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Research Article

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Mechanistic and Statistical Analysis of Possible Defense Mechanisms for Probabilistic Failures in Robustness, Stability and Sustainability of Static and Dynamic Complex Networks

Rakib Hassan Pran*

M.Sc. in Applied Statistics with Network Analysis, International Laboratory for Applied Network Research, NRU HSE, Moscow, Russia

***Corresponding Author**
Rakib Hassan Pran, M.Sc. in Applied Statistics with Network Analysis, International Laboratory for Applied Network Research, NRU HSE, Moscow, Russia

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Abstract

Complex networks are ubiquitous even in the form of defense networks in many fields such as ecological immune systems, biological immune systems, brain networks, air defense systems, national security, cyber security, peace and conflict studies, and so on. In the real world, complex networks exist in both static and dynamic forms. By understanding probabilistic failures and their possible defense mechanisms in static and dynamic complex networks, scientists and researchers can make decisions in developing and implementing methodologies of many fields as we can say, from the fields of biology to fields of technology. In this research, static and dynamic scale-free and small world networks have been generated as complex networks with network generation algorithms. Besides that, probabilistic failures and their possible defense mechanisms have also been generated and designed for previously generated complex networks. On these generated probabilistic failures and their possible defense mechanism, several mechanistic and statistical analysis methods have been applied. To generate these static and dynamic complex networks and their probabilistic failures and possible defense mechanism, two modified network generation algorithms along with several network measurements algorithms have been implemented where parameters have been chosen to produce possible types of static and dynamic scale-free and small world networks, and their corresponding probabilistic failures and possible defense mechanisms. Besides analyzing probabilistic failures and possible defense mechanisms for generated complex networks; the robustness, stability and sustainability of those networks have been studied and analyzed for probabilistic failures and their possible defense mechanisms.

Keywords: Mechanistic Analysis, Statistical Analysis, Defense Networks, Static Complex Networks, Dynamic Complex Networks, Network Generation Algorithms, Scale-Free Networks, Small World Networks, Stability, Sustainability, Robustness, Network Data Computation

1. Introduction

Complex networks exist in many forms of real life networks such as the networks of the World Wide Web, power grid, mobile phone calls, emails, science collaboration, E. Coli Metabolism, protein interactions, brain networks, air defense system, etc [1-5]. In most of the cases, complex networks follow mathematical models like scale-free networks and small world networks where scale-free network's nodes follow power law degree distributions and small world networks follow a high degree of clustering [1-2]. The study of complex networks is preferential in many fields of study where the network topology changes over time and sometimes, along with network propagation [1]. For changing topology over time in dynamic complex networks such as dynamic scale-free networks and dynamic small world networks, the connections among nodes also follow mathematical models [6-10].

Previously, several researches have been done where the synchronizations of topologies in dynamic networks were analyzed and modeled [6-10]. Robustness, stability and sustainability in static networks are also studied in several existing literature, meanwhile, few researches have done on the robustness, stability and sustainability in dynamic networks [10]. To understand robustness, stability and sustainability in a static or dynamic network, it is also necessary to understand which nodes in a network are more dependable for robustness, stability and sustainability in a static or dynamic network.

In this research paper, two network generation algorithms have been selected and implemented to generate static and dynamic scale-free and small world networks [2,13]. After generating static and dynamic scale-free and small world networks, several network measurement algorithms have been applied to identify probabilistic failures in robustness, stability and sustainability of generated static and dynamic scale-free and small world networks [14]. Besides that, several defense mechanisms have been designed to analyze the prevention of the failures in robustness, stability and sustainability of generated static and

dynamic scale-free and small world networks by following previously existing literatures [5,11,12].

2. Methodology & Computational Experiment

For generating undirected small world networks, a small world network generation algorithm [2] and for generating directed scale-free networks, a directed scale-free graph generation algorithm [13], have been implemented in this research. To implement these two network generation algorithms, the probabilities of connections among nodes (the probability of each edge existence) in generated networks have also been computed. Each node's value and each edge's weight have been generated randomly but overall node values and overall edge values are following power law distribution. For synchronization in dynamic scale-free networks and dynamic small world networks, state of node has been computed for each node where state of each node is based on coupling configuration matrix and the probabilities of connections of that considered node with all other nodes [8,9]. For directed scale-free networks, states of nodes are considered as states of nodes for in-degree and states of nodes for out-degree. Meanwhile, for undirected small world networks, only states of nodes are considered as there is no in-degree and out-degree edge exists in undirected small world networks. In synchronization, to generate next phase structure of dynamic directed scale-free network; four parameters have been chosen which are: previous phase's normalized states of nodes for in-degree, previous phase's normalized states of nodes for out-degree, previous phase's normalized nodes connection matrix which is based on shortest path frequency among nodes and next phase's evolving scale-free network matrix which is based on tendency of previous phase's scale-free be more scale-free in next phase which implies degree of next phase's hubs will be more than the degree of previous phase's hubs. In synchronization of dynamic undirected small world networks, three parameters have been chosen which are: previous phase's normalized states of nodes for degree, previous phase's normalized nodes connection matrix which is based on shortest path frequency among nodes and next phase's evolving small world which is based on tendency of previous phase's small world be more small world in next phase which implies density of network getting high so that neighbor of any node's neighbor is also neighbor of that node. Coefficients of all parameters need to be considered so that the next phase structure of any dynamic complex network can be adjusted by increasing or decreasing coefficients. For example, if we want to increase scale free attribute in the next phase of dynamic directed scale-free network, we need to increase the coefficient of the evolving scale free matrix value.

