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New Definition of Prime Numbers with Sppn Tables and Proofs by Induction

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Abstract

Previously published work is summarized showing that the prime numbers form a unique number system independent of other numbers. Rather than being the odd numbers that do not appear in the multiplication table, or identified by a Sieve, they in fact form an elegant mathematical Algebraic Group structure and are calculated directly using only addition and subtraction. A new definition of prime numbers is given, and a new number system results called Nature's Number System. The prime numbers are calculated in groups of increasingly larger size from prior calculated groups, forming families with unique ancestors and descendants. Group properties include closure, reciprocity, symmetry, completeness and present a wave nature whose wavelengths repeat to infinity. Prime number structures such as twin primes are predictable to infinity because they are built from prior twin prime pairs as ancestors. Unlike other attempts to determine primes such as formulas (e.g. Mersenne Primes), multiplication wheels, sieves, etc., this method becomes more accurate as the numbers increase in size using just a few simple rules. This provides new tools for understanding the complex nature of prime numbers and to create proofs by induction for problems that previously were out of reach.

Keywords: Prime Numbers, Twin Prime Conjecture, Goldbach Conjecture, Riemann Hypothesis, Induction

1. Introduction

Since Euclid proved that there are an infinite number of prime numbers over 2500 years ago, there has been little progress in understanding the illusive nature of primes. There have been dozens of attempts at creating formulas, "wheels", sieves, etc., however, these tend to work for small numbers and break down quickly as you go to larger numbers. Mersenne primes are a good example of a formula whose success rate diminishes rapidly with increasing size. They do not predict primes but ultimately depend on brute force calculations to finally determine primality. Reliance on solving the Reimann Hypothesis has shrouded the issue and [1-3]. Even if it were proven that the Reimann zeros all lie on the vertical ½ line, it would still not give any predictive value to the problem of identifying prime numbers to infinity or solving other outstanding problems like the Goldbach or Twin Prime Conjectures. Lastly, supercomputer and GIMPS calculations lead to occasional large primes and [4-7]. These do not give any understanding since all the intermittent primes are not identified nor does it provide a way to predict where the next prime will be found.

For 25 years the author worked in high level engineering positions dealing with encryption and computer/telecommunications protocols. Another 8 years were spent at university level positions in Physics, Mathematics and Computer Science. One of the main issues encountered dealt with computer security and at the base of this were the prime numbers from both a pure mathematical and practical perspective. One of the major problems was always protecting the location of RSA codes in both hardware and software. With compartmentalization in civilian and military applications, it became increasingly difficult to know where various prime number based public key encryption protocols had been placed or how they were implemented. The assumption was always that prime numbers are undeterminable except by huge amounts of super computing time. The current work changes this because now prime numbers are directly calculated and there are associated formulae which predict large prime numbers with great accuracy.

My own personal research in this field involved studying prime numbers since I was in high school. What was needed was a new set of tools that 1) allowed rapid direct calculation of prime numbers and 2) on the pure mathematics side, allowed proofs by induction. The vulnerability of RSA encryption exposed other issues with prime numbers. The lack of understanding was amplified by the fact that although prime numbers are the basis of all numbers, nothing was known about them. At best they were believed to be random (or pseudorandom) or as one professor put it "they are like so many random lottery numbers".

In 2007 my first public release was the book "Calculate Primes" defining the "Generator Function" and a new definition of prime numbers based on addition and subtraction. It was released on an international radio program, other public venues as well as within the mathematical and encryption industry. Prime numbers could now be calculated in groups and had families with each prime number having a unique set of ancestors and descendants [8]. A new theorem was proven that each prime number had a unique ancestry of prime numbers and a unique downline of "offspring" primes. This was the equivalent of the fundamental theorem of arithmetic (that all numbers had a unique factorization in prime numbers). However, this theorem states that all prime numbers have a unique ancestry of prime numbers that created them via addition using the Generator Function. It additionally stated that all future primes were calculated uniquely from this prior existing set of prime numbers. The complete mathematical system started with Peano's Postulates and proceeded to generate the prime numbers in Algebraic Groups of rapidly increasing sizes. This first book was followed in 2010 by "Principles of Prime Numbers – Volume I" which formalized the Generator Function and presented the visualization in tables called "Sppn Tables". All the prime numbers that had been directly calculated were found in these tables and no other tests or calculations (factorization) were needed to show primality [9].

The new number structure "Nature's Number System" (notation modNt) that had been defined in the original text, visually illustrated the unique prime number ancestry of every prime number in the digits of the number. This system of generating prime numbers and the related number system modNt have properties of symmetry, reciprocity, closure, completeness (all primes are found) and shows wave function properties that predict all prime numbers to infinity with high accuracy. These wave patterns add like other wave systems, and this is visible in the prime number tables once you know how to see them. Prime numbers are calculated in Algebraic Groups using previously calculated prime groups starting with only 0 and 1 (creating the bridge between the primes and Peano's Postulates to form a complete mathematical structure). This gave the basis for proofs by induction since one group of prime numbers generates the next group. The primes were calculated from base numbers known as Sequential Prime Products (notation Sppn) and multiples of these numbers. It is around these numbers that the prime number symmetries occur. In 2014 another book was released "Breaking RSA Codes" which used the principles to quickly break RSA codes. This was the public release of the mathematical methods, although the inner circle encryption community knew this upon release of the Calculate Primes book in 2007 [10].

The new number system showed that attempts to categorize or understand prime numbers based on the number of digits or which digit they began or ended with were quirks of the number system and had nothing to do with the prime numbers themselves. A major result is that the modulo 10 number system (or any other modulo n system) was limited and created major stumbling blocks to the true understanding of prime numbers. The other benefit of the new number system is that all digits of the numbers have meaning showing the ancestry of the number. Each digit shows the history of the number in relation to prime number building blocks as opposed to large modulo 10 (or any modulo n) numbers where the digits are meaningless. The new number system uses far fewer digits to express a given number especially for large numbers. The conversions between modNt and mod10 are given below.

