814487663</td></tr>
<tr><td>SF</td><td>LF</td><td>Bn</td><td>SRi</td><td>(SRi x 10^{-9})/SF</td><td>9.45SF - (SRi x 10^{-9})</td><td>9.45SF</td></tr>
<tr><td>10780251859
59431232990
7551187</td><td>956478064792
755281357337
812662039047
944195630644
07</td><td>103110744366
432692787769
570948735978
666103180950
244877199079
356660902693
0301109</td><td>10154346082
66001010577
60046910822
5957770</td><td>9.4193959611
646247622</td><td>329919246566
150459866311
8063</td><td>101873380073
166251517626
3587171.5</td></tr>
<tr><td colspan="7">16356523386605064689019366172800 (Kd2") = $2^8 \times 3^3 \times 5^3 \times 7^4 \times 11^3 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 53$
16356523386605064689019366172800 x 1031107443664326927877695709487359786661031809502448771990793566609026930301109 =
168653330163981276345273748179618754268957139381396147326143518369510602293588945526304179190121658329256 352000 : 10
18733800731662515176263587171.5 = 1655519136037826042933754433731058179745776380620840962120070673183522543377280000 =
$2^9 \times 5^3 \times 7^3 \times 11^3 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 53 \times$ 95647806479275528135733781266203904794419563064407</td></tr>
</table>

Tables 11: Decomposition by multiplication of the number Bn to factorize by a decompositon key and dividing the product by the integer part of SR (Sri). SF: small factor. LF : large factor. SR : square root. Sri = integer part of SR. Kd2 (decompositon key for two-digit odd numbers). Kd2' or Kd2'' the key decomposition for two-digts numbers extended by its multplication with more prime factors.
Table 11A ,Table 11B and Table 11C

In the first example in Table 11A, we have the number Bn = 129490320518144110022503 = 144598763437 (p) x 895514715619 (q). As explained above, we start from numbers with known factors to explain how the method works. The SR of this Bn = 359847635143.18683100696289337317. We will consider Sri = 359847635143 and ignoring the decimal part. But the question that interests us in the first place is the following: how far is the SRi from a multiple of the smallest factor of Bn, i.e. p (Bn = p x q such q > p)? p is called SF (small factor) in this section. In truth we don't know, but we will decide to set the square root at a predictive value, that is to say we will, for example, look for the case where the SRi/SF = N.X (like N.5 thus we look at the two-digits at the end of the quotient digits). We therefore need a key Kd2 which decomposes all the two-digit numbers NX. We know that the SRi is > SF, but we are primarily interested in a ratio of type N.X. In our example (Table 11A), we have SRi/SF = 2.48859413863370574904. We retain 2.48 and we will then look for SF x 2.5 which is one ratio closest to 2.48. The calculation shows that SF x 2.5 = 361496908592.5. It takes 1649273449 operations or decimal fractions before reaching this value from the SRi.

The Kd2 = $2^8 \times 3^2 \times 5^2 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47$. Therefore, Bn x Kd2 =34599550043192188081623069876111856980585885893 7600 and so 345995500431921880816230698761118569805858858937600 : 361496908592.5 = $2^9 \times 3^2 \times 5 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times$ 895514715619 . We therefore have the largest factor (LF or q) of Bn and we can break it down.

Indeed, the detailed calculation is as follows: 34599550043192188081623069876111856980585885893760 0 : 361496908592.5 = 345995500431921880816230698761118569805858858937600 : (3614969085925 : 10) = (345995500431921880816230698761118569805858858937600 x 10) : 3614969085925. Because 3614969085925 must be a multiple of SF (which is= 144598763437) and indeed 3614969085925 = 25 x 144598763437. Therefore, we have:

(345995500431921880816230698761118569805858858937600 x 10) : 3614969085925 = $2^9 \times 3^2 \times 5^3 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 144598763437 \times 895514715619$: ($5^2 \times 144598763437$) = $2^9 \times 3^2 \times 5 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 895514715619$.

