ISSN: 2836-8495

Research Article

Journal of Current Trends in Computer Science Research

Natural Equidistant Primes (NEEP) and Cryptographic Coding of the Goldbach's Strong Conjecture

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Bahbouhi Bouchaib, Natural Equidistant Primes (NEEP) and Cryptographic Coding of the Goldbach's Strong Conjecture.

Submitted: 2024, Dec 23; **Accepted:** 2025, Jan 30 **Published:** 2025, Feb 20

Citation: Bouchaib, B. (2025). Natural Equidistant Primes (NEEP) and Cryptographic Coding of the Goldbach's Strong Conjecture. *J Curr Trends Comp Sci Res, 4*(1), 01-09.

Abstract

For the first time, this article introduces the notion of natural equidistant-equiranked prime numbers (NEEP) which are the only ones to verify the strong Goldbach conjecture naturally in the set of natural integers. If E is an even ≥ 4 and E = p + q such that q > p, NEEP are the equidistant primes which also have the same ranking for p between 0 and E/2 and for q between E and E/2. Primes are counted from 0 to E/2 on one hand, and inversely from E to E/2 on the other hand. Therefore, primes having the same ranking face each other on a same line and if equidistant relatively to E/2 then their sum = E. From the NEEP, we calculate the deducible equidistant prime numbers (DEP) and it is only from NEEP + DEP that we obtain all the possible sums of two prime numbers of a given even number. No current algorithm for converting even numbers to the sum of two prime numbers distinguishes NEEP from DEP. There are evens like 30 or 90 which don't have NEEP and therefore not satisfying naturally Goldbach's strong conjecture (GSC) unless DEP is deduced by calculation. This is a new matter of thinking : should GSC be refuted because there are evens not having NEEP ? Is this conjecture only deducible by calculation ? Normally one expects GSC to be true with NEEP before getting to DEP.

The natural presence of NEEP has been exploited here to set up for the first time a system of coding and deciphering even numbers which allows a calculator to deduce all their possible sums of two prime numbers. This article then has two originalities not published before which will certainly be subject to debate.

Keywords: Goldbach's Strong Conjecture, Equidistant primes, Encoding, Dicephering, Cryptology, Prime Numbers, Prime Number Countig Function, Algorithm

Abbreviations
GSC: Goldbach's Strong Conjecture.
PN: Prime Number.
NEEP: Natural Equidistant-Equiranked Primes.
DEP: Deducible Equidistant Primes.

1. Introduction

For a even E ≥ 8 to be sum of two PN p and q such that q > p ; E/ 2 – p = q – E/2 so that p and q are said to be equidistant. However, there is a subtle difference between equidistance and equiranking of two PN p and q. First let us recall that the prime number counting function, called π(E), which aims to count prime numbers less than or equal to a number E. Calculating π(E) allows you to position a prime number in relation to another, by knowing its rank in the list of prime numbers. If π(a) < π(b) then a < b. How to know if p and q are equiranked ? First determine π(E) by the PN counting function, then separate the prime numbers (PN) < E/2 and those > E/2. Then draw a table with 3 columns, the first of which is used for PN < E/2; the second to note the value of E/2 at each line and the third to mark the PN > E/2. The most important thing in this process is that the PN < E/2 are in ascending order and those > E/2 are in descending order from the top line of the table (Figure 1 and Tables 1A-1F) because this is how the PN add up to give a value closest to E. The smallest PN which is 3 must be opposite the largest prime number > E/2. The equiranked primes are those on the same line but they are not always equidistant. The postulate of this article is the following "For the GSC to be naturally true in the set N, there must be two equidistant and equiranked PN". Naturally in mathematics here means a fact which appears instantly without recourse to calculation and these equidistant and equiranked PNs are designated here NEEP (natural equidistant-equiranked primes). While all other equidistant PNs which are not equiranked are called DEP (deducible equidistant primes). DEPs require calculation from NEEPs. If not possible, from odd numbers not multiples of 2 and 3 whose sum makes E (see the next section). This is the first time that these notions are published here which distinguishes NEEPs from DEPs.