Another aspect of understanding is that an upper limit could now be defined which limits what one might call "rogue primes". Previously it was unknown if there could somewhere be primes that have huge gaps or small clusters which would affect solutions such as the Twin Prime and Goldbach Conjectures. The Generator Function provides an alternative proof of the counting function and more importantly puts an upper limit on the location and number of primes that is monotonically decreasing (an advanced topic not covered in this paper). The techniques developed also have been used in esoteric topics like snowflake creation, galactic symmetry, N-Body problems, large scale quantum systems or the construction of DNA since many of the same principles are at work (using building blocks to create large scale systems). We no longer have to depend on analysis statistical methods to put limits on the prime numbers but can directly calculate the boundary conditions with certainty.

The power of the system can be seen in the ability to directly determine future twin primes (or prime pairs of any gap size or gap sequence) since they are directly calculated from ancestor pairs. A result is that not only can one determine all future twin primes and determine that there will be an infinite number of them, but also prime pairs of any gap size or even prime gap patterns of any length. Finally, the system is very simple with only one basic formula with only one simple boundary condition rule that generates the large array of properties of primes that were heretofore unexpected. A note on the text below … The following sections are primarily mouse copied from the books noted above and [8-10]. The portions that are mouse copied are enclosed in boxes and are reprinted with copyright permissions from the original texts (any use requires written permission from the original author and publisher). This is a basic summary of the major points in the referenced texts and for a complete understanding please refer to those books using the links in the reference section below. Much of the material cannot be reproduced here because it is not possible to format for Section 508 standards. The reader is referred to the referenced 3 books which are currently available (the second volume of the Principles of Prime Numbers is being released which covers proofs and advanced topics).

2. Discussion of Multiplication Tables, Sieves, Wheels, Formulas/Categories of Primes, The Riemann Hypothesis and Computational Methods

Before looking at the Generator Function and related solutions, it is important to define terms and identify prior methods of determining prime numbers so there is no confusion or false claims that the current work is one of these. These prior methods tend to work with small prime numbers but soon lose their ability to locate primes and one then has to revert to division by prime numbers (factorization) to determine primality. Multiplication Tables: The most fundamental method of locating prime numbers is to create a multiplication table and note that all the numbers missing are the prime numbers. The draw back is that you have to write all the natural numbers to find a decreasing number or primes. Finding these missing numbers becomes increasingly difficult and time consuming as you grow to larger numbers. You also have to repeat many iterations (all multiples of 2, 3, 5, etc). Of all the primitive methods this is the least productive and as I recall was my first introduction to prime numbers in grade school. Unfortunately, that situation has not changed to this day.

Sieves: The next stage to streamlining the multiplication table process in locating primes is called a Sieve. There are numerous forms of Sieves, but all are basically variations on the same theme. You take a list of all natural numbers and begin crossing out the numbers that are multiples of known small prime numbers, leaving a very small select list of primes relative to the large number of numbers at the beginning of this process. The issues with a sieve are that 1) you need to know the prime numbers before you start the process, so it only is valid for small numbers for which you already have the prime number solutions and 2) you have to list all the natural numbers (as with the multiplication table method *it is a process of elimination starting with all numbers)* so it is not a number system based on just prime numbers but likewise an elimination from the entire list of natural numbers. Most importantly, it ultimately resorts to factorization to identify the prime numbers for more eliminations and lastly 5) IT HAS NO PREDICTIVE PROPERTIES. All of these issues are what separate the Sieves from the work presented in this paper. To be clear, in the current paper, we only use prime numbers to generate more prime numbers using just addition and subtraction, you do not use the non-prime numbers, you start with the smallest possible set (beginning with just 0 and 1) and build to larger groups (which grow very rapidly). Primes are discovered by direct calculation from prior known primes. The method works on patterns of prime numbers and predicts prime numbers to infinity including individual prime numbers, twin prime pairs, prime pairs of any gap size and creates an alternative proof to the prime counting function and additionally puts an upper limit on the counting function (something that had not existed previously). This is stressed here because the first utterance from some people too incumbered by sloth to read the entire paper would try to negate the current work as a Sieve prior to reading and understanding it. Then those who are even more incumbered by sloth would rely on those false claims to revert to their comfort zone. To be clear, *the current work is not a Sieve.*

Wheels: There are dozens of attempts to organize or predict prime numbers based on patterns that can be represented as consecutive squares, complex tables, multisided objects or concentric circles (usually based on basic number associations). They seem to work for small numbers but very soon break down or develop large numbers of false (non-prime) numbers. They generally are followed by long lists of "rules" that break down as numbers get larger. As a result, one has to resort to factorization to determine primality. Many amateurs have fallen into the traps of using such limited models. One such "wheel" depends on multiples of 6 and another recent attempt relied on prime numbers less than 210 with long lists of convoluted "rules". These arise from people noting nuances in the lists of small prime numbers but which fade quickly. One very interesting wheel or circle pattern involves making concentric circles with 24 divisions (the first inner circle numbered from 1 to 24). The next outer circle likewise has 24 divisions numbered from 25 to 48 with the next circle numbered from 49 to 72 and so on. This is not a new discovery but simply restating the long known fact that if you create a circle with 24 divisions, all of the squares of primes will be found on the radial line above the number 1. That is, the squares of prime numbers all differ by a multiple of 24. An equation can be written which is the opposite of this known fact of prime numbers and as you will see is not very useful in predicting prime numbers. It quickly fades to having many false predictions. It has some value in predicting the squares of primes which is related to certain analysis results. To create the circles ... draw a series of concentric circles. In the first circle divide the outer circumference into 24 divisions. Number these from 1 to 24 in the clockwise direction. On the next circle outwards (also divided into 24 sections), the numbers should be the number on the first circle + 24. So the radial line extending out from 1 will have the values as follows: ... (notice that it skips 2^2 and 3^2) starting on the next page.