To identify probabilistic failure in complex networks, probabilistic failures have been considered into three types which are probabilistic failure of robustness, probabilistic failure of stability and probabilistic failure of sustainability. To be more specific and accurate, we need to consider these three types as probabilistic failure of robustness in betweenness, probabilistic failure of stability in betweenness, probabilistic failure of sustainability in betweenness. For computing these three probabilistic failures, betweenness centrality has been selected to compute each node's importance in robustness, stability and sustaibability [15]. In robustness, nodes which are weakly connected in a network have been considered vulnerable to failure in robustness. So, nodes with one degree or two degree and which are not in shortest paths of any other nodes are considered as nodes of probabilistic failure of robustness in betweenness. Accordingly, nodes which are neither strongly connected nor weakly connected and which are shortest paths among few nodes are considered as nodes of probabilistic failure of stability in betweenness. Likewise, nodes which are strongly connected and in the middle of the shortest path of most other nodes are considered as nodes of probabilistic failure of sustainability in betweenness.

For defense mechanisms, several defense mechanisms have been followed where network measurement algorithms have been implemented to identify optimal places (nodes) for putting defense instruments which will protect other places (nodes) based on given defensive range [5,11 and 12].

For experiment, python packages have been used in google colab and documentations done in google docs and overleaf [16-26].

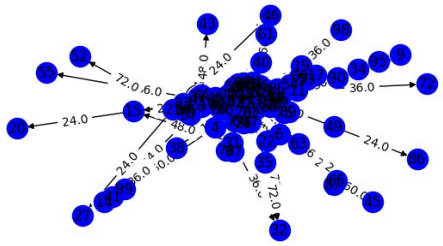
3. Computational Results

In this section, the results of the experiment have been shown where some mechanistic and statistical analysis are included.

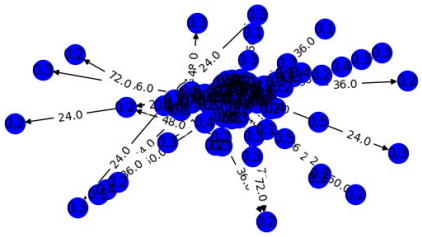
At first, six network algorithms have been applied to generated static scale-free and small world network where network_ algorithm_1's basement is measuring a node as the fraction of nodes that it is connected to , network_algorithm_2's basement is depending on a ranking of nodes in graph based on the structure of links , network_algorithm_3's basement is depending on the sum of the fraction of all-pairs shortest paths , network_algorithm_4's basement is depending on the reciprocal of the average shortest path distance, network_algorithm_5's basement is depending on the centrality of neighbors, and for static small world, network_algorithm_6 is based on measuring a node as having information. For the static scale-free network, network_algorithm_6 and network_algorithm_7 are sequentially in-degree and out-degree the fraction of nodes that it is connected to.

Analysis of applying six algorithms into generated static scale-free network:

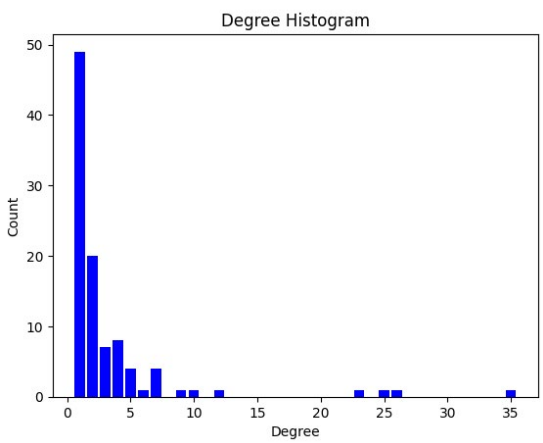
(1)



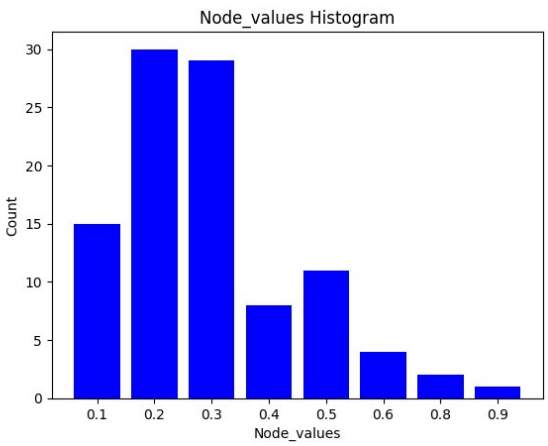
(2)



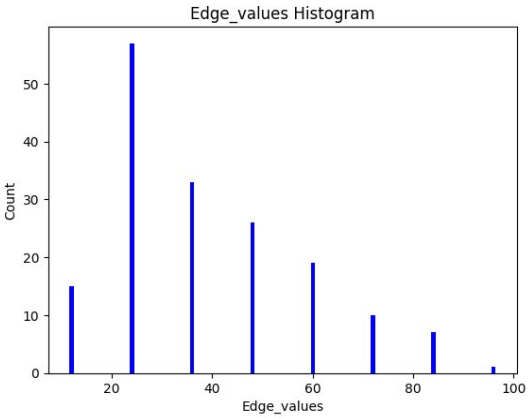
(3)