Therefore, there are two types of equidistant primes: those that occur naturally and are equiranked and those that are deducible by calculation. In fact, only natural equidistant- equiranked PN (NEEP) can be used to prove the GSC in the set N just by following Figure 1 and Tables 1A-1F without any calculation.

2. Results

2.1 The Natural Equidistant-Equiranked Primes (NEEP) and the Deducible Equidistant Primes (DEP)

The NEEP are colored gray (Tables 1A-F). The two NEEP p and q appear naturally, so that p + q = E. The line corresponding to the smallest odd PN which is 3 is coloured yellow. As is well known, every even number has a number of possible sums p + q, but we don't see all of them naturally because the density of PN between 0 and E/2 is > that between E/2 and E, which always results in a mismatch between all possible equidistant primes when using the prime counting function. That's why Goldbach's verification must occur naturally with NEEP, since they're the only ones we can see in the set of integers. However, by calculation, they will give all the other PED (by deduction). We can see that the number of possible sums p + q is not all natural, but mostly a result of calculation that we deduce. But how are we going to deduce the DEP? I explained this method in a more recent article [1-2]. Interested readers can consult it for more details, but very briefly, there are two categories of PN: 6x - 1 or 6x + 5 and those that are 6x + 1. Between two PN 6x - 1 and between two PN 6x + 1 there is a difference of 6n (n ≥ 1). But between PN 6x - 1 and 6x + 1 there are variable gaps of 2n (n ≥ 1).

There are also three categories of even numbers 6x; 6x + 2 and 6x + 4. The 6x are obtained by adding an PN 6x + 1 and another 6x - 1, or vice versa. The 6x + 2 require two 6x + 1 PN. Whereas 6x + 4 are also 6x - 2 and require two 6x - 1 PN. In all cases, the GSC always follows the 6x ± 1 equations, and the sum of the PN is based on the category of the even.

Example of deduction of DEP from NEEP. Let's take the example of the even number E = 44 and so E/2 = 22 (Table 1B) has practically three possible sums 3 + 41; 7 + 37 and 13 + 31.

However, there is only one pair of NEEP visible in Table 1B and it's 7 + 37 from which we deduce the other two. So (7 - 4) + (37 + 4) = 3 + 41. And (7 + 6) + (37 - 6) = 13 + 31. The deduction always follows the same calculation: if an even number E = p + q , the deduction is made according to E = (p - 6n) + (q + 6n) or E = (p + 6n) + (q - 6n). Globally, the deduction is made according to E = (p - 2n) + (q + 2n) or E = (p + 2n) + (q - 2n). In case there are no NEEP, then we follow the same equation with C being a composite number not multiple of 2 and 3 such that E = C + C'. Hence E = (C - 6n) + (C' + 6n) or E = (C + 6n) + (C' – 6n) on one hand. On the other hand, E = (C - 2n) + (C' + 2n) or E = (C + 2n) + (C' – 2n) ↔ E = p + q. Using this process, we get all DEP and all possible sums of two primes for a given number E.

There is also another method using the smallest PN < E/2 which is 3 and the largest PN > E/2. Note in passing that an even number multiple of 3 denoted 3n will have E/2 which is also 3n and E - 3 will not be prime. On the other hand, a non-3n number could also give a prime or composite E – 3 number. Now let's take the example of E = 44 and E/2 = 22 (Table 1B). We see that 3 + 43 = 46 and therefore we have 46 - 44 = 2. We can then remove 2 from the other PNs for example we have 3 + (43 - 2) = 3 + 41. Or add 2 for example 13 + 29 becomes 13 + (29 + 2) = 13 + 31 = 44. We can easily deduce all the possible sums by following the equations above.

