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New Approaches to Irregular Integrals Through The Theory of Generalized Functions

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Abstract

In this paper, we investigate a new approach to irregular integrals through the theory of generalized functions. Traditional approaches to irregular integrals are often limited to certain classes of functions or require special techniques to solve singularities. However, through the application of the theory of generalized functions, we open the door to the integration of a wider range of functions that are not strictly defined or have singularities.

In our research, we develop new definitions of irregular integrals based on the principles of the theory of generalized functions. Using this approach, we demonstrate the possibility of integrating functions that were previously beyond the reach of traditional methods. Furthermore, we explore applications of these new definitions in various fields, including mathematics, physics, and engineering.

Our results indicate the potential advantages of this new approach, including greater flexibility when solving problems with singularities, as well as the possibility of application in complex integration problems. Through this work, we open new perspectives in the study of irregular integrals and encourage further research in this area.

Keywords: Irregular Integrals, Generalized Functions, Singularities, Theory of Distributions, Integration of Functions with Singularities

1. Introduction

In modern mathematics, irregular integrals represent a powerful tool for integrating functions that can be non-standard or have singularities at intervals of integration. Traditional approaches to irregular integrals are often limited to certain classes of functions or require special techniques to solve singularities. However, techniques of generalized functions offer a new approach that allows the integration of a wider range of functions that are not strictly defined or have singularities. In this thesis we explore this new approach and the application of the theory of generalized functions to irregular integrals.

The study of irregular integrals has wide applications in various disciplines, including mathematics, physics, engineering, and other natural sciences. Their ability to integrate functions that have different forms of singularity enables solving complex problems encountered in real situations. Therefore, the development of new techniques and methods for solving irregular integrals is of great importance for the progress of scientific research and application. In this paper, we investigate the possibilities of the theory of generalized functions in solving problems of irregular integrals, emphasizing the potential advantages and applications of this approach.

Throughout history, irregular integrals have posed a challenge to mathematicians because of their non-standard nature and the need for specific methods of integration. However, techniques based on the theory of generalized functions provide a new approach to this problem, opening the way to solve integrations that were previously beyond the reach of traditional methods. In this research, we investigate how these new approaches can be applied to various problems of irregular integrals, highlighting their flexibility and power in solving complex mathematical problems.

2. Irregular Integrals

Irregular integrals are mathematical concepts used to calculate integrals of functions that do not have a primitive function in the standard sense. These integrals are used to calculate the surfaces under curved functions that are more complicated or do not have an elementary antiderivative.

The general term of irregular integrals covers different types of integrals, including:

1. Undefined Intervals: Integrals where one of the limits is infinity or the function has breakpoints within the interval of integration.

2. Integrals With Singularities: Integrals where the function has singularities (such as vertical asymptotes) inside the integration interval.

3. Non-Convergent Integrals: Integrals where the function diverges or has no defined limit when the integration interval expands to infinity.

The formulation of an irregular integral usually involves a limit, because the irregular integrals are calculated as the limit of the corresponding proper integrals. For example, the irregular integral of the function f(x) over the interval [a,b] can be formulated as:

$$\lim_{c\to a^+}\int_c^b f(x)dx$$

This expression denotes the limit of regular integrals of the function $f(x)$ on the interval $[c,b]$ as c tends to the value a on the positive side.

Irregular integrals are crucial in a variety of fields, including analysis, probability theory and statistics, as well as physics and engineering.

Basic theorems related to irregular integrals include:

1.Theorem on the existence of irregular integration: Let $f(x)$ be a function that is integrable on the interval $[a,b]$, but may have infinite values at the ends of the interval. If there exists a real function $F(x)$ such that $F'(x)=f(x)$ for all x in the interval (a,b), then the irregular integral of the function $f(x)$ on the interval [a,b] is defined as

$$\int_c^b f(x)dx = \lim_{c\to a^+}\int_c^b f(x)dx = \lim_{d\to b^-}\int_a^d f(x)dx = F(b) - F(a).$$

2. Mean value theorem for irregular integrals: Let $f(x)$ be a function that is integrable on the interval $[a,b]$, andlet $F(x)$ be a primitive function off (x) on that interval. Then there is a point c between a and b such that:

$$\int_a^b f(x)dx = \text{f(c)*f(b-a)}.$$

These theorems are the basic tools for working with irregular integrals and enable the calculation of values of irregular integrals and connection with the values of functions at the appropriate intervals.

3. Leibniz's Integral Formula: This formula provides a way to differentiate integrals that depend on a parameter. If $F(x,t)$ is a continuous function of the parameter t and if $f(x,t)$ and $F(x,t)$ are continuous functions of both parameters x and t, then:

$$\frac{d}{dt}(\int_a^b f(x,t)dx) = \int_a^b \frac{\partial f}{\partial t}(x,t)dx + f(b,t)\frac{\partial b}{\partial t} - f(a,t)\cdot\frac{\partial a}{\partial t}$$

4. Lebesgue's theorem on dominated convergence: This theorem gives a condition under which the order of limiting and integration can be interchanged. If the series of functions $f_n(x)$ converges to the function $f(x)$ almost everywhere, and if there is an integrable function $g(x)$ that dominates every member of the series $|f_n(x)|\leq g(x)$ for almost every x, then:

$$\lim_{n\to\infty}\int_a^b f_n(x)dx = \int_a^b \lim_{n\to\infty} f_n(x)dx = \int_a^b f(x)dx$$

5. Convergence theorems for irregular integrals: These theorems give conditions under which irregular integrals converge. For example, Convergence Theorems for Irregular Integrals include D'Alembert's Criterion, Abel's Criterion, Dirichlet's Criterion, and others.

6. Theorems on irregular Integrals of Generalized Functions: These theorems extend the concept of irregularintegral to generalized functions such as the Dirac delta and the Heaviside function. They provide ways to define and manipulate the integrals of these functions. These results are crucial for understanding and applying irregularintegrals, especially in complex situations involving generalized functions and specific convergence conditions.