Degree Histogram
Count
Degree

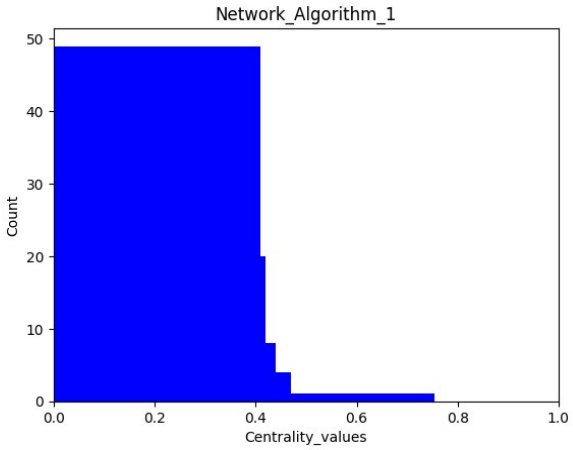
(4)


Node_values Histogram
Count
Node_values

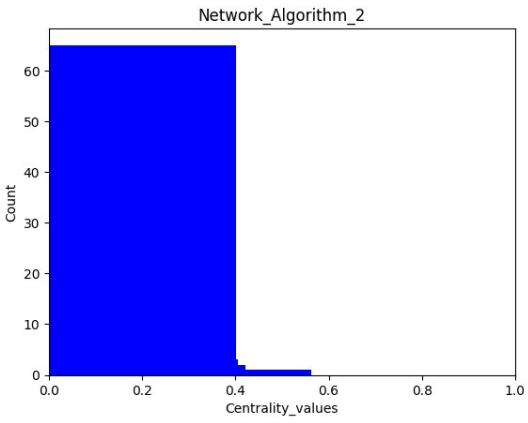
(5)


Edge_values Histogram
Count
Edge_values

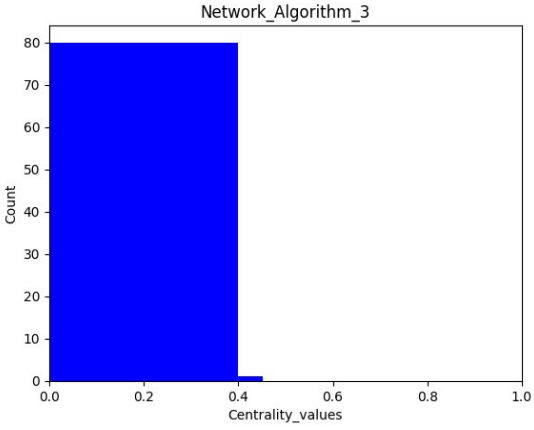
(6)


Network_Algorithm_1
Count
Centrality_values

(7)


Network_Algorithm_2
Count
Centrality_values

(8)