For example, 44 = 9 + 35 = (9 – 6) + (35 + 6)= 3 + 41. Or 44 = 9 + 35 = (9 + 4) + (35 – 4) = 13+ 31.

Use the gaps 6n in E = (C - 6n) + (C' + 6n) or E = (C + 6n) + (C' – 6n) when C and C' are non-3n. Another example E = 74 et E/2 = 37 (Table 1D) which has practically 4 possibles p + q sums including 3 + 71 ; 7 + 67 ; 13 + 61 ; 31 + 43 ; 37 + 37 (in this paper we only focus on two NEEP p and q such that q > p so the latter sum is excluded). The single NEEP is 13 + 61 = 74 visible in Table 1D. The DEP can all be deduced from the NEEP like for exampe 13 + 61 = (13 – 10) + (61 + 10) = 3 + 71 or 13 + 61 = (13 + 18) + (61 - 18) = 31 + 43. This is true for all evens E ≥4. The table 1A-F show 6 examples used for illusttration.

2.2 New Cryptological Coding of GSC

The NEEP can also be used to encode even numbers, allowing us to deduce DEP and therefore all possible p + q sums. It seems that every even number E ≥4 in the set N has a unique configuration of NEEP (Figures 2A-F), and even if we find two even numbers E with the same configuration, the NEEP and DEP will not be the same. This is a good material for cryptology and all those interested in it, as each number is associated with a specific configuration of its PN and NEEP. Mathematically, this coding will enable you to deduce all possible sums p + q by calculation or by using a program that performs E = (p - 6n) + (q + 6n) or E = (p + 6n) + (q - 6n) or E = (p - 2n) + (q + 2n) or E = (p + 2n) + (q - 2n).

How does this coding work? Let's take two examples from **Figure 2**. First, the figure is read from the top ; and the NEEP line is marked with 0, above which the total number of preceding lines is marked, and so on. For example, E = 24 (**Figure 2A**) is associated with the number 1000 because there are three NEEP lines preceded by one PN line devoid of NEEP . The NEEP and PN of the even numbers E can be used to encode the even number E by associating it with a line and number configuration. Afterwards, a number is associated with it, which, when deciphered, makes it possible to deduce all possible sums p + q = E. Example E = 44 et E/2 = 22 (Figure 2B) is coded 203Ø which means that it has a pair of NEEPs marked with 0 preceded by two lines of PN and followed by 3 lines of PN devoid of NEEP. The Ø sign means that there is a PN < E/2 which has no PN > E/2 in front of it. Let's not forget that first of all we must put the number as explained in Figure 1.

The examples given in the figure will help one to understand this encoding and decryption system. The Ø sign always corresponds to single PN close to E/2 on both sides.

For example, let's decipher the code 12080706øø (Figure 2F) which means that this number has 12 pairs of PNs (which are not NEEPs) followed by a pair of NEEPs; then 8 pairs of PN ; a NEEP line marked by zero; then 7 pairs of PN; a third NEEP line; and finally 6 lines of pairs of PNs, two of which do not have a PN > E/2 opposite marked with the Ø sign. The reader could practice encoding and deciphering numbers. This encoding and decryption system described for the first time in this paper shows its potential usefulness in a cryptological application. Mathematically, it allows you to encode an even number in such a way as to be able to deduce all possible sums p + q.

3. Evens without NEEP Might Signify Mathematical Rejection of GSC.

Tables 2A-C show one number with one NEEP line (**3A**) and two examples without NEEP (**Tables 2B + 2C**). The number 40 and E/2 = 20 (3A) has one NEEP line 3 + 37 = 40 from which we can deduce all DEP like for example 3 + 37 = (3 + 8) + (37 – 8) = 11 + 29 = (11 + 12) + (29 – 12) = 23 + 17 and so on. In contrast the numbers with no NEEP require first putting E = C + C' such that C + C' are odds composite and non-3n. Let us take the number 30 (**2B**).