1 + 0x24 = 1 (the first circle)
1 + 1x24 = 25 = 5^2 (the second circle)
1 + 2x24 = 1 + 48 = 49 = 7^2
1 + 3x24 = 73 (prime)
1 + 4x24 = 97 (prime)
1 + 5x24 = 121 = 11^2
1 + 6x24 = 145 = 5x29 (composite of primes)
1 + 7x24 = 169 = 13^2 and so one. ALL of the squares of primes fall on this radial line. By subtracting 1 you get an integer divisible by 24. The problem with this (or other "predictive" formulas such as 6 ± 1) is that they seem to work well for small numbers but as the prime numbers become scarce, you are getting many more false outcomes than real primes and you must then resort to the old brute force method of factorization to see if the numbers are prime or not. It is really not very useful. On this theme I have proven that all squares of prime numbers in fact fall on this line using induction using the Generator Function (something that was conjectured before but not proven). Wheels with divisions of 6, 24, 30, 210 and others have been attempted and all fail. Formulas and Categories of Primes: It would take pages to list all of the different small equations that have been presented over the centuries in attempts to predict prime numbers. Some of the more famous include the category of Mersenne Primes which was developed relative to the search for "Perfect Numbers". Volume II of the Principles of Prime Numbers (about to be released) covers these categories in detail, comparing them to the Generator Function presented in the current work to show their limitations. The simple result is that some equations work for small numbers but end up generating far more false solutions as the iterations become larger. Mersenne Primes are a primary example which has an extremely low success rate with increasing number size. Despite this, they are used as a basis for calculations using super computers and GIMP calculation networks. As with all the simple formulas, they work well for small numbers but immediately break down and start producing large quantities of false results. As a result, they are relatively useless in the understanding of prime numbers and one must then revert to factorization to determine if the number is truly prime or not. The current paper shows that the Generator Function becomes more accurate with larger numbers.

The Riemann Hypothesis: It is incorrectly assumed by many that the so called "solution" to the Riemann Hypothesis would solve the issues of understanding prime numbers. To this end, I refer to the seminal book on the subject by [4] John Derbyshire "Prime Obsession" (written for the professional and layman alike) where he addresses this topic. The proof that all Riemann Zeros lay on the vertical ½ line will not bring any additional understanding of locating prime numbers. The status is that if anyone discovers a large arbitrary prime number, mathematical methods can be employed to show that it in fact also creates a zero point on the ½ vertical line. It is the belief of this author that the Riemann Hypothesis solution exists and the methods being disclosed here provide the tools for a proof by induction. That was one of the primary motivators for developing the Generator Function in the first place, to create new tools that will allow proofs by induction relative to prime numbers, something that had not existed previously.

Computational Methods: Computer methods use short cuts in eliminating numbers as prime numbers but they generally begin with Mersenne's simple formula as a starting point because it is thought to produce a higher percentage of results than just picking odd numbers at random or in succession. Ultimately, it is brute force computing that will find a new large prime number. When the numbers are announced, they are generally given in relation to the nth Mersenne number related to the discovery. The numbers leave immense quantities of primes undiscovered, and the discovery does not provide any insight into where the next prime will be found. When the numbers are published in base 10 digit format, they truly are a list of meaningless digits and do in fact look like so many random lottery numbers hooked together. The base 10 number system is a hindrance when it comes to understanding prime numbers. The elegance that comes from the new modNt Nature's Number System is that every digit signifies the ancestry of the number. Evey digit has meaning, and every digit relates the number back to its ancestry and is then used to build a future set of primes that are unique to infinity that have the same base digits. If new primes are discovered using the modNt system, they could be used to predict the location of larger or smaller primes (a field that some may pursue when computational methods are required to claim prize money).

In the development of the Generator Function, which directly calculates prime numbers with increasing accuracy, it was necessary to have all these failed systems in mind. It is important to repeat here that the Generator Function and the visualization of primes in what are known as the Sppn Tables, one only deals with prime numbers as a complete and distinct number system and mathematical Algebraic Group structure. It has predictive abilities and sees prime numbers as a distinct class of numbers completely independent of the rest of the non-prime numbers. The prime numbers are generated in groups of ever-increasing size and use the new definition of prime numbers illustrated below. The traditional definition of prime numbers, that prime numbers are numbers only divisible by themselves and 1, as well as the base 10 number system are hinderances to understanding the true nature of prime numbers.

One last thought will be given before summarizing the Generator Function and its vast number of implications. Many will be caught up in the one aspect that this may allow solutions to unsolved problems, but the most important aspect is that this finally creates a sound mathematical structure for prime numbers in a pure mathematical sense. Similar previous works of this fundamental nature would be Peano's Postulates, George Cantor's work defining infinities, or Russel's work on logic. The idea that prime numbers form in groups that follow Algebraic Group theory is a fundamental discovery that has been missing since the inception of prime numbers. The fact that prime numbers have a different definition which leads to these results now augments the importance of relative primes. Previously, there were primes and non-primes only. The new definition shows the new understanding of relative primes (of which the prime numbers are a subset), in the generation of true prime numbers. The new structure shows the prime number Groups have properties of symmetry, reciprocity, closure, completeness and a wave nature that was never understood. Also recall that what is presented below is a limited summary of major points and the complete definition and explanations are found in references and [8-10].

3. The Generator Function – New Definition of Prime Numbers

The following material is mouse copied from the book "Calculate Primes" (2007) and forms the new definition of prime numbers. It creates the basis for prime numbers being directly calculated and the Algebraic Group structure of prime numbers that will become more evident when visualized in the Spp_n Tables. This simple equation holds the key to understanding the complex mathematical nature of prime numbers as a number system completely independent of other number systems. The prime numbers are directly calculated without using any other numbers. A major result is that this gives rise to proofs by induction [8].

The basic concept is a simple application of the distributive property of algebra and an extension of the Euclid proof that prime numbers are infinite. The product of sequential primes (notation the nth Sequential Prime Product = Spp_n) is a product that contains one of each of the prime numbers up to and including the nth prime number pn. If you add or subtract any number N that contains any of these prime factors, then that prime can be factored out of both Sppn and N and therefore (Sppn + N) is not prime. This means that the only numbers that can be prime are Sppn ± 1 and Sppn ± M where M is **relatively prime to Sppn.** The follow up is that all numbers generated by this method less than the square of pn will be prime. Note that relative primes are as much a prime as "real" primes with respect to these numbers. This is a basic alternative definition of prime numbers which generates prime numbers from existing groups of relative prime numbers without the use of factorization. There are simple methods of identifying all the prime numbers in the group and not just those less than the square of the nth prime (the boundary condition) as an advanced topic not covered in this paper. The force of this method is that it can begin with the two numbers generated in Peano's Postulates 0 and 1 and can be used to generate groups that rapidly grow in size to generate all of the prime numbers. These can be represented in visual tables called Sppn Tables which contain only prime numbers in the nth Group. Since each Group is generated from a prior group, and since the properties of the original group are carried to the new group, this allows for proofs by induction. These are the tools that have been missing from prime number mathematics. The following is mouse copied from the book [8] which must be referred to for the complete explanation. The entire process is based on this simple equation but has many far reaching effects. Current work on this topic is continuing to discover subtle nuances to understand prime numbers.