Network_Algorithm_3
Count
Centrality_values

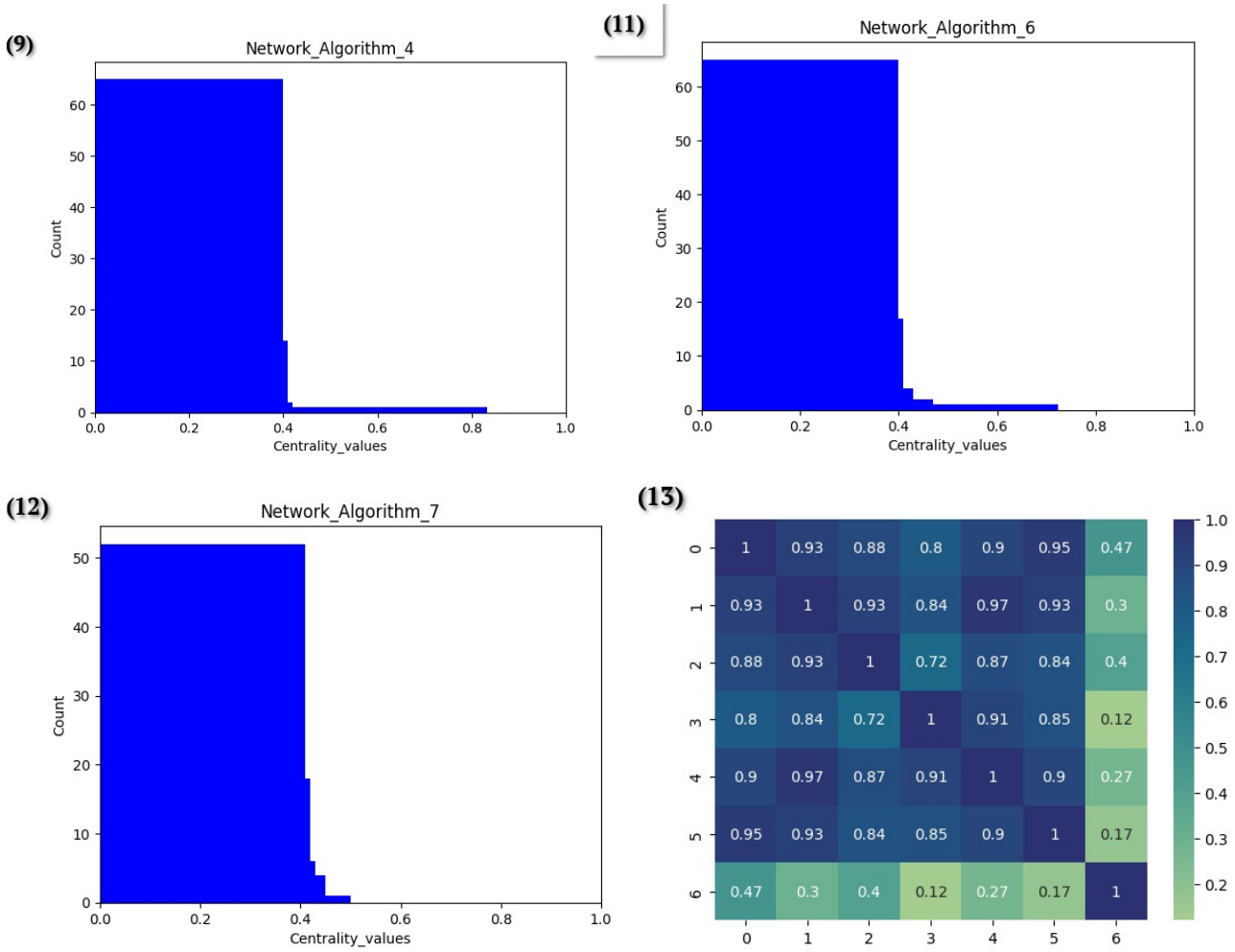
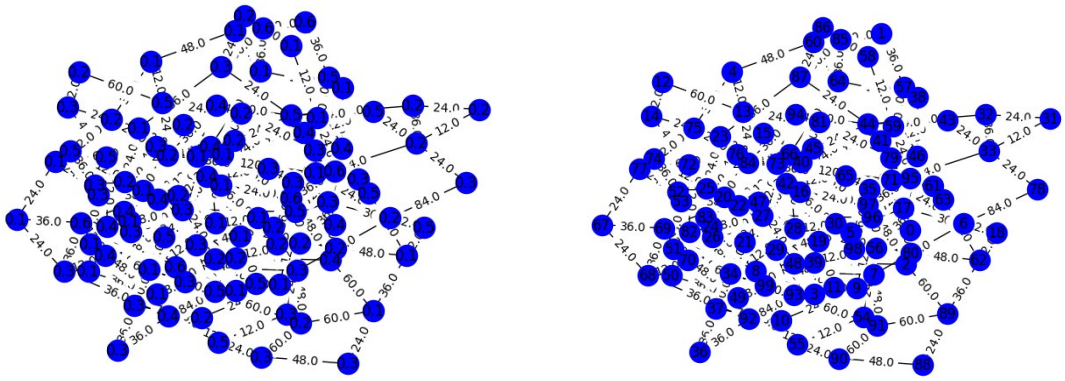
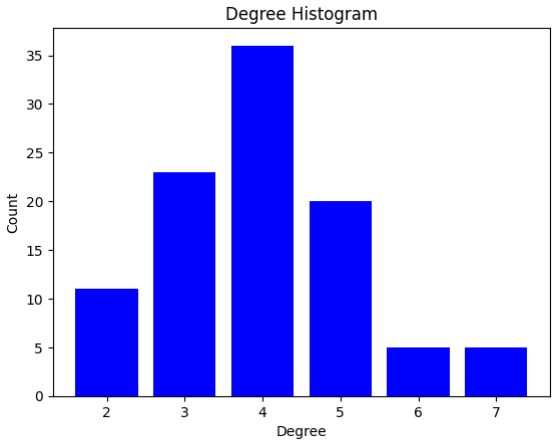
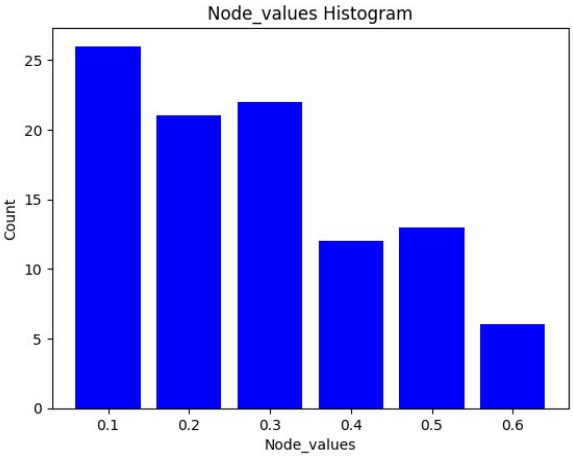