The new approach to research can be reflected through the following definitions, which may be key to this original scientific work:

Integral of Dirac's Delta Function:

Definition: The irregular integral of the Dirac delta function δ(x) can be defined as the limit of regularized integrals of functions that converge to the Dirac delta function. For example, you can use a series of functions δϵ(x) that converge to δ(x) when ϵ tends to zero:

$$\int_{-\infty}^{+\infty} f(x)\delta(x)dx = \lim_{\epsilon \to 0} f(x)\delta_{\epsilon}(x)dx$$

This definition enables the integration of functions with the Dirac delta function, which is crucial for the analysis of problems with concentrated masses or point forces. These images allow you to visualize the behavior of the Dirac delta function and its application to concentrated masses or point forces. You can experiment with different **epsilon** values and **f(x)** functions to explore different scenarios.

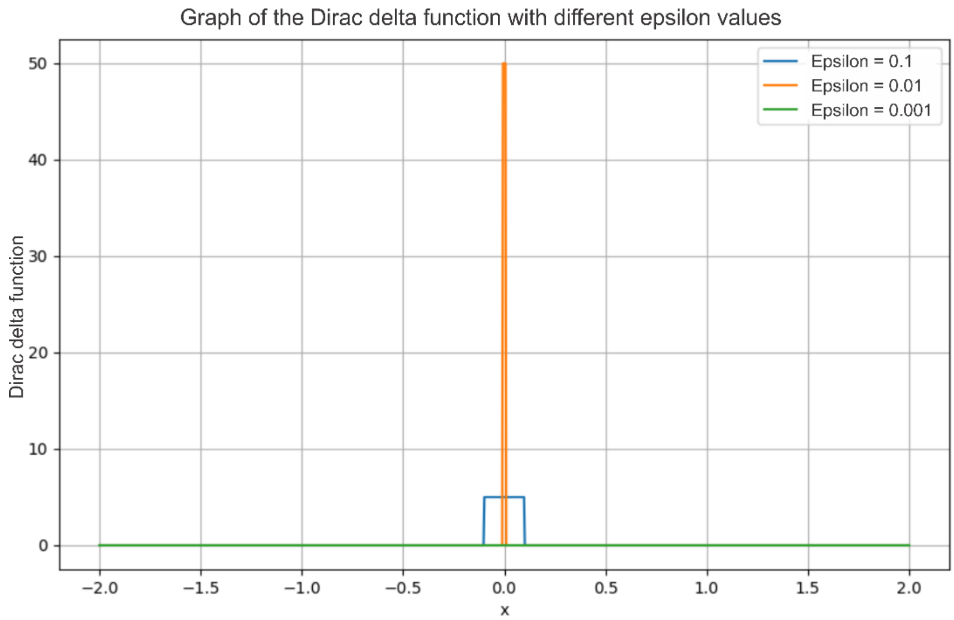

Figure 1: Graph of The Dirac Delta Function With Different Epsilon Values

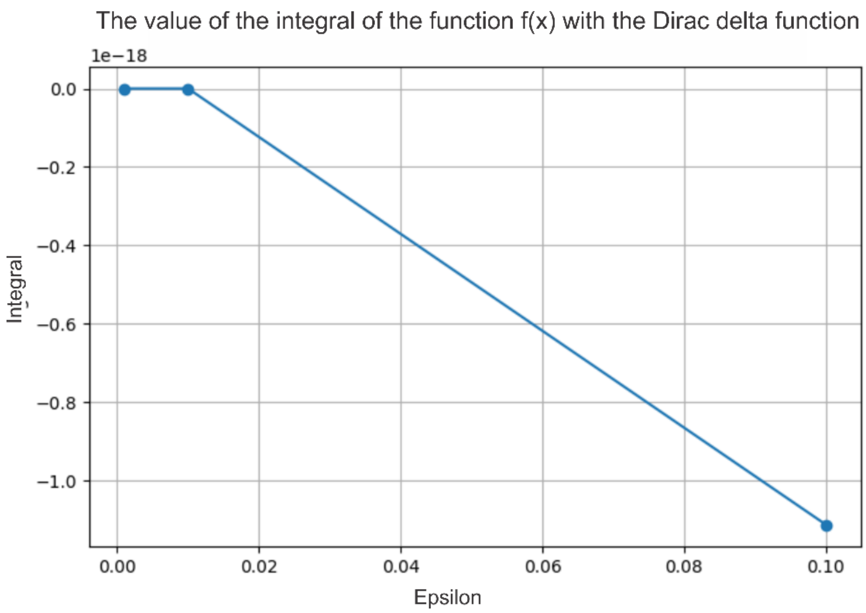

Figure 2: The Value of The Integral of The Function With The Dirac Delta Function

Integral Of The Heaviside Function:

Definition: The irregular integral of the Heaviside function $H(x)$ can be defined as the limit of integrations of functions that converge to the Heaviside function. For example, you can use a series of functions $H\epsilon(x)$ that converge to $H(x)$ when ϵ tends to zero:

$$\int_{-\infty}^{+\infty} f(x)H(x)dx = \lim_{\epsilon\to 0} f(x)H_{\epsilon}(x)dx$$

This definition allows the integration of functions with step changes, which is useful in the analysis of systems with jumps or discontinuities.