******** THE GENERATOR FUNCTION *********

The notational representation for an "umbrella group" is as follows (the symbols ε and $\underline{\varepsilon}$ represent "are an element of " and "are not an element of" respectively):

$$U_{gn} = \Pi_1^n p_i \pm 1 \text{ and } \Pi_1^n p_i \pm p_{cm}$$

$$p_{cm} \ \varepsilon \ U_{g(n-1)} \text{ and } p_{cm} \ \underline{\varepsilon} \ R_{gn} \ \text{(for all } p_{cm} > 0)$$

Prime Number Selection Rule – $p_{real} < p_n^2$

(Where p_i is the ith prime number of the nth "sequential prime product" Spp $_n$ and the Greek letter Π represents the operation of multiplying all terms defined after it).

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Each term will now be separated out and examined individually.
In this definition the "sequential prime product" is written by the following expression.

$Spp_n = \Pi_1^n p_i =$ "repetition group length" = wavelength

Within the wavelength is a series of relatively prime numbers (prime candidates) that repeats to infinity by repeating this wave – all future prime numbers are contained within this repeating wave. All twin primes and prime pairs of any gap size repeat and therefore generate all future pairs of similar gap sizes. The prime numbers are predicted without the need for factorization. The waves repeating will add similarly to other wave systems to give prime numbers a seemingly pseudorandom nature, with patterns coming and seemingly fading as other patterns emerge. Each succeeding Spp_n group wavelength becomes larger and defines an upper limit of the location of prime numbers; something that has not existed previously. Additionally, this wave is symmetrical. If you apply the same wave of prime candidates in the negative direction it likewise identifies all the prior prime numbers to the left of Spp_n on the number line. This is one of the most amazing and unexpected properties of the Generator Function (note that ± is used in the Generator Function definition and not just addition). Prime numbers have symmetrical properties. It creates a new number system based on Spp_n and a mathematical Group structure.

4. Sppn Tables

The Sppn Tables are the visual representation of the Generator Function. The following is mouse copied from the book [9] which must be referred to for the complete explanation. Please refer in that book to the sample tables on pages 75 and 79 (copied below). Page 75 develops the first tables up to n = 4, pn = 7 and Sppn = 210. Page 79 presents the table for n = 5, pn = 11 and Sppn = 2310 with 11 rows and 48 columns. It has already uniquely isolated 35 prime numbers. The n = 11 table cannot legally be reprinted here because it does not comply with Section 508 format rules but is included below for illustration. The size of the tables grows faster than exponential which is one of the powers of this method. For example, the table for n = 13, $p_n = 41$, and $Spp_n = 3{,}042{,}632{,}473{,}527{,}210$. If this number were represented in a table with 1 cm cubes, it would stretch to 5 times the distance to the sun. This shows the rapid expansion of the method to rapidly find large numbers of prime numbers without factorization since they are directly calculated. The boundary condition for identifying prime numbers in the Sppn tables is all numbers in the "red columns" less than p_n^2. For example, below, in the n = 3 table with $p_n = 5$, all numbers in the red columns less that $p_n^2 = 25$ are certified as prime. The number of identified primes grows faster than exponentially as the number Spp_n grows faster than exponentially. To build an Spp_n table, take the rows from the prior table and link them together to form row 1 of the nth table. Next add Spp_{n-1} to each number in row 1 to form the second row, add 2 Spp_{n-1} to form the third row and N Sppn up to and including the p_n row. The tables have p_n rows. All prime numbers are generated in the red columns so one can eliminate the white rows and only deal with the red columns. The reverse process is to find the ancestry of each prime number back to its origins.

0

n = α
$P_n = 0$
$Spp_n = 0$

0	1

n = 0
$P_n = 1$
$Spp_n = 1$

Red columns only contain relative and true primes

0	1
1	2

n = 1
$P_n = 2$
$Spp_n = 2$

0	1	2
2	3	4
4	5	6

n = 2
$P_n = 3$
$Spp_n = 6$

0	1	2	3	4	5	6
6	7	8	9	10	11	12
12	13	14	15	16	17	18
18	19	20	21	22	23	24
24	25	26	27	28	29	30

n = 3 $P_n = 5$ $Spp_n = 30$

000 000	000 001	000 002	000 003	000 004	000 005	000 006	000 007	000 008	000 009	000 010	000 011	000 012	000 013	000 014	000 015	000 016	000 017	000 018	000 019	000 020	000 021	000 022	000 023	000 024	000 025	000 026	000 027	000 028	000 029	000 030
30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150
150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210

n = 4 $P_n = 7$ $Spp_n = 210$

"Raw" Tables - Examples of table dimensions using square cells

Tables have p_n rows. Prior table rows build row 1 of next table. Add multiples of Spp_{n-1} to row 1 creates rows 2 to p_n .
Exercise – construct tables n = 5, 6, 7 and 8 using square cells (do not have to be to scale)
Every prime has a unique "ancestry" of primes. Prime numbers generating prime numbers.

5. Nature's Number System and Conversion to And from Modulo 10

One of the major developments of the Generator Function is the creation of the new number system "Nature's Number System" or ModNt. The major stumbling block in understanding prime numbers has been the universal use of modulo 10 (or other modulo systems). With mod10 numbers there is no relationship between digits in the number and the primes. Gauss studied various modulo number systems attempting to see patterns [11]. The most one can surmise from mod10 numbers is the final digit (if the number ends in a 5 or 0 it is not prime). Other mod N number systems do not offer any more than this less than rudimentary observation, and do not give any information about whether a number is prime or not.

The new number system is called "Nature's Number System" because it is a natural system based on prime numbers and if discovered in another part of the universe, would have the same properties irrespective of the symbols used to describe the numbers. It is based on Sppn values. Rather than 10, 100, 1000 as in mod10, the columns in modNt are 1, 2, 6, 30, 210, 2310, etc. The lowest order digits are 0 or 1 (from Spp_n = 1, table n = 0). The second order digits are 0, 1, or 2 (from Spp_n = 2, table n=1). The third order digits are from 0 to 4 (from Sppn = 6, table n=2), etc.. The formula for digits in each column n are as follows: 0, 1, … to p_{n+1} -1 (see table below for details). The beauty of this number system is that each digit represents the position in the Spp_n Table and gives the complete ancestry of the number. The following is mouse copied from the book which must be referred to for the complete explanation of the modNt number system [9].

The following table shows modNt vs mod10 number system between 0 and 72 mod10. Recognize the typical counting model but using the values of Spp_n as "powers" to higher order digits rather than powers of 10 as in the mod10 number system. Look at the lower order digit patterns of the modNt numbers. After the table are more details of the modNt number system including how to convert between the two systems. Most important below is to see the modNt values for the Spp_n values of 1, 2, 6, 30, etc. (values in modNt are as follows 1, 10, 100, 1000, etc.) and their multiples (e.g. 6 x 2 = 12 mod10 = 200 modNt; 30 x 2 = 60 mod10 = 2000 modNt). Even seasoned mathematicians will take some time to understand all the subtleties of this new number system. A word of warning … the first appearance is that the modNt numbers are larger and more complicated, however because the factors between successive digits of modNt grow as 1, 2, 6, 30, 210, 2310, etc., whereas the mod10 numbers grow as 10, 10, 10, 10, etc., the modNt numbers soon use far fewer digits to represent a given number than mod10 as detailed below. See modNt values for 1, 2, 6 & 30.

000	000		200	012		400	024		1100	036		1300	048		2000	060
001	001		201	013		401	025		1101	037		1301	049		2001	061
010	002		210	014		410	026		1110	038		1310	050		2010	062
011	003		211	015		411	027		1111	039		1311	051		2011	063
020	004		220	016		420	028		1120	040		1320	052		2020	064
021	005		221	017		421	029		1121	041		1321	053		2021	065
100	006		300	018		1000	030		1200	042		1400	054		2100	066
101	007		301	019		1001	031		1201	043		1401	055		2101	067
110	008		310	020		1010	032		1210	044		1410	056		2110	068
111	009		311	021		1011	033		1211	045		1411	057		2111	069
120	010		320	022		1020	034		1220	046		1420	058		2120	070
121	011		321	023		1021	035		1221	047		1421	059		2121	071
200	012		400	024		1100	036		1300	048		2000	060		2200	072

Table - Nature's Number System modNt vs mod10 Number System to 72

Below is a table which defines the modNt number system with relation to the magic numbers and associated values of n, p_n , Spp_n , the maximum digit value for each "power" and the patterns of each digit. This table defines the modNt number system up to Spp_7 - 1 = 510510 - 1 . Each pattern of digits in each column repeats as shown then repeats again and again as with any counting process. NOTE that the first two rows in the table below are subtle so understand that row 1 is the column of the digit (from right to left), whereas row 2 represents the counting subscript "n" in the nth prime p_n, the nth Sequential Prime Product Spp_n and is also the exponent of 10 in modNt.

Digit in column	7	6	5	4	3	2	1
n mod10	6	5	4	3	2	1	0
n modNt	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$p_n$mod10	13	11	7	5	3	2	1
$p_n$modNt	201	121	101	21	11	10	1
Spp_n mod10	30030	2310	210	30	6	2	1
Spp_n modNt = 10^n	1,000,000 = 10^6	100,000 = 10^5	10,000 = 10^4	1000 = 10^3	100 = 10^2	10 = 10^1	1 = 10^0
Max digit Value in mod10 = p_{n+1} -1	16	12	10	6	4	2	1
Pattern each digit repeats the given pattern over and over (values given in mod10)	0 x 30030 1 x 30030 2 x 30030 3 x 30030 4 x 30030 5 x 30030 6 x 30030 7 x 30030 8 x 30030 9 x 30030 10x30030 11x30030 12x30030 13x30030 14x30030 15x30030 16x30030	0 x 2310 1 x 2310 2 x 2310 3 x 2310 4 x 2310 5 x 2310 6 x 2310 7 x 2310 8 x 2310 9 x 2310 10x2310 11x2310 12x2310	0 x 210 1 x 210 2 x 210 3 x 210 4 x 210 5 x 210 6 x 210 7 x 210 8 x 210 9 x 210 10 x 210	0 x 30 1 x 30 2 x 30 3 x 30 4 x 30 5 x 30 6 x 30	0 x 6 1 x 6 2 x 6 3 x 6 4 x 6	00=0x2 11=1x2 22=2x2	0=0x1 1=1x1
Max number in pattern mod10	510509	30029	2309	209	29	5	1
Max number pattern modNt	16.12.10.6.4.2.1	12.10.6.4.2.1	10.6.4.2.1	6.4.2.1	4.2.1	2.1	1

Table – modNt definition – to Spp_7 – 1 = 510510 – 1 = 510509

Note that decimal mod10 numbers (16, 12, 10, 6, etc.) are used to designate digits (separated by "." periods) rather than create individual symbols to represent each digit. This is commonly done in mathematical number systems such as hex numbers or especially where the base number is greater than 10. We could have created other symbols or used the various alphabets or Greek alphabet but even they would eventually run out of symbols, so it is easier to use the familiar base 10 system with dots to separate the digits. Eventually we will not use numbers at all but will use tables with indicators in key locations, so this problem goes away as the tables get larger. The following section is also mouse copied from book [8] and discusses the size of Spp_n tables. For example, examine table n = 6, $p_n = 13$, $Spp_n = 30,030$ mod10 = 1000000 modNt and $Spp_{n-1} = 2310$ mod10 = 100000 modNt. If all numbers (prime and non-prime) were included, the table would be 2310 columns and p_n=13 rows deep = 2310x13 = 30,030 cells. However, when separating out only "red" columns which contain the prime numbers in the Group, the red cell only Spp_n table has only 480 columns and 13 rows = 6240 cells ALL OF WHICH contain prime numbers. This ratio decreases with larger tables since we are only dealing with prime numbers. This is the opposite effect from other simple equations, sieves or multiplcation wheels etc which become much less productive at discovering prime numbers.