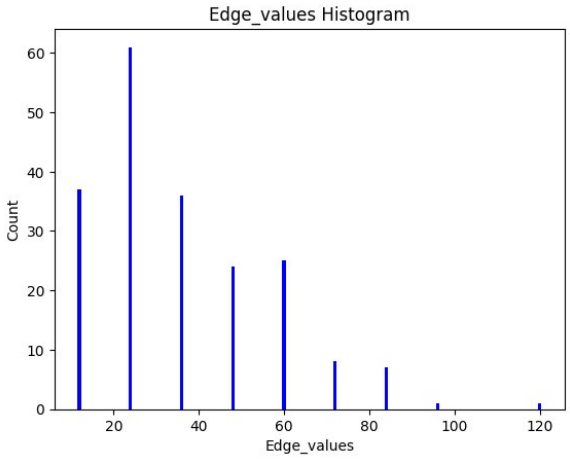
Figure 1: (1) Scale free network's node number with assigned weight values, (2) Scale free networks assigned node value with assigned weight values, (3) Degree histogram of scale free network, (4) Assigned weighted node histogram of scale free network, (5) Assigned weighted edge histogram of scale free network, (6) values of network_algorithm_1's histogram of scale free network, (7) values of network_algorithm_2's histogram of scale free network, (8) values of network_algorithm_3's histogram of scale free network, (9) values of network_algorithm_4's histogram of scale free network, (10) values of network_algorithm_5's histogram of scale free network, (11) values of network_algorithm_6's histogram of scale free network, (12) values of network_algorithm_7's histogram of scale free network, (13) correlation among network_algorithms.

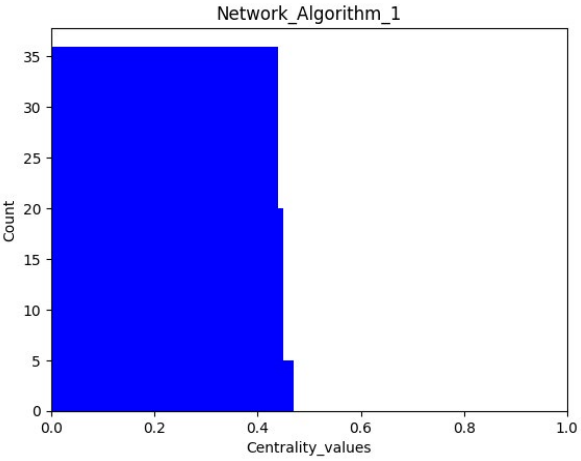
Analysis of applying six algorithms into generated static small world network:

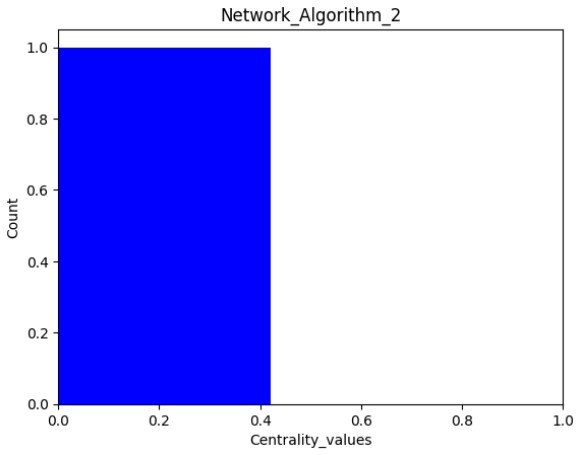


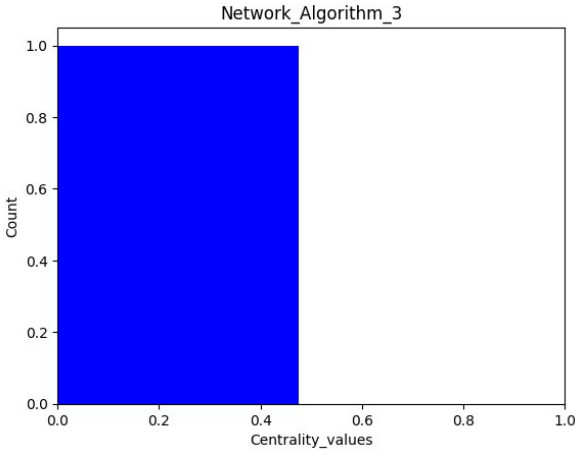

Degree Histogram
Count
Degree

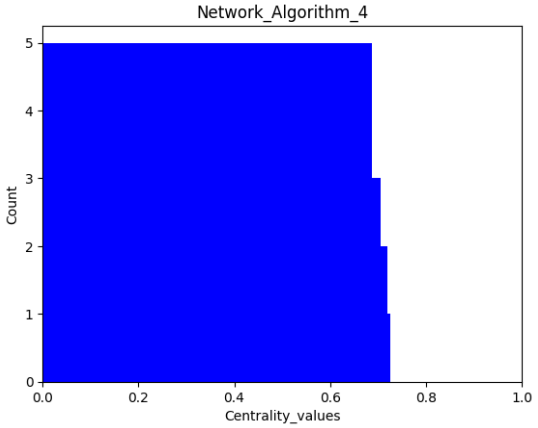

Node_values Histogram
Count
Node_values

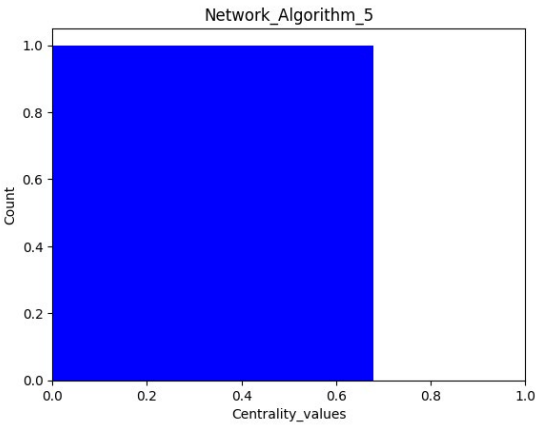

Edge_values Histogram
Count
Edge_values


Network_Algorithm_1
Count
Centrality_values


Network_Algorithm_2
Count
Centrality_values


Network_Algorithm_3
Count
Centrality_values


Network_Algorithm_4
Count
Centrality_values


Network_Algorithm_5
Count
Centrality_values

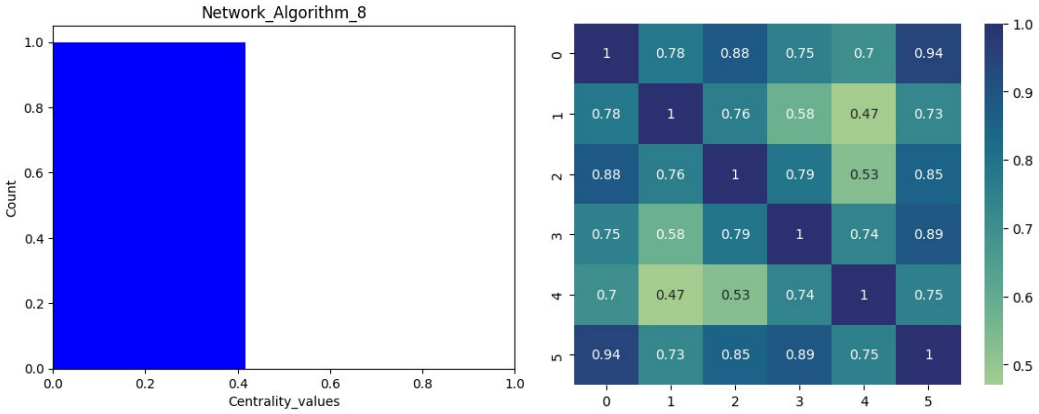


Figure 2: (From top to bottom and left to right) (1) Small world network's node number with assigned weight values, (2)Small world networks assigned node value with assigned weight values, (3) Degree histogram of small world network, (4) Assigned weighted node histogram of small world network, (5) Assigned weighted edge histogram of small world network, (6) values of network_algorithm_1's histogram of small world network, (7) values of network_algorithm_2's histogram of small world network, (8) values of network_algorithm_3's histogram of small world network, (9) values of network_algorithm_4's histogram of small world network, (10) values of network_algorithm_5's histogram of small world network, (11) values of network_algorithm_8's histogram of small world network, (12) correlation among network algorithms.

Here, Newly generated scale-free network and small world network have been demonstrated below:
Example of adjacency matrix:
Scale Free Networks:

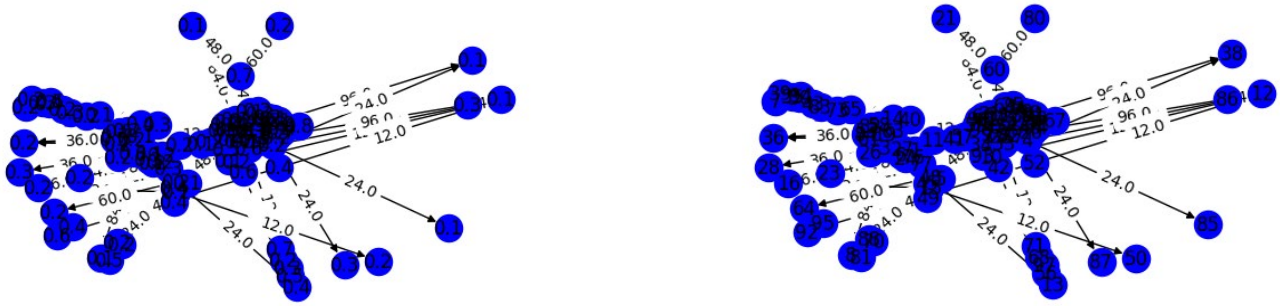


Figure 3: (Left side) weighted Scale Free Network with Node value, (right side) weighted Scale Free Network with Node number
Adjacency matrix of Figure (3):

Adjacency Matrix with weight:
[[36. 36. 12. ... 0. 0. 0.]
[24. 0. 36. ... 0. 0. 0.]
[24. 0. 0. ... 0. 0. 0.]

...

[0. 0. 0. ... 0. 0. 0.]
[0. 0. 36. ... 0. 0. 0.] [0. 0. 48. ... 0. 0. 0.]]

Adjacency Matrix without weight:
[[1. 1. 1. ... 0. 0. 0.]
[1. 1. 1. ... 0. 0. 0.]
[1. 0. 1. ... 0. 0. 0.]

...

[0. 0. 0. ... 1. 0. 0.]
[0. 0. 1. ... 0. 1. 0.]
[0. 0. 1. ... 0. 0. 1.]]

Coupling Configuration Matrix:
[[-9. 1. 1. ... 0. 0. 0.]
[1. -3. 1. ... 0. 0. 0.]
[1. 0. -1. ... 0. 0. 0.]