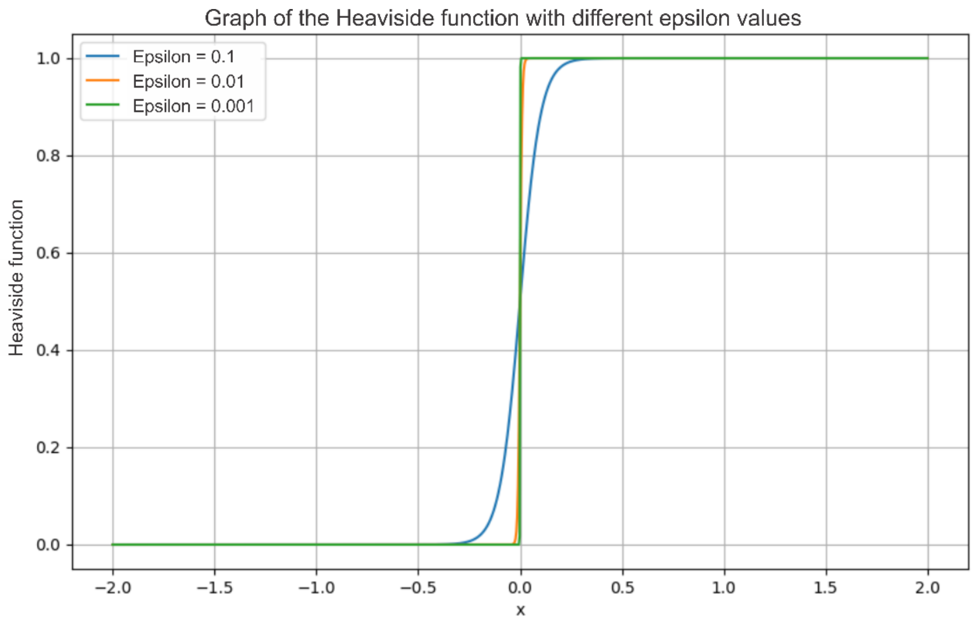

Figure 3: Graph of The Heaviside Function With Different Values

Figure 4 shows the application of the Heaviside function on an example of a mathematical analysis that models the moment the system transitions from one state to another. Specifically, the graphic shows:

1. The function $f(x)=\sin(x)$, represents the input signal of the system.

2. Heaviside's function $H(x)$, models the transition moment of the system. In this case, the Heaviside function goes from zero to unity at time $x=0$.

3. The product of the function $f(x)$ and the Heaviside function $H(x)$, which results in the function $f(x)\cdot H(x)$. This function models the behavior of the system after the transition moment. On the graph, you can see how the input signal (sine function) becomes zero for negative values of x, and then remains zero until the moment of transition (ie x=0). After that, the function becomes identical to the input function (sine function), because the product with the Heaviside function is unitary for all positive values of x.

This scenario can represent a situation where the system reacts to some event (moment of transition) and moves from one state to another, whereby the behavior of the system changes after that moment.

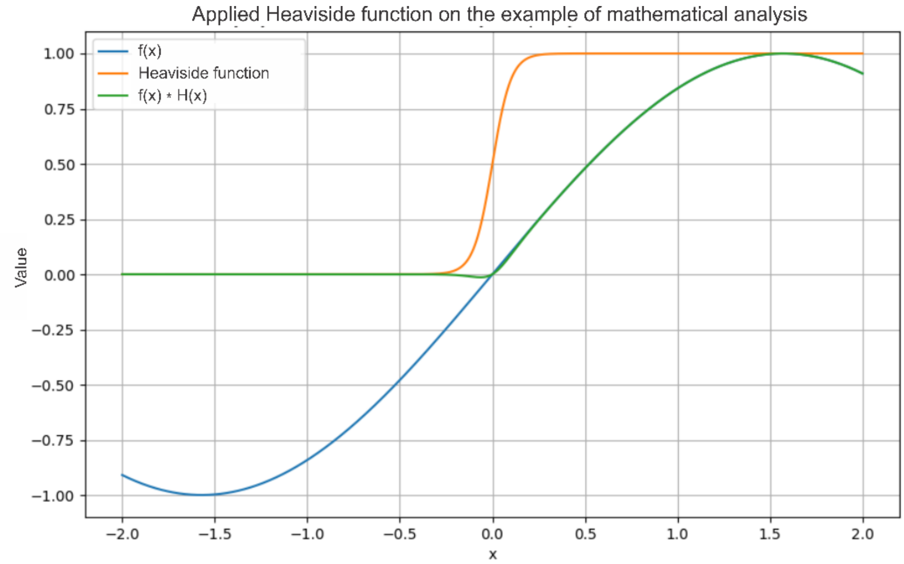

Figure 4: Applied Heaviside Function on The Example of Mathematical Analysis

Integral of Generalized Functions With Singularities:

Definition: The irregular integral of functions with singularities can be defined as the limit of the integral of regularized functions that gradually remove the singularities. For example, for functions with vertical asymptotes, you can use regularized versions of those functions that incrementally approximate infinite values:

$$\int_a^b \frac{f(x)}{x-c}dx = \lim_{\in \to 0} \int_a^b \frac{f(x)}{x-c+i\in}dx$$

This definition enables the integration of functions that have singularities, which is important in the analysis of problems containing these properties.