For example, 30 = 5 + 25 = (5 + 6) + (25 – 6) = 11 + 19 = (11 + 6) + (19 – 6) = 17 + 13 ad so one. The same applies to E = 90 and E/2 = 45 in **Table 2C**. For their encoding, we can use the first line corresponding to PN = 3 and denote it by a capital letter followed by the total of PN line like for example for 30 we have A4Ø meaning the line of PN 3 is followed by four lines one which has only one prime > E/2 (the last one). By contrast, if we take two of these numbers (E = 24, E/2 = 12 ; E = 30 ; E/2 = 15) in addition to a new one (E = 60 ; E/2 = 30) ; and we take all the natural integers from (E/2 - 1) to 1 (decreasing order) and from (E/2 + 1) to E - 1 (ascending order), we see the equidistant PNs reappear, the sum of which is equal to E **(Tables 3A-C). But all these equidistant additive PN are not equiranked at E/2 and are not true NEEP but DEP (because integers are placed in a specific order before and after E/2 and this is this positioning that helps recovering equidistant primes. Hence they are DEP)**, and this is the most important point that this article rises. By taking only PN in their natural ranks before and after E/2, GSC is merely deducible by calculation and depends upon gaps that separate the PNs. But if we take the whole integers, we no longer have equiranked primes NEEP, but the equidistant ones at E/2 reapper on the same lines and increase the numbers of p + q sums. This result deserves further research for the moment. However, the great advantage is that cryptographic encoding of GSC is much more easier as we have many equidistant PN lines and no single PN (the sign Ø is thus useless here). Numbers are enough to encode all the information about equidistant primes and non-equidistant ones.

This shows that Goldbach' strong conjecture (GSC) is not naturally true in the set N. The GSC would not be natural in all cases of even numbers, but would be deducible by the calculation as seen above. The absence of NEEP in numbers like 30 or 90 and probably an infinity of others raises the question: Is the GSC naturally true? Does it have a meaning since it disappears in some numbers when we use the counting function of the PNs and their natural orders. If the GSC is absolutely deducible this means that it could be solved by algorithms and calculation programs looking for a calculation equation like those seen above. This article is the first to raise this question because if the GSC is not naturally verified, it is because the equidistant and equiranked PNs are not always present, and therefore the GSC loses its mathematical meaning at this level.

4. Discussion

Some and likely an infinity of even numbers E ≥4 do not have NEEP, and therefore Goldbach's strong conjecture (GSC) is not naturally true. An even E that doesn't have a NEEP doesn't check GSC naturally. Current algorithms that provide us with all possible p + q sums in one click confuse NEEP and DEP, and this article raises this point for the first time.

But in fact, if an even number does not have a NEEP, this means that it does not naturally verify the GSC. Does GSC need to be demonstrated with NEEP or NEEP+DEP or with one of the two?

In fact, if no NEEP, no natural GSC, and in this case, the GSC would be deduced by the calculation by looking for the DEPs. But deduction by the calculation will never be proof of its veracity which explains why GSC remains unsolved for centuries. Now the central question that needs to be addressed further is to determine why some evens do not have NEEP while others do. Very likely there are some hidden rules that lead to NEEP or not. Another important idea is the fact that GSC is absolutely a function of gaps between PN or between PN and composite odds that are not multiple of 3. This is this function that allows us to convert an even lacking or having NEEP in all possible sums of two primes p + q.

In addition, this article presents for the first time a coding of even numbers having NEEP which makes it possible to deduce all possible sums either by a calculation or by a computer program. Another encoding is suggested for evens without NEEPs. This encoding is suggested here for the first time and might very likely be improved with time.

If we do not use the PN counting function and their natural ordrer, we lose the NEEP but we increase the possibilities of p + q sums of the evens denoted E because we recover equidistant primes (that are not equiranked). This result needs more investigation in future.