...

[0. 0. 0. ... -1. 0. 0.]
[0. 0. 1. ... 0. -1. 0.]
[0. 0. 1. ... 0. 0. -1.]]

scale_free_probability_matrix:
[[0. 0.54 0.49975643 ... 0.001 0.001 0.001]
[0.7236071 0. 0.7965207 ... 0.001 0.001 0.001]
[0.90326731 0.001 0. ... 0.001 0.001 0.001]

...

[0.001 0.001 0.001 ... 0. 0.001 0.001]
[0.001 0.001 0.16906101 ... 0.001 0. 0.001]
[0.001 0.001 0.02749893 ... 0.001 0.001 0.]]

States of nodes need to be considered as states of nodes for in degree and state of nodes for out degree.

State of nodes for outdegree: [3.801729387093247, 1.521127799779646, 0.9032673123580325, 3.5686612481581705, 5.362028355627901, 1.6949904484696336, 0.04694031073161342, 0.0, 0.0, 0.3996076573089916, 3.916001209308355, 0.43799352121552615, 0.0, 0.0, 2.265055084141128, 2.4870663074678725, 0.0, 0.11558579239906608, 0.8805797996226874, 0.7708471768404422, 0.3627195684303347, 0.0, 0.6961224484045014, 0.3920523070185845, 0.07579480888263568, 0.31119283021742883, 0.01057808198489929, 0.3123899935464478, 0.0, 0.3268404596438198, 0.0, 0.31666102633662807, 3.316076671616878, 1.9247393392297387, 0.3148264804506139, 1.2378640208432312, 0.0, 0.3097208188597528, 0.0, 0.21183673874303255, 2.169514531969143, 1.2406481994549157, 0.14947770255982074, 0.18542060559928308, 0.08829229388772519, 1.623682350387618, 0.031936565785456894, 0.09313235812402454, 1.7408553780009108, 0.3705558830387463, 0.0, 0.24079710064577753, 0.22639344276868445, 0.04220101366994389, 0.9866971064793161, 0.0, 0.05924844838388621, 0.7423586404764091, 0.1990445540532767, 0.0011145233523295328, 1.7297953134287698, 0.059097629977413746, 0.14089280378834268, 0.9714417985239749, 0.0, 0.3278348579994852, 0.7294699214099613, 0.3299912717992981, 0.17084162821155058, 0.28867338276592225, 0.0871638439055138, 0.04145376454641092, 0.1608819039202518, 0.07328392875645462, 0.11689766022456516, 0.007584258914974162, 0.2264487934209598, 0.1258569742346236, 0.06670491727414829, 0.20593677970795232, 0.0, 0.3757278625822197, 0.2343331655051848, 0.1722761697516908, 0.047389059651351206, 0.0, 0.1396066961694048, 0.0, 0.15016972017080854, 0.01999057648131608, 0.40905965184097626, 0.026918319568253524, 0.0, 0.15552440714560056, 0.17533534322527133, 0.3243232400888777, 0.3836726018627814, 0.06736390923482594, 0.16906101461637946, 0.02749892648072072]

State of nodes for indegree: [15.738791199350741, 3.2628965167834236, 10.181169116273095, 1.006901806779294, 0.0, 0.0, 0.0, 3.6004458480951835, 2.6425442734448197, 1.3844406029619911, 0.5883993694141141, 0.0, 1.8733174801866, 2.4033926739875238, 0.0, 0.3705558830387463, 2.0255500034283536, 0.0, 1.9773864012638422, 3.917768354717599, 0.0, 0.8358550829335347, 0.0, 0.0, 0.0, 0.0, 0.433562890073909, 0.0, 0.001, 0.3299912717992981, 0.7403186168252657, 0.0, 0.0, 0.0, 0.15552440714560056, 0.0, 0.6591895668367961, 0.0, 0.8109519060470967, 0.0, 0.0, 0.0, 0.8805761640963191, 0.0, 0.0, 0.0, 0.0, 0.9430601507476991, 0.0, 0.0, 0.001, 0.0, 0.0, 0.0, 0.0, 0.001, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.001, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.1258569742346236, 0.0, 0.0, 0.0, 0.001, 0.0, 0.0, 0.0, 0.0, 0.001, 0.0, 0.001, 0.0, 0.0, 0.0, 0.0, 0.3253232400888777, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]

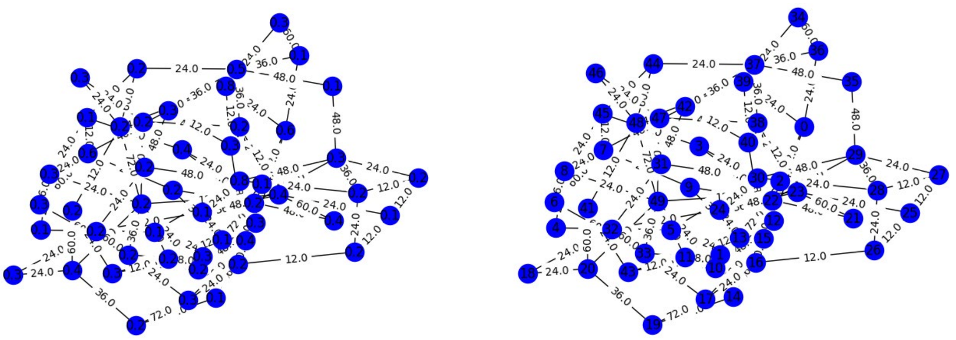


Figure 4: (Left side) weighted Small World Network with Node value, (right side) weighted Small World Network with Node number