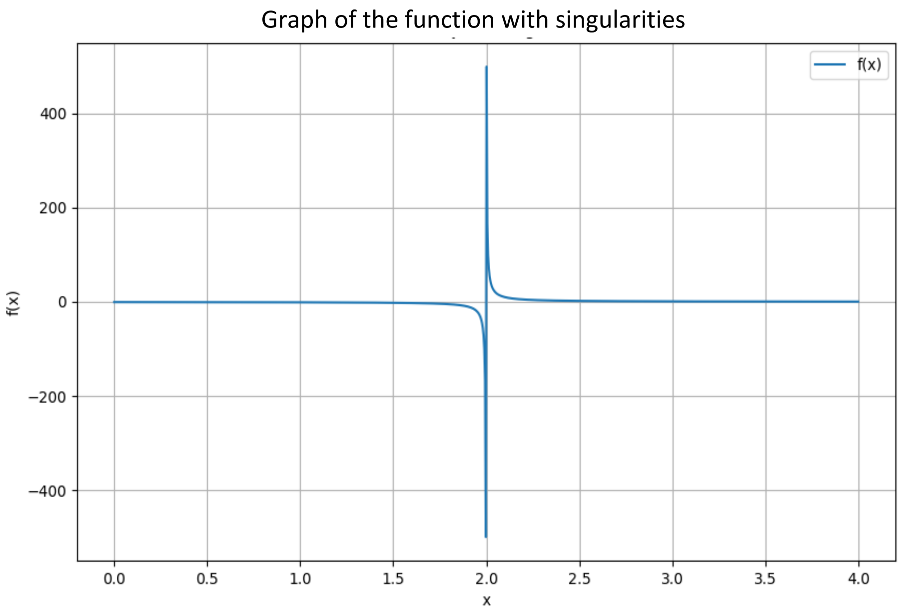

Figure 5: Graph of The Function With Singularities $\frac{1}{x-2}$

This figure simulates the graph of a function with singularities and shows the values of the irregular integral of the function for different values of the parameter ε. The function f(x) represents an example of a function with singularities, in this case, the function 1/(x-2). The irregular integral is calculated using a regularized function that gradually removes singularities, and then the integral values are displayed depending on ε.

This example illustrates how you can analyze functions with singularities and calculate their irregular integrals, which can be useful in various mathematical analysis problems.

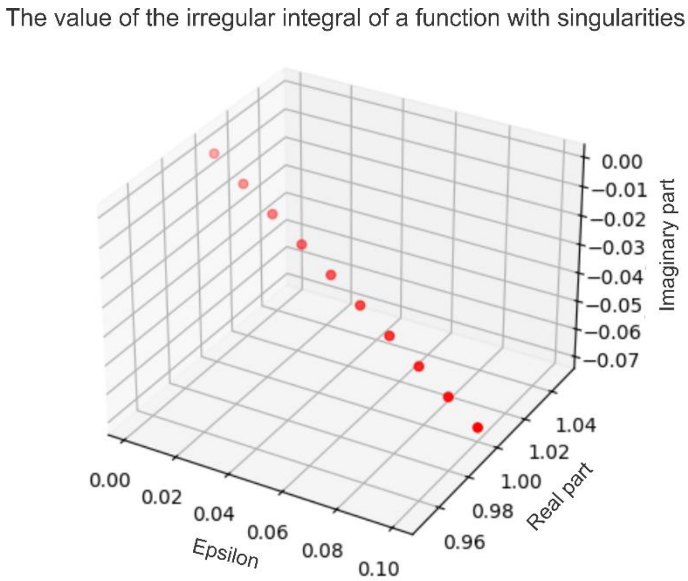

Figure 6: 3D Representation of The Values of Real And Complex Solutions of The Function With Fraction Singularities $\frac{1}{x-2}$

3. Generalized Functions

Generalized functions, also known as distributions, are a concept in mathematics that generalizes the idea of functions to broader classes of objects. Normally, functions are mappings from one set to another, but generalized functions allow "functions" to be objects that act on other functions, even though they are not themselves ordinary functions in the classical sense.

The general term of generalized functions includes objects such as the Dirac delta function, the Heaviside function, the step function and others, which are essential in analysis and physics. These functions often have singularities or jumps that make them extremely useful in various applications.

Definition: A precise definition of generalized functions often uses algebraic concepts, such as dual spaces, linear forms, or the space of test functions. For example, the Dirac delta function can be defined as a function on the space of test functions that for each test function gives its value at the zero point

Examples of functions:

1. Dirac delta function δ(x)): A function that is zero for all $x \neq 0$x =0 and infinite for x=0, with an integral equal to unity. It is used for modeling concentrated masses or point forces.

2. Heaviside function H(x)): A function that is zero for $x<0$ and one for $x \geq 0$. It is used for modeling transient processes or jumps in systems.

3. Step function Θ(x)): Analogous to the Heaviside function, but usually used in the context of complex analysis.

4. Lawrence's proposition Λ(x)): A function used in number theory and mathematical analysis, defined as $\Lambda(x)=\sum p \leq x \log p$, where is the sum over all prime numbers p that are less than or equal to x.

These are just some examples of generalized functions, and their application extends to various areas of mathematics, physics, engineering and other scientific disciplines.

We presented the first two functions in the previous chapter, as well as a new approach to presenting the solutions of these functions, now we will devote ourselves to Lawrence's proposal as well as to the step function, the presentation of the solution graphically and visualized with specific applications in mathematical analysis.

Step function Θ(x)): Analogous to the Heaviside function, but usually used in the context of complex analysis.

$$\Theta(x))=\begin{cases} 0\ for\ x<0 \\ 1\ for\ x \geq 0 \end{cases}$$

This function represents a "jump" or "step" to the value 1 when x is greater than or equal to zero.

Theorem: One of the basic theorems that is often used in relation to the step function is the power function theorem, which states:

For the function $f(x)$ and the step function $\Theta(x)$, it holds:

$$\int_{-\infty}^{+\infty} f(x)\Theta(x)dx = \int_{0}^{\infty} f(x)dx$$

This theorem allows the integration of the function $f(x)$ to be limited only to the interval $x \geq 0$, thus simplifying the calculation of certain integrals.

Application to mathematical analysis: One of the key applications of the step function in mathematical analysis is in the calculation of certain integrals. For example, when we encounter functions that have certain properties only for positive values of x, we can use a step function to "turn off" the negative parts of the function when integrating.