For numbers having too much NEEPs, we might simply add 0s near the lines of NEEPs which allows a calculator to know their total number.

This debate deserves close attention in the future.

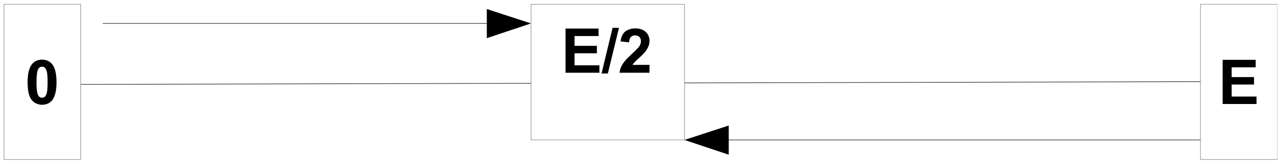

Figure 1: Primes numbers (PN) < E/2 are in ascending order while those > E/2 are in a descending order from the closest PN to E to E/2. The results obtained with this system are shown in tables 1A-F.

p	E/2	q
3	12	23
5	12	19
7	12	17
11	12	13

Table 1A

p	E/2	q
3	22	43
5	22	41
7	22	37
11	22	31
13	22	29
17	22	23
19	22	

Table 1B

p	E/2	q
3	24	47
5	24	43
7	24	41
11	24	37
13	24	31
17	24	29
19	24	ø
23	24	ø

Table 1C

p	E/2	q
3	37	79
5	37	73
7	37	71
11	37	67
13	37	61
17	37	59
19	37	53

23	37	47
29	37	43
31	37	41
37	37	37

Table 1D

p	E/2	q
3	80	179
5	80	173
7	80	167
11	80	163
13	80	157
17	80	151
19	80	149
23	80	139
29	80	137
31	80	131
37	80	127
41	80	113
43	80	109
47	80	107
53	80	103
59	80	101
61	80	97
67	80	89
71	80	Ø
73	80	Ø
79	80	Ø

Table 1E

p	E/2	q
3	180	397
5	180	389
7	180	383
11	180	379
13	180	373
17	180	367
19	180	359
23	180	353
29	180	349
31	180	347
37	180	337
41	180	331
43	180	317
47	180	313
53	180	311
59	180	307

61	180	293
67	180	283
71	180	281
73	180	277
79	180	271
89	180	269
97	180	263
101	180	257
103	180	251
107	180	241
109	180	239
113	180	233
127	180	229
131	180	227
137	180	223
139	180	211
149	180	199
151	180	197
157	180	193
163	180	191
167	180	181
173	180	Ø
179	180	Ø

Table 1F

Tables 1A-F : Positions of natural equidistant primes (grey) which form the basis of the calculation to find the other equidistant primes deducible by the equations 6x ± 1 by gaps of 6 or by variable gaps of 2n (n ≥ 1).