Small_world_probability_matrix of Figure (4):

[[0.0.01 0.99 ... 0.001 0.01 0.01]
[0.01 0. 0.99 ... 0.001 0.001 0.01] [0.99 0.99 0. ... 0.001 0.001 0.001]
...
[0.001 0.001 0.001 ... 0.0.99 0.99]
[0.01 0.001 0.001 ... 0.99 0.0.99]
[0.01 0.01 0.001 ... 0.99 0.99 0.]]

Small world networks are having more tendency to connect with most other nodes while nodes of scale free networks are having less tendency to connect with most other nodes except for hubs in scale free networks.

Synchronization of scale-free network is based on the scale-free network generation algorithm and the states of nodes in the immediate previous phase of dynamic scale-free Network. For synchronization, path's importance, state of nodes and probability of connection among nodes including predetermined and evolving scale-free matrix are considered. Highly frequent path's frequency score, normalized node connection matrix based on path frequency and probabilistic matrix of connections among nodes has been considered in synchronization of scale free network.

Synchronization of the small world network is based on the small world network generation algorithm and the states of nodes in the immediate previous phase of dynamic small world network. For Synchronization, Path's importance, state of nodes and probability of connection among nodes including predetermined and evolving small world matrices are considered.

Several phases of sequential next phase structures of dynamic scale-free networks have been shown in Figure (5).

Synchronization of scale-free network:

Components needed for synchronization:

Here, components are used as synonyms "part of", not components in a network.

Component_1: Scale_free_probability_matrix:

[[0.0.540.49975643 ... 0.0010.0010.001]
[0.7236071 0.0.7965207...0.0010.0010.001]
[0.90326731 0.0010.... 0.0010.0010.001]
...
[0.0010.0010.001 ... 0.0.0010.001]
[0.0010.0010.16906101...0.0010.0.001]
[0.0010.0010.02749893...0.0010.0010.]]

State of nodes for outdegree: [3.801729387093247, 1.521127799779646, 0.9032673123580325, 3.5686612481581705, 5.362028355627901, 1.6949904484696336, 0.04694031073161342, 0.0, 0.0, 0.3996076573089916, 3.916001209308355, 0.43799352121552615, 0.0, 0.0, 2.265055084141128, 2.4870663074678725, 0.0, 0.11558579239906608, 0.8805797996226874, 0.7708471768404422, 0.3627195684303347, 0.0, 0.6961224484045014, 0.3920523070185845, 0.07579480888263568, 0.31119283021742883, 0.01057808198489929, 0.3123899935464478, 0.0, 0.3268404596438198, 0.0, 0.31666102633662807, 3.316076671616878, 1.9247393392297387, 0.3148264804506139, 1.2378640208432312, 0.0, 0.3097208188597528, 0.0, 0.21183673874303255, 2.169514531969143, 1.2406481994549157, 0.14947770255982074, 0.18542060559928308, 0.08829229388772519, 1.623682350387618, 0.031936565785456894, 0.09313235812402454, 1.7408553780009108, 0.3705558830387463, 0.0, 0.24079710064577753, 0.22639344276868445, 0.04220101366994389, 0.9866971064793161, 0.0, 0.05924844838388621, 0.7423586404764091, 0.1990445540532767, 0.0011145233523295328, 1.7297953134287698, 0.059097629977413746, 0.14089280378834268, 0.9714417985239749, 0.0, 0.3278348579994852, 0.7294699214099613, 0.3299912717992981, 0.17084162821155058, 0.28867338276592225, 0.0871638439055138, 0.04145376454641092,

0.1608819039202518, 0.07328392875645462, 0.11689766022456516, 0.007584258914974162, 0.2264487934209598, 0.1258569742346236, 0.06670491727414829, 0.20593677970795232, 0.0, 0.3757278625822197, 0.2343331655051848, 0.1722761697516908, 0.047389059651351206, 0.0, 0.1396066961694048, 0.0, 0.15016972017080854, 0.01999057648131608, 0.40905965184097626, 0.026918319568253524, 0.0, 0.15552440714560056, 0.17533534322527133, 0.3243232400888777, 0.3836726018627814, 0.06736390923482594, 0.16906101461637946, 0.02749892648072072]

State of nodes for indegree: [15.738791199350741, 3.2628965167834236, 10.181169116273095, 1.006901806779294, 0.0, 0.0, 0.0, 3.6004458480951835, 2.6425442734448197, 1.3844406029619911, 0.5883993694141141, 0.0, 1.8733174801866, 2.4033926739875238, 0.0, 0.3705558830387463, 2.0255500034283536, 0.0, 1.9773864012638422, 3.917768354717599, 0.0, 0.8358550829335347, 0.0, 0.0, 0.0, 0.0, 0.433562890073909, 0.0, 0.001, 0.3299912717992981, 0.7403186168252657, 0.0