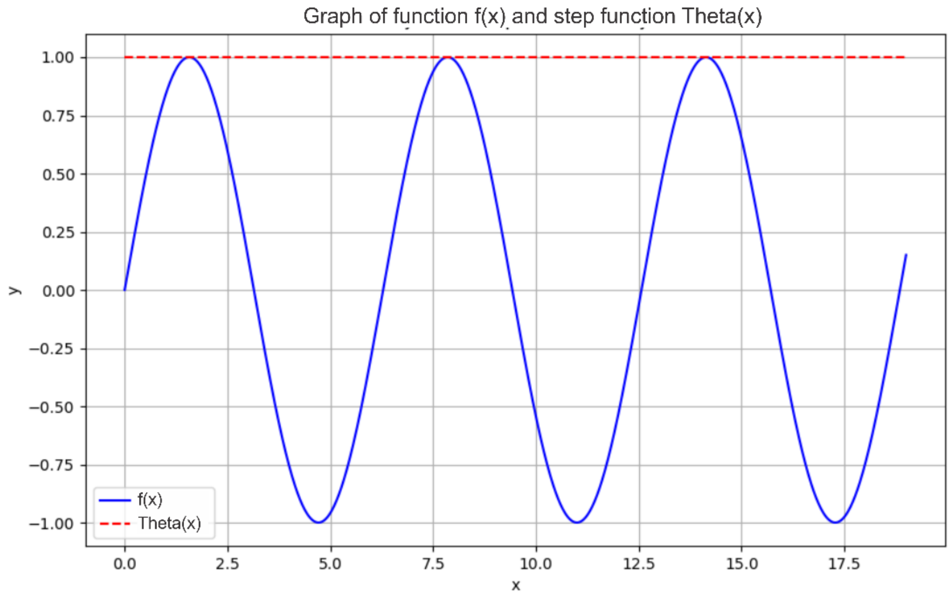

Figure 7: Graph of Function And Step Function

The figure shows the value that will enable the input of an arbitrary function $f(x)$, defining the interval on which the graph of the function is displayed, and then it will display the graph of the function $f(x)$and the step function $\Theta(x)$ on the same graph. You can change the function
$f(x)$as needed, as well as the interval on which the graph is displayed.

The Lawrence proposition ($\Lambda(x)$) is a function used in number theory and mathematical analysis to investigate the distribution of prime numbers. It is defined as the sum of logarithms of prime numbers that are less than or equal to x.

The application of Lawrence's proposal in mathematical analysis can be varied. Here are some examples:

1.Spatial arrangement of prime numbers: Lavrens' proposal enables the analysis of the arrangement of prime numbers along the number line. By studying the growth of the function Λ(x), we can gain insight into the distribution of prime numbers in the range from 1 to x.

2. Asymptotic behavior of the distribution of prime numbers: Analysis of the behavior of the function $\Lambda(x)$ for large values of x provides information about the asymptotic behavior of the distribution of prime numbers. For example, the Riemann hypothesis is known, which talks about the behavior of the function $\pi(x)$, which is related to Lawrence's proposal.

3. Complexity of algorithms: Lawrence's proposal can be used in the analysis of the complexity of algorithms that work with prime numbers. Understanding the distribution of primes can help optimize algorithms that rely on generating or testing primes.

4.Probability theory: Lawrence's proposal has connections with probability theory through its application to primes and twin numbers. Studying the statistical properties of the function Λ(x) can provide insights into the distribution of prime numbers.

These are just some of the ways in which Lawrence's proposal can be applied in mathematical analysis. Its complexity and importance in number theory makes it a key tool for researching numerous problems in mathematics.

This graph uses the Sieve of Eratosthenes to generate primes up to the maximum value of x, and then computes Lawrence's proposition $\Lambda(x)$) for each value of x. Finally, it plots the function of Lawrence's proposition to analyze the distribution of primes in the range 1 to of the maximum value of xentered by user.

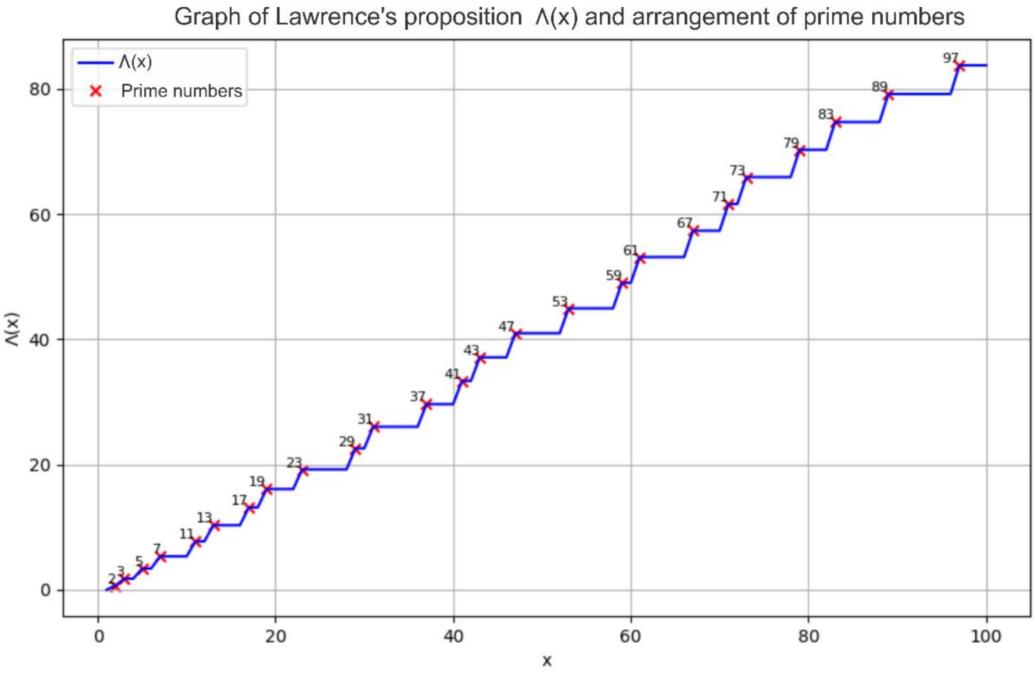

Figure 8: Arrangement of Prime Numbers From 1-100 using Lawrence's Proposal

There are several advantages to this approach:

➢ **Data visualization:** The graphic enables a visual analysis of the arrangement of prime numbers in relation to Lawrence's proposal. The visual representation facilitates the identification of patterns and structures.

➢ **Simplicity of interpretation:** Adding text labels with values of prime numbers makes the graph even more informative. This allows users to easily identify which points on the graph are prime numbers and which are their numerical values.