Note that in this example we have the ratio 25/10 = 2.5 and therefore the two-digit number 25 = 5^2 and we have 10 = 2 x 5. The method works just as well with all decimals generating natural quotients like 2.1 or 2.2, and even 2.01, 2.001 except that in the last two cases we must have Kd3 and Kd4. Here the Kd2 has been optimized for the example and we must always have a Kd2 capable of decomposing any two-digit number. In this example we have set the value 2.5 x SF closest to the SRi/SF ratio, but in reality we ignore the factor of the number to be decomposed and therefore we will just put one digit or two or more after the decimal point of the decimal fraction and wait to have an integer as the quotient. Another key element is how to reduce the number of operations to get the desired value? The only solution is to use higher Kd and thus increase the number of decimals. We have as said above SRi/SF = 2.48859413863370574904 (Table 11A, first example): if we use 2.49 instead of 2.5 we will have less operations 203285815 (compared to 1649273449 with 2.5). However 249 = 3 x 83 and we have to use Kd3.

- If we use 2.4885 instead of 2.5 we will have 13612330 operations that are much less than 1649273449 with 2.5. However 24885 = $3^2 \times 5 \times 7 \times 79$ meaning that we have to use Kd5.
- If we use 2.48859 instead of 2.5 we will have 598441 operations that are much lesser than 1649273449 with 2.5 or 13612330 with 2.4885. Nevertheless, 248859 = $3^3 \times 13 \times 709$. Hence a need for a Kd6.
- If we use 2.4885941386 instead of 2.5 the number of operations decreases drastically because we only have 4 operations to go through from the Sri to decompose the number. However, 24885941386 = 2×12442970693 and thus we need a Kd11.
- If we use two kd for a same calculation with Sri, we will have much less operation with the higher one because this signifies the use of more decimals after the decimal separator.
- In the example cited above in Table 11A, 2.5 is the closest values to Sri by using Kd2. For each key used, we have one closest value. Requiring the least number of operations. Note that Kd2 factors all numbers between 1 and 99. If we set NX as being any two-digit number, thus any product Sri x NX such that $1 \leq NX \leq 99$ can be factorized by Kd2, we therefore have 99 chances of decomposing Bn using Kd2.
- For comparison, and if we pose NXX as being any three-digit number, Kd3 will offer more chances because if we have Sri x NXX such that $100 \leq NXX \leq 999$, we have 899 chances of factoring Bn. To express it differently Bn x Kd2/SRi x NX offers 99 possible factorizations and Bn x Kd3/SRi x NXX offers 899. This is why this method is robust and safe and this is above all the major advantage offered by the use of Kd. Unlike the method of division by series which has only one solution, this method offers several factorization solutions and therefore accelerates the decomposition process.

Here are the instructions to follow for this method

a) Calculte SR of Bn to factorize and take only the integer part by ignoring the decimal one (SRi). Let's name it Sri.Choose your Kd. Then calculate Bn x Kd and put it as a numerator in the calculator of decimal fractions. Put Sri as the denominator.
b) Let us therefore set the decimal fraction Bn x K/SRi = N.X or N.XX (in this article we only consider one or two digits after the decimal point and the corresponding Kd2 and Kd3). You might use two calculators : one to increase SRi (Bn x K/↑SRi↑ = N.X or N.XX) and the other to decrease it (Bn x K/↓SRi↓ = N.X or N.XX). If one gives the desired quotient, all the calculations stop all at once.
c) Start the calculation by reducing and/or increasing Sri which must then be set as N.0 or N.00. For example if we startfrom 359847635143 from the example cited above, we then start with 359847635142.9 or 359847635142.99. We let it scroll towards 0 or to higher values.
d) Stop the calculation as soon as you obtain an integer, decompose it and take the prime factor p or q in order to break down the number Bn.