The importance here is that we are able to build a mathematical structure to predict the size of future tables and the number of products, and therefore cross into the realm of analysis that has never existed previously to determine the number or primes, twin primes, and other patterns as the tables are created from prior tables. (NOTE: the table dimensions below are based on 1 cm x 1 cm cell sizes each of which contains just one number … for simplicity all numbers below are in mod10). Also note that the measurement in km of the table refers not to the prime number table with only red columns which is much smaller, but the complete Spp_n table which includes all numbers).

n = 6 $p_n = 13$ $Spp_n = 30,030$ $Spp_{n-1} = 2310$ $\sqrt{Spp_n} = 173.29$ $\sqrt{Spp_n} / Spp_{n-1} = 0.075$
table height = 13 cm table length = 2310 cm = 23.1 meters = .0231 km

n = 10 $p_n = 29$ $Spp_n = 6,469,693,230$ $Spp_{n-1} = 223,092,870$ $\sqrt{Spp_n} = 80,434.4$
$\sqrt{Spp_n} / Spp_{n-1} = .0003606$ (note is much smaller than for n = 6)
table height = 29 cm table length = 223,092,870 cm = 2,230,928 m = 2230 km

n = 13 $p_n = 41$ $Spp_n = 3,042,632,473,527,210$ $Spp_{n-1} = 74,210,548,134,810$
$\sqrt{Spp_n} = 55,160,062.3$ $\sqrt{Spp_n} / Spp_{n-1} = 0.00000074329$… (compare to above)
table height = 41 cm table length = 74,210,548,134,810 cm = 742,105,481 km
This table for n = 13 is about 5 times the distance from earth to the sun.

The concept of infinity and understanding the building blocks of all numbers being calculated directly in the tables may be difficult to imagine since it is a foreign concept on first sight. When translated into modNt, the values of Spp_n are as follows.

n = 6 $Spp_n = 10^6 = 1,000,000$ (compare to mod10 = 30,030)

n = 10 $Spp_n = 10^{10} = 10,000,000,000$ (mod10 = 6,469,693,230)

It is between n = 10 and n = 11 where the modNt numbers become more efficient at representing a given number. Note how n = 13 modNt has fewer digits below.

n = 13 $Spp_n = 10^{13} = 10,000,000,000,000$ (mod10 = 3,042,632,473,527,210)

n = 169 $Spp_n = 10^{169}$ (with 169 modNt digits compare to the mod10 number with 422 digits =
2002362020904452815951174109202611157572109628795203328089021691102946630929
9244261280131263405759302102973368274392117584712154554147852782521923413761
3370618124449226098176039020148146252608779838701145634546415654684175675729
7645106831441674354112486302483982702884250297015432485426918812250715726057
5794215703271997986950447857877037520410269496235087062782560401130103628214
34482930709936770277182142246605394175147 0
See the Appendix end of this text [9] which lists the Spp_n numbers up to n = 169.

Conversion between ModNt and Mod10 number systems

It is relatively easy to convert between the two number systems and is necessary for many operations and understanding of numbers (especially large numbers). A note is to be conscious of the subscripts as we will be talking about tables using "n", "n + 1", "n + 2", "n – 1", etc.

Conversion from mod10 to modNt

Given a mod10 number N = abcde… xyz where the letters represent decimal digits … we are going to use both a mathematical as well as visual approach to finding the properties of N

1) Determine the largest number Spp_n less than the given number (this corresponds to the highest value or rightmost cell of row 1 of table Spp_{n+1} in which the number N lies)

2) Divide N by the number Spp_n and determine the result A_1 plus a remainder R_1
3) A_1 is the value of the highest order modNt digit (the multiple of Spp_n) added to the value in the top cell of the table. The remainder R_1 is the top number in the column where the number would lie in the table Spp_{n+1} (R_1 is the value in row 1 of table Spp_{n+1}; if this is a grey column the number cannot be prime)
4) Next divide R_1 by Spp_{n-1} resulting in A_2 with remainder R_2.
5) A_2 is the second highest order digit modNt.
6) Continue this process until the entire modNt number is calculated = $A_1A_2A_3 \ldots A_{n+1}$
7) If there is a "0" it must be placed to hold the digit as with any long division.

Example: N = 737,269,373 mod10 (see section below from table Spp_{10} = 6,469,693,230 mod10 showing just rows 1, 2, 3, 4 and 5 with the local rows and columns)

67,990,761	67,990,762	67,990,763	67,990,764	67,990,765
291,083,631	291,083,632	291,083,633	291,083,634	291,083,635
514,176,501	514,176,502	514,176,503	514,176,504	514,176,505
737,269,371	737,269,372	737,269,373	737,269,374	737,269,375
960,362,241	960,362,242	960,362,243	960,362,244	960,362,245

Values in rows (mod10) increment by 1 →
The difference between numbers in the columns is Spp_9 = 223,092,870 mod10

1) Select Spp_9 = 223,092,870 the first Spp_n number less than N.
2) 737,269,373 ÷ 223,092,870 = 3 + R_1 = 67,990,763
3) N lies in table Spp_{n+1} = Spp_{10} = 6,469,693,230 in the column with top number = 67,990,763. N lies in row 4 because you have added 3 times the value of Spp_n to the value in the top cell of the column 67,990,763 and puts N in row 4 of table Spp_{n+1} = Spp_{10}. 737,269,373 = (3 x Spp_9) + 67,990,763
4) The highest order digit = A_1 = 3.
5) Divide 67,990,763 by Spp_{n-1} = Spp_8 = 9,699,690.
6) 67,990,764 ÷ 9,699,690 = 7 + R_2 = 92,933
7) The second highest order digit A_2 = 7 and this number lies in the table Spp_9 which is the next lower table. 67,990,763 resides in row 8 and in column with cell value = 92,933 in the top cell. Draw the section of the table that corresponds to this. Your table section will have the number 92,922 in the top cell and in the 8th row down will contain the number 67,990,763. As in the sample table above draw a few columns to the right and left of this column.
8) Divide 92,933 by Spp_7. But Spp_7 is 510,510 which is greater than 92,923 therefore the third digit modNt A_3 = 0. It appears in row 1 of table Spp_7.
9) Divide 92,933 by Spp_6 = 30030
10) 92,933 ÷ 30030 = 3 + R_4 = 2,843 so 92,934 lies in table Spp_7 in row 4 in the column with 2,843 in the top cell. A_4 = 3.
11) Divide 2,843 by Spp_5 = 2310.
12) 2843 ÷ 2310 = 1 + R_5 = 533 so 2843 lies in row 2 of table Spp_6 = 30030 in column with 533 top cell. A_5 = 1
13) 533 ÷ 210 = 2 + R_6 = 113 so A_6 = 2
14) 113 ÷ 30 = 3 + R_7 = 23 A_7 = 3
15) 23 ÷ 6 = 3 + R_8 = 5 A_8 = 3
16) 5 ÷ 2 = 2 + R_9 = 1 A_9 = 2
17) 1 ÷ 1 = 1 + R_{10} = 0 A_{10} = 1
18) The modNt number is 3,703,123,321

Note that the modNt number still has more digits than the equivalent mod10 number but this changes as the "powers" of the modNt system are continually increasing. To convert modNt to mod10, multiply each modNt digit by its corresponding Spp_n value and add them together.

Ancestry of Twin Prime Pairs

The following examples show the ancestry of twin prime pairs. First begin with the twin prime pair (97841,97843) (numbers given in mod10). Using the method above of finding the ancestry (the values or R_i) to locate the unique path back to the alpha prime 0.

97841 ÷ 30030 = 3 + R_1 = 7751 (R_1 = 7751 in Spp_7 table 97841 is in the 7751st column and 4th row)

7751 ÷ 2310 = 3 + R_2 = 821

821 ÷ 210 = 3 + R_3 = 191

191 ÷ 30 = 6 + R_4 = 11

11 ÷ 6 = 1 + R_5 = 5

5 ÷ 2 = 2 + R_6 = 1

1 ÷ 1 = 1 + R_7 = 0

Notice how this refers ultimately to the prime ancestor 1 and then to the alpha prime 0. The complete unique ancestry path is 7751, 821, 191, 11, 5, 1, and 0. The equivalent modNt number for 97841 is 3336121 (derived from the factors resulting from the division by values of Spp_i). Looking at the second number of the pair 97843, its ancestry path is 7753, 823, 193, 13, 7, 3, 1. All of the corresponding numbers form true twin prime pairs (821,823), (191,193), (11,13), (5,7), and (1,3) with the exception of (7751,7753) for which 7751 is ultimately found to be not prime (in a future table). So why is it included in the ancestry of this twin prime pair? Because in the table 30030 the number 7751 is relatively prime. The factors of 7751 = 23 x 337 are greater than the prime associated with that table's Spp_{n-1} value of 11 and thus is in the 4th row of a red relatively prime column and therefore carries to the next table to generate more prime numbers. In the red column tables, the relative primes are as much a prime number as "real" prime numbers. They are needed to generate "real" primes until they are eliminated naturally by the boundary condition rules of the Generator Function. This realization, that relative primes are as "real" as real primes in their respective tables, is one piece of the 7 part rigorous proof of the Generator Function.

7751 is finally "eliminated" in the table Spp_9 = 223,092,870 with p_n = 23 with 7751 residing in row 1. Before 7751 is eliminated it was able to generate the following list of prime numbers: 37781, 97841 (the 1st of our twin prime pair) and many others in Spp_n tables 30030, 510510, 9699690 and 223092870. It carried to these next tables generating prime numbers and twin prime pairs with its pair 97843. As an exercise see how many other twin prime pairs you can find generated by the pair (97841,97843). You will find an infinite number of them if you continue long enough, each of which will in turn generate more offspring twin primes. This will lead to the formal proof that states that **"Every twin prime pair will generate an infinite number of subsequent twin prime pairs".** Note that all the twin prime pairs in this generation path have the exact same ancestry going back along the same path to 0 and 1 as (97841, 97843). All of the new twin prime pairs in turn will continue to generate twin prime pairs including those which have one or both of the members as relative primes in a given table. Only when 97843 becomes the prime number associated with its own table Spp_n will it cease to generate prime numbers as it will then fall into the DZ_n "Dead Zone" region. But the large number of twin primes that it has generated will continue on generating future twin primes all with ancestry back to (97841,97843) which have their ancestry back to the alpha prime 0 by their own unique paths. This illustrates another of the properties of the Generator Function that is equivalent to the Fundamental Theorem of Arithmetic in which every number has a unique set of prime factors. In the current case it is stated that **"Every prime number has a unique ancestry of prime numbers going back to 0 the "alpha prime"".** Of course, included here are numbers that are relatively prime to the table in which they are found in this ancestral path since in that table they are as much a prime number as true prime numbers. There is a formal proof for this but this should be fairly obvious at this point. The process efficiently finds the real primes in each table with simple selection rule $p_{real} < p_n^2$ with no factorization required. The same is true for every twin prime pair, and every prime pair of gap k_i and every series or pattern of gaps no matter how large and complex. This will allow us to prove many previously unproven aspects of prime numbers and will show the method of finding any defined gap patterns to infinity (something that previously could only be accomplished in limited scope with super computers or with analysis statistical methods).

I chose the twin prime pair (97841, 97843) for this example because it is part of a triple twin prime pair sequence. The other two successive twin prime pairs are (97847,97849) with a gap of 4