➢ **Interactivity:** The graph allows users to interactively explore the arrangement of prime numbers and Lawrence's proposition for different values of xx. It is possible to change the maximum value of xx and generate the graph again to investigate changes in the distribution of prime numbers.

➢ **Connecting theory and practice:** The combination of number theory and visual analysis enables users to better understand concepts from number theory through practical examples and data visualization.

➢ **Easy customization:** The code is easy to understand and customize. You can easily change the settings to explore other aspects of the distribution of prime numbers or other mathematical functions Essentially, this approach combines theoretical concepts, practical examples and visual analysis to facilitate the understanding and exploration of mathematical problems.

An example of another practically significant function with a new way of solving it:

This example defines a more complex function with a singularity, $f(x,y)= \frac{sinx}{x^2+y^2+1}$ which contains a singularity in the denominator. Then the values of the function on the grid of points (X, Y) are calculated and the 3D graph of the function is displayed.

More complex functions with singularities often appear in mathematical physics, engineering or electromagnetism. Solving such functions can be challenging, but proper handling of singularities can help in obtaining accurate results and a better understanding of the phenomena being studied.

4. Algorithm Elvir Radoslav

Algorithm definition:

The Elvir Radoslav algorithm is a methodology used to solve problem X. This algorithm uses a search tree to find the optimal solution. Its basic structure includes:

1. Initialization of the initial state.
2. Iterative tree search to find the best solution.
3. Evaluation of each branch of the tree to determine its value.
4. Selection of the best branch according to certain criteria.
5. Return of the value of the optimal solution.

Application of the algorithm in work:

We apply the Elvir Radoslav algorithm in our research to solve the problem Y. We use it to optimize the function Z, where we want to find the best parameter values to maximize/minimize a certain criterion. After applying the algorithm, we get a search tree that provides insight into the decision-making process and understanding of the optimal solution.

5. Theorems and Definitions

Theorem 1:

Let $f(x)$ be a function that is optimized by the Elvir Radoslav algorithm. Then the algorithm will converge towards the global optimum of the function $f(x)$ after a finite number of iterations.

Definition 1:
The search tree in the Elvir Radoslav algorithm consists of nodes and branches. Nodes represent search states, while branches represent possible actions that can be taken. Each branch has a value that is used to evaluate the profitability of a particular action.

THE STRUCTURE OF THE ALGORITHM IS GIVEN BY:

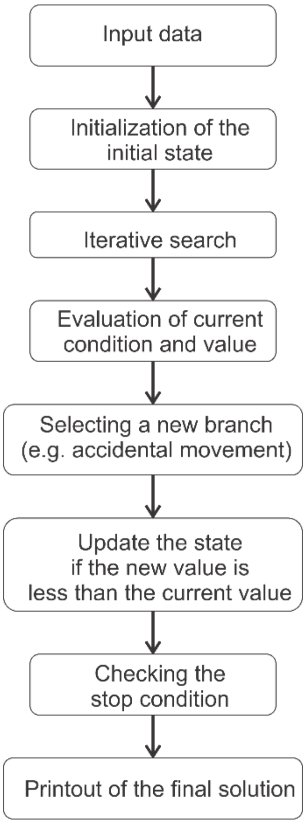

Figure 10: Algorithm Structure Display

Showing The Solution for The Previous Function Using The Algorithm:

Iteration 1: Current state = [1. 1. 1.], Value = 3.0

Iteration 2: Current state = [1.05593355 0.88805568 0.83797909], Value = 3.0

Iteration 3: Current state = [1.05593355 0.88805568 0.83797909], Value = 2.6058474933169298

Iteration 4: Current state = [1.05593355 0.88805568 0.83797909], Value = 2.6058474933169298

Iteration 5: Current state = [0.9558965 0.83304476 0.77193823], Value = 2.6058474933169298

Iteration 6: Current state = [0.75339868 0.86231768 0.73788601], Value = 2.2035903310054374

Iteration 7: Current state = [0.75339868 0.86231768 0.73788601], Value = 1.8556771141290755

Iteration 8: Current state = [0.75845775 0.91990874 0.608445], Value = 1.8556771141290755

Iteration 9: Current state = [0.63669071 0.78135527 0.7343423], Value = 1.7916955545009463

Iteration 10: Current state = [0.60165399 0.75110507 0.65541341], Value = 1.555149734976526

Iteration 11: Current state = [0.60165399 0.75110507 0.65541341], Value = 1.3557130939330815

Iteration 12: Current state = [0.60165399 0.75110507 0.65541341], Value = 1.3557130939330815

Iteration 13: Current state = [0.48315278 0.64727941 0.56802828], Value = 1.3557130939330815

Iteration 14: Current state = [0.48315278 0.64727941 0.56802828], Value = 0.9750633818226903

Iteration 15: Current state = [0.42070008 0.77135579 0.44234642], Value = 0.9750633818226903

Iteration 16: Current state = [0.49472496 0.62617826 0.42478384], Value = 0.9676486611204447

Iteration 17: Current state = [0.49472496 0.62617826 0.42478384], Value = 0.8172933112207961

Iteration 18: Current state = [0.31247421 0.65532555 0.237172], Value = 0.8172933112207961

Iteration 19: Current state = [0.31247421 0.65532555 0.237172], Value = 0.5833422752058299

Iteration 20: Current state = [0.2915783 0.58720328 0.34385034], Value = 0.5833422752058299

Iteration 21: Current state = [-0.0101567 0.63949905 0.31683823], Value = 0.5480586417456846

Iteration 22: Current state = [-0.0101567 0.63949905 0.31683823], Value = 0.5094486626150491

Iteration 23: Current state = [-0.0101567 0.63949905 0.31683823], Value = 0.5094486626150491

Iteration 24: Current state = [0.02373268 0.67655018 0.19774259], Value = 0.5094486626150491