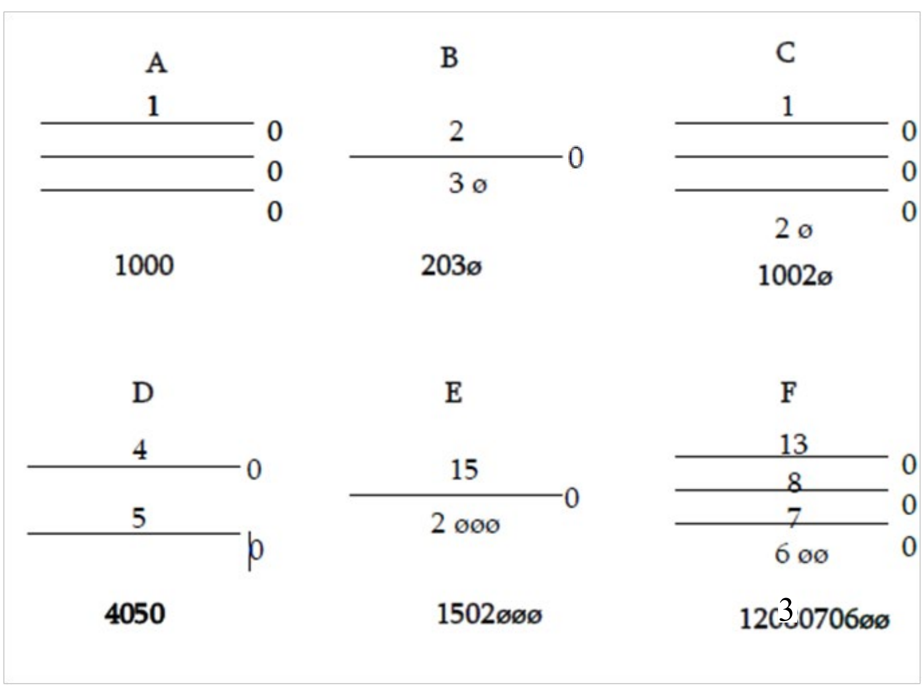

Figure 2 : Coding and deciphering of even numbers based on GSC.

The figures correspond in order to Tables 1A-F. It is read from top to bottom.

Each line marked with 0 corresponds to a NEEP pair. The number at the bottom or top of the line gives the number of PN pairs that precede or follow the NEEP pair. The sign means that there is no PN on the right, i.e. > E/2. The coded number at the bottom brings together all the information about the even number. We speak of coding because with the coded number an independent calculator can deduce all the possible sums p + q satisfying the GSC. The encoding number is obtained by reading the figure from top to bottom.

p	E/2	q
3	20	37
5	20	31
7	20	29
11	20	23
13	20	ϕ
17	20	ϕ
19	20	ϕ

Table A

p	E/2	q
3	15	29
5	15	23
7	15	19
11	15	17
13	15	ϕ

Table B

p	E/2	q
3	45	89
5	45	83
7	45	79
11	45	73
13	45	71
17	45	67
19	45	61
23	45	59
29	45	53
31	45	47
37	45	ϕ
41	45	ϕ
43	45	ϕ

Table C

Tables 2A-C. Two evens that do not have NEEPs (B and C). For comparison, an even having a pair of NEEPs (A). This shows that Goldbach's strong conjecture is not naturally true for all evens if we use prime counting function and their natural ranks [1,2].

11 → 1	**E/2**	**13 → 23**
11	12	13
10	12	14
9	12	15
8	12	16
7	12	17
6	12	18
5	12	19
4	12	20
3	12	21
2	12	22
1	12	23

Table A

29 → 1	**E/2**	**31 → 60**
29	30	31
28	30	32
27	30	33
26	30	34
25	30	35
24	30	36
23	30	37
22	30	38
21	30	39
20	30	40
19	30	41
18	30	42
17	30	43
16	30	44
15	30	45
14	30	46
13	30	47
12	30	48
11	30	49
10	30	50
9	30	51
8	30	52
7	30	53
6	30	54
5	30	55
4	30	56
3	30	57
2	30	58
1	30	59

Table B

14 → 1	E/2	16 → 29
14	15	16
13	15	17
12	15	18
11	15	19
10	15	20
9	15	21
8	15	22
7	15	23
6	15	24
5	15	25
4	15	26
3	15	27
2	15	28
1	15	29

Table C

Table 3A-C. Numbers not having NEEP (B, 30 ; and C, 60) can give sums of two primes when not only prime numbers are taken in function of their natural rank. Integers before and after E/2 should placed as shown. The cryptograhic encoding remains the same as with NEEP, but it is much more easy. For instance Table 3A (E = 24, E = 12) is encoded 030104 whereas 3B (E = 30) is encoded 1010306. The table C (E = 60) is encoded 050301030506. Note this time we have no longer Ø because there are similar lengths of integers on the first and third columns.

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