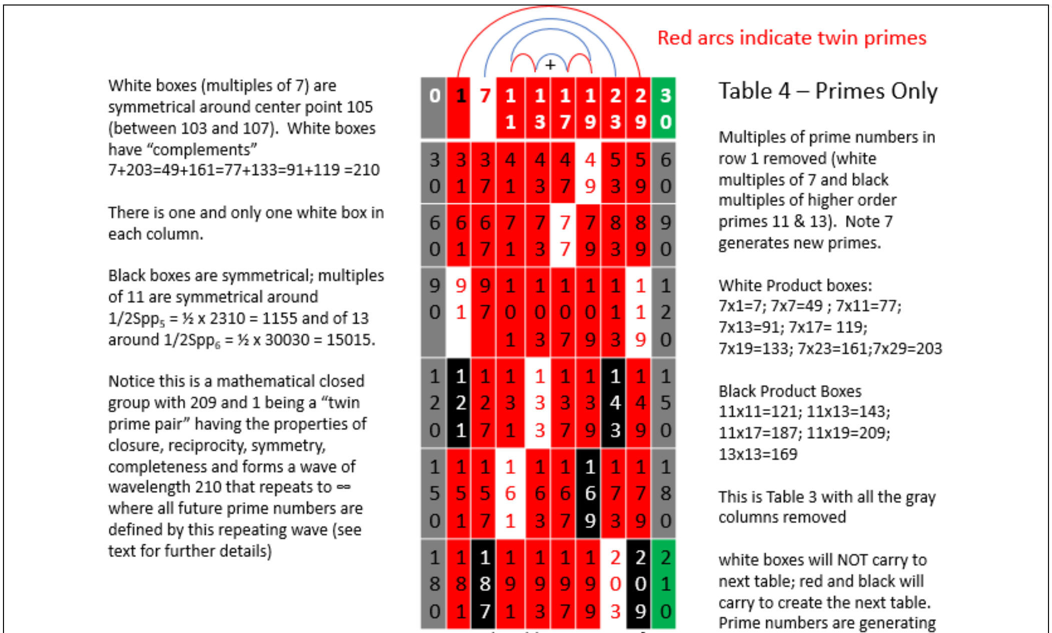
between these and the following member of the triplet (97859,97861) with a gap of 10 between the second pair. The ancestry sequence of these are as follows (non-primes are noted with an *). See the next page for details. (97841,97843), (7751*,7753), (821,823), (191,193), (11,13), (5,7), (1,3) (97847,97849), (7757,7759), (827,829), (197,199), (17,19), (5,7), (1,3) (97859,97861), (7769*,7771*), (839,841*), (209*,211), (29,1), (5,7), (1,3) The ancestries all converge at (5,7) and (1,3) but this is due to different remainders. For example, 11 ÷ 6 = 1 + R = 5 ... whereas 17 ÷ 6 = 2 + R = 5 and lastly 29 ÷ 6 = 4 + R = 5 so all ultimately converge to the same ancestors (5,7) and therefore (1,3). This example shows that two relative primes (7769*,7771*) are able to generate true twin primes. These generate many other twin prime pairs in this table. Note in the 3rd row above (ancestry of (97859,97861)), that the twin prime pair (29,1) appears. In a given table the end prime number can wrap to column 1 to create a twin prime pair that in fact carries forward to generate future twin primes. This is from the second prime of pair (209*,211). 211 ÷ 210 = 1 + R = 1 where R1 = 1 so technically by the rules it results in the pair (29,1). We have to refer to higher abstract concepts of Abstract Algebra Group Theory for a complete explanation of this issue.
The following note is added for clarity. Converting the numbers above to modNt, it is clear that these numbers have meaning in every digit of the new number system. The numbers are built from smaller numbers that are in the same columns of the Spp_n table. A good exercise for interested readers would be to spend some time converting prime numbers in mod10 to realize their root numbers in modNt. It will become apparent that the prime numbers are not random lottery numbers but a complete number system with ancestors and descendants. Here is the sequence noted above of twin primes in mod10 (97841,97843), (7751*,7753), (821,823), (191,193), (11,13), (5,7), (1,3) and here is the same sequence modNt (3336121,3336201), (336121*,336201), (36121,36201), (6121,6201), (121,201), (21,101), (1,11). Look at the first numbers of the pairs in descending order. They are 3336121, 336121, 36121, 6121, 121, 21 and 1. Prime numbers are generating prime numbers and each digit has meaning. Each successive number is found by simply removing the higher order digit. In reverse, each higher order number is found by placing a digit. In other words, prime numbers are generating prime numbers. The number system modNt shows that that mod10 number system is a hindrance to understanding the true order of the prime numbers. ModNt numbers show the complete ancestry of any given prime number.

6. A Useful Application Related to The Goldbach Conjecture

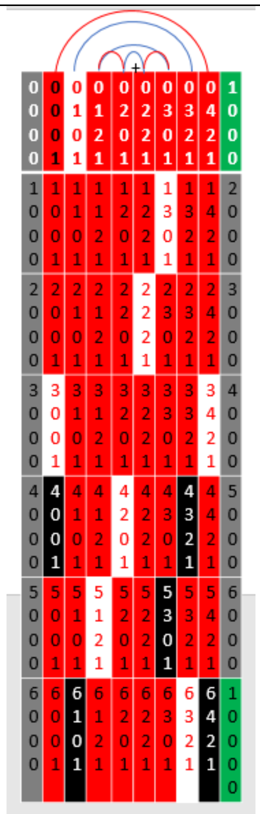
There has been a longstanding observation that certain even integers have more than a random preference for higher than normal solutions of Goldbach pairs (pairs of prime numbers that add to this particular even number). With the Sppn tables, one can visually see why this is true (which relates to the mathematical nature of the Generator Function). In a recent article in sciencenews the author noted that "for example, the number of partitions (Goldbach pairs) of 30,030 is 905, whereas the neighboring even integer 30,028 has 237 partitions and 30,032 has 225 partitions." According to the Generator Function one notes that 30,030 = Spp6 for n = 6 and p6 = 13. When you look at the symmetry in the Sppn tables, every prime number greater that ½ Sppn has a "complement" prime that adds to Sppn. Additionally, for every integral multiple of Sppn there is a secondary peak in the number of Goldbach pairs. The new method of understanding primes shows that previously mysterious relationships are easily explained (see table in section 7 below) [12].

7. Nature's Number System Modnt Vs Mod10

The following tables have been mouse copied from reference [9] and show the Sppn = 210 table with mod10 first followed by the same table with modNt numbers. See the progression of numbers in the rows and columns. ModNt numbers show order and a clear logical progression whereas mod10 numbers do not. This becomes more pronounced as the tables get larger. In another analysis, one has only to put the entire numbers in the first row and column when using Nature's Number System because all numbers in the rows and columns are built from these root numbers. Refer to reference for a complete [9]. Also note the symmetries in the tables (indicated by the red and blue arches above prime pairs with gaps of 2 and 4). Since the lower order tables generate the next larger Sppn table, this symmetry is carried from table to table. The fact that prime numbers have symmetry is possibly the most unexpected aspects of this work and an advanced topic.



Primes Only Table $Spp_n = 210$ for n = 4; $P_n = 7$; 7 rows; $Spp_{n-1} = 30$; mod10



n = 4; $p_n = 7$; $Spp_n = 210$; modNt number system; red columns only

8. Summary

The initial information relative to the Generator Function as a definition of prime numbers has been presented. Although the basic definition with its single boundary condition for directly calculating prime numbers is simple, it has far reaching implications. The reader is encouraged to pursue the books presented in references from which this paper was [8-10]. The current paper is a summary of some of the major results and not meant to be a comprehensive presentation of the topic.

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