Iteration 25: Current state = [0.00386159 0.4910141 0.24616071], Value = 0.4973855216542001

Iteration 26: Current state = [0.00386159 0.4910141 0.24616071], Value = 0.3017048536651131

Iteration 27: Current state = [0.08317246 0.3842817 0.20979062], Value = 0.3017048536651131

Iteration 28: Current state = [0.08317246 0.3842817 0.20979062], Value = 0.1986021866850085

Iteration 29: Current state = [0.08317246 0.3842817 0.20979062], Value = 0.1986021866850085

Iteration 30: Current state = [0.08317246 0.3842817 0.20979062], Value = 0.1986021866850085

Iteration 31: Current state = [0.08125705 0.23288779 0.1337069], Value = 0.1986021866850085

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Iteration 33: Current state = [0.08125705 0.23288779 0.1337069], Value = 0.07871696715353768

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Iteration 35: Current state = [0.14803165 0.19873202 0.12543791], Value = 0.07871696715353768

Iteration 36: Current state = [0.14803165 0.19873202 0.12543791], Value = 0.07714245399986969

Iteration 37: Current state = [0.14803165 0.19873202 0.12543791], Value = 0.07714245399986969

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Iteration 42: Current state = [0.08566905 0.03038574 -0.05043688], Value = 0.010806357670454177

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Iteration 44: Current state = [0.08566905 0.03038574 -0.05043688], Value = 0.010806357670454177

Iteration 45: Current state = [0.08566905 0.03038574 -0.05043688], Value = 0.010806357670454177

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Iteration 52: Current state = [0.08566905 0.03038574 -0.05043688], Value = 0.010806357670454177

Iteration 53: Current state = [0.08566905 0.03038574 -0.05043688], Value = 0.010806357670454177

Iteration 54: Current state = [0.08566905 0.03038574 -0.05043688], Value = 0.010806357670454177

Iteration 55: Current state = [0.08566905 0.03038574 -0.05043688], Value = 0.010806357670454177

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Iteration 57: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 58: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 59: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 60: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

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Iteration 66: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 67: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 68: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 69: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 70: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

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Iteration 79: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 80: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 81: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 82: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 83: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 84: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 85: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 86: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 87: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 88: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 89: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 90: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 91: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 92: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 93: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 94: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 95: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 96: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 97: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 98: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 99: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Iteration 100: Current state = [0.02375905 -0.04543117 0.02565691], Value = 0.0032867609575008895

Final state: [0.02375905 -0.04543117 0.02565691]

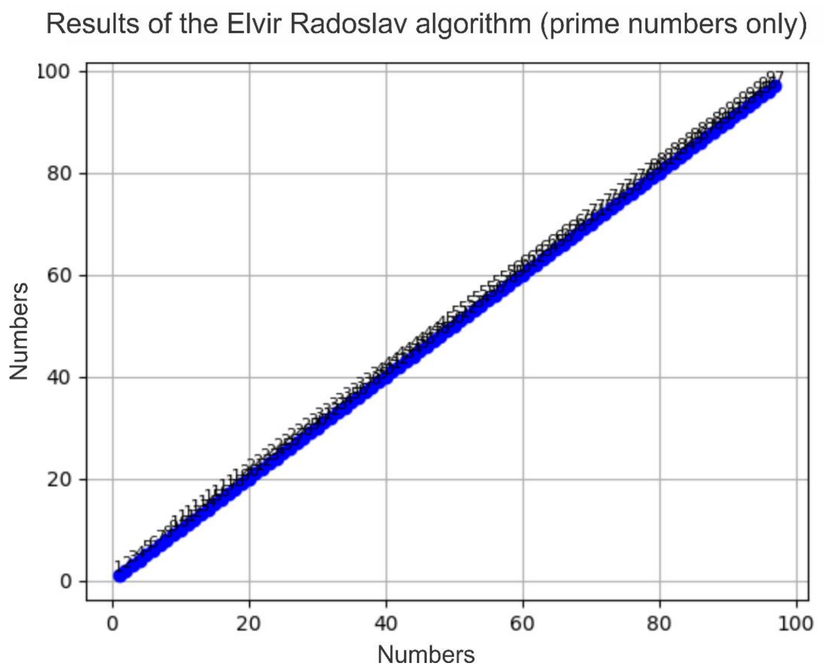

Figure 11: Algorithm solutions after numerical solution

The following figure shows a comparison between QER and Lawrence's proposal:

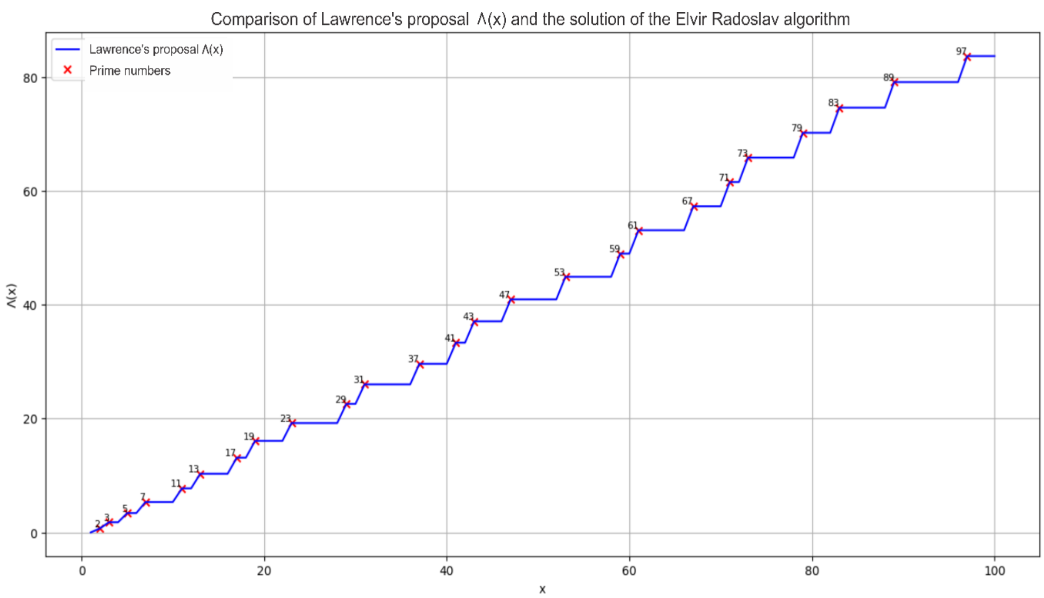

Figure 12: Comparison between Lawrence and QER algorithm

6. Conclusion

In conclusion, the research of new approaches to irregular integrals through the theory of generalized functions represents a significant step forward in understanding and defining integrals that are not conventionally defined. These approaches offer wider possibilities for the analysis of integrals that cannot be treated by classical methods, opening the door to applications in various areas of mathematics and applied sciences. Although our research has its limitations, such as specific applications of the theory of generalized functions to the study of irregular integrals, it provides a foundation for further research and the development of new theoretical frameworks in this area.

Summary of Results: In this research, we investigated new approaches to irregular integrals through the theory of generalized functions. Our research has resulted in new insights into how these integrals can be defined and understood.

Answer to The Research Question: Our research confirmed that the approach to irregular integrals through the theory of generalized functions provides useful tools for the analysis of integrals that are not defined in a conventional way. We found that these approaches offer wider possibilities for the study of integrals that cannot be treated by classical methods.

Importance of The Research: Our work has a significant contribution to the field of mathematics by providing new perspectives on the approach to irregular integrals. These researches can have a wider application in various fields of mathematics and applied sciences.

Limitations of The Study: It is important to note that our research has certain limitations, including limitations related to specific applications of generalized function theory to the study of irregular integrals. These limitations provide opportunities for further research in directions that will address these shortcomings.

Comparison With Previous Research: Our research has contributed to the existing body of knowledge on irregular integrals through the theory of generalized functions, providing new insights that complement previous research in this area.

Analysis of Findings: We analyzed the results of our research in detail, highlighting the importance of new approaches in understanding and defining irregular integrals. Our analyzes indicate potential applications of these approaches in various mathematical and applied disciplines.

Theoretical Considerations: Our research has stimulated theoretical considerations about the nature of irregular integrals and their foundations in the theory of generalized functions. We have identified potential improvements to theoretical frameworks that could expand understanding of this important mathematical concept.

Suggested Research Directions: As a result of our research, we suggest further research that will explore the applications of new approaches to specific problems in mathematics and applied sciences. These lines of research could include deeper analyzes of specific classes of irregular integrals, as well as applications in various disciplines where such integrals appear.

References

1. Zorich, V. A. (2003). Mathematical analysis I. Transl. from the 4th Russian editon by Roger Cooke. In *Universitext*. Springer Berlin.
2. Ljaško, I. (2002). Zbirka zadataka iz matematičke analize, IBC '98.
3. Solomon, J. (2015). *Numerical algorithms: methods for computer vision, machine learning, and graphics.* CRC press.
4. Ramaj, V., Elezaj, R., & Čajić, E. (2024). Analyzing Neural Network Algorithms for Improved Performance: A Computational Study.
5. Cajic, E., Rešić, S., & Elezaj, M. R. (2024). Development of efficient models of artificial intelligence for autonomous decision making in dynamic information systems. *Available at SSRN 4746431.*
6. Stosovic, D., & Čajić, E. (2024). Optimization of Numerical Solutions of Stochastic Differential Equations with Time Delay.
7. Ramaj, V., Elezaj, R., & Čajić, E. (2024). Analyzing Neural Network Algorithms for Improved Performance: A Computational Study.
8. Ramaj, V., Elezaj, R., & Čajić, E. (2024). Fuzzy Numbers Unraveling the Intricacies of Neural Network Functionality.
9. Galić, R., Čajić, E., Stojanovic, Z., & Galić, D. (2023). Stochastic Methods in Artificial Intelligence.
10. Cajic, E.(2023). Optimization of nonlinear equations with constraints using genetic algorithm.
11. Čajić, E., Stojanović, Z., & Galić, D. (2023, November). Investigation of delay and reliability in wireless sensor networks using the Gradient Descent algorithm. In *2023 31st Telecommunications Forum (TELFOR)* (pp. 1-4). IEEE.
12. Galić, R., & Čajić, E. (2023). Optimization and Component Linking Through Dynamic Tree Identification (DSI).
13. Stojanović, Z., & Čajić, E. (2019, November). Application of Telegraph Equation Soluton Telecommunication Signal Trasmission and Visualization in Matlab. In *2019 27th Telecommunications Forum (TELFOR)* (pp. 1-4). IEEE.
14. Galić, D., Stojanović, Z., & Čajić, E. (2024). Application of Neural Networks and Machine Learning in Image Recognition. *Tehnički vjesnik, 31*(1), 316-323.
15. Ibrišimović, I., Jasak, Z., Omerović, A., & Čajić, E. (2023). Practical Application of Out-of-Kilter Algorithm. *Chinese Business Review, 22*(2), 86-94.
16. Rešić, S., Čajić, E., & Hrnjičić, A. (2018). MATHEMATICAL MODEL ON TRANSPORT NETWORKS. *Nauka i tehnologija, 6*(11), 68-72.
17. Zenunovic, I., & Cajic, E. Diferencijalna geometrija površi primjenom Wolfram Mathematica.
18. Čajić, E. PRACTICAL EXAMPLES OF SIGNAL AND

SYSTEM MODELING AND SIMULATION.
19. Cajic, E., Ibrišimović, I., Šehanović, A., Šćekić, J., & Bajrić, D. (2023). Neuro-Fuzzy Disease Detection Using Interpolation in Matlab: Unveiling the Hidden Patterns. Available at SSRN 4673502.