The second example of the Table 11A is the number Bn = 3691077221389216605059 = 9471240377 x 389714237467 has for SRi = 60754236242 and the ratio SRi/SF = 6.41460187089495089055 and therefore the closest number is SF x 6.5. And we therefore have the ratio 65/10 which will impose itself and therefore 65 = 5 x 13 and 10 = 2 x 5 and thus the Kd2 defined above is able to decompose it. It takes 808826208 decimal fractions to go from SRi = 60754236242.0 up to SRi x 6.5 = 61563062450.5. In table 11B, we will use the same Kd2 for the decomposition of a three-digit number. As said above the use of a Kd3 is avoided to save space and not to interrupt the text with a large gap, but in reality we have to use a Kd3 for a three-digit number but the methods works the same anyway. Indeed, a Kd2 can work for some three-digit numbers which are the products of smaller prime factors and in this study it was extended to be able to provide explanatory demonstrations.

In table 11B, we have the number 278603302758072676738091 = 398714257013 x 698754303007 and whose SRi = 527828857451. The SRi/SF ratio = 1.32382739811029692832. Here we set the ratio Bn/SRi = N.XX. You have to count or scroll through 1526038193 decimal fractions on calculators before arriving at 1.32SF = 526302819257.16. We therefore have the ratio 132/100 and then 132 = $2^2 \times 3 \times 11$ and 100 = 2^2 x 5^2. We will use to simplify the Kd2 cited above because it works. Kd2 = $2^8 \times 3^2 \times 5^2 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47$ and after calculation as shown in table 11B, we arrive at the decomposition which gives the LF (the largest factor of the number Bn) as follows = 74442235353227592257173111915367167756492266707200 : 526302819257.16 = 1414437328272298725648035663718757920000 = $2^8 \times 3 \times 5^4 \times 7^3 \times 11 \times 13^2 \times 17^2 \times 19^2 \times$

23 × 29 × 31 × 37 × 41 × 43 × 47 × 698754303007.

In the second example of Table 11B, we use a more extended Kd2 noted Kd2' (meaning that we multiply the initial Kd2 with two more prime factors to enlarge it). Kd2' = Kd2 x 53 x 59 = $2^8 \times 3^2 \times 5^2 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 53 \times 59$ = 8355280344672717027291277958400. We see that we have the ratio 295/100 in this example with 295 = 5 x 59 et 100 = $2^2 \times 5^2$. The number is broken down as explained in the table 11B with the quotient = 8788038544977674004361905048997925718 40663105152000 = $2^{10} \times 3^2 \times 5^3 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 41 \times 43 \times 59 \times 484703753292603814487663$.

In the example described in Table 11C we are dealing with an SR which is too large compared to the prime factor SF. This case is to be mentioned in giant numbers and especially when the two prime factors are large with a large gap between them and also far from each other from the SR. In this case, it will be necessary to provide a certain number of calculators which operate in parallel but interrelated so that if one displays an integer, the calculation stops on all of them simultaneously. We multiply the SR with 10-n depending on its value. The higher Kd, the fewer operations are required. But if we configure several interconnected calculators, we could work with Kd2 and Kd3.
The number Bn in Table 11C that we want to decompose consists of 79 digits. You have to count 3299192465661504598663118063 decimal fractions to get to 9.45 x SF = 10187338007316625151 76263587171.5.
Here we use a Kd2'' extended (meaning that we multiply the initial Kd2 with more prime factors to enlarge it). Kd2'' = 16356523386 6050646890193661728000 = $2^8 \times 3^3 \times 5^3 \times 7^4 \times 11^3 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 53$. We then have the ratio 945/100 with 945 = $3^3 \times 5 \times 7$ and 100 = $2^2 \times 5^2$. The breakdown of the number is explicit in table 11C.

5. Conclusion on the Method

The method which uses the decomposition key is a new method which can be promising. It can be used for numbers to decompose by starting with their square roots. It works with all numbers. It is necessary to have calculators for very large numbers.

6. Discussion

This article provides original material and proposes new methods for decomposing numbers. It draws its originality from 1) proposing decimal divisors instead of natural integer divisors as in the case of serial divisions known as Trial division algorithm; 2) it also proposes a calculation trick which consists of fixing two decimal digits by varying the rest of the digits of the number which makes it possible to accelerate the decomposition of the odd number into its prime factors; 3) it varies the methods which proves that they are robust since they are flexible and determinate or semideterministic; 4) it takes advantage of the decimal parts of decimal or irrational numbers to decompose numbers. 5) it proposes a new notion that of decomposition key (Kd) which is only the product of consecutive prime numbers and their powers (only certain ones) which decomposes all numbers having a fixed number of digits. Here the article uses those specific to two- and three-digit numbers.The Erastothenes'sieve is ineffective when it comes to carrying out an indeterminate number of steps to decompose a number going multiple by multiple, the method described in this article in fact makes it possible to exploit this sieve by a careful calculation of decimal fractions and go up or down to find the submultiples or supermultiples of a number to be decomposed. This sieve gives us all the numbers having a common factor. A number to be decomposed then has an infinity of numbers having a common factor with it. In this article, those having a common fact < the number to be decomposed are called submultiples; and those > are called supermultiples.

The methods described in this article do not only allow the factorization of biprime numbers or any multiples of prime factors, they make it possible to find a whole set of numbers having a common factor with the number to be decomposed, unlike the unique solution offered by trial division where we only have one prime factor that works; or by comparison to other methods such as that of Fermat based on the subtraction between two perfect squares. Here, the decomposition of the number has limitless solutions or outcomes and therefore we can now argue that these methods described in this paper have the particularity of offering several solutions (even unlimited) and therefore of considerably increasing the chances and the speed of decomposition of an odd number whose prime factors we do not know.

The method described here based on the divisions of the number to be decomposed Bn = p x q by 2^n or 10^n which gives submultiples (or symmetrically supermultiples of Bn) should not be confused with the classic methods of searching for common factors between two numbers, whether manual or using a specific calculator. Let us recall here that a submultiple is of the form n x p or n' x q which are both < Bn, and a supermultiple is m x p or m' x q that both are > Bn. Firstly, we are assumed not to know the p and q factors of Bn and secondly the common factor calculator will not be of any help since the p and q factors of Bn are unknown. We cannot list the factors of Bn neither since it only has two which are unknown. The method described here is above all a method of decomposition and factorization like any other algorithm. The decomposition method of this paper has its own specificity and cannot be deduced from any known algorithms which are more complex and based on more advanced mathematical notions. The relative simplicity of this article's methods is an additional advantage because it can be used by a very wide range of audiences.

By using decimal divisors and playing with the location of digits and decimal commas or by allowing the square root to unfold towards 0 or infinity, this study shows that it is possible to decompose numbers with an efficiency that is not negligible compared to existing algorithms whether it is The trial division or others such as Pollard's. These are all limitated by the length of the number and the time of calculation or analysis [6]. Unless there is an unintentional error, this type of calculation detailed in this article has not been reported before. The calculation methods are all based on the calculation of decimal fractions with the number to be decomposed into the numerator and the denominator is a decimal divider concocted to result in a sub- or super-multiple of the number to be decomposed which contains one of its prime factors.

The calculator best suited to the methods described here would be that which allows continuous calculation of decimal fractions with decimal numbers as denominators. It is also possible to perform that calculation with specific program on a computer. By analogy, the methods decribed here would amount to going back through Erastothenes'sieve upstream or downstream to find multiples having a common factor with the number to be decomposed. The idea of decomposing a number by looking for limitless numbers that have a common factor with it is a new idea of this paper.

The methods described are also limited by the length of the operations like those known, but are very promising since they offer possiblities to shorten the paths and save time required for the decomposition of a number. Dramatically increasing the exponent n of 2^n or 10^n which are used to fix the decimal part of the denominator results in decimal fractions tending towards 0 meaning that the decomposition of a giant number could be done quickly. These new methods decribed in this article and the concepts they convey might lead to new algorithms or programs to decompose an integer into its prime factors. More importantly, this article attests that the decomposition of a biprime number or multiple of prime factors, does not have only one solution (finding the right prime divisor) but can have an infinity of solutions. This idea has always been present in Erastothenes' sieve if we follow multiple by multiple, but it was necessary to develop a calculation method that puts it into practice, and that is what this article was dedicated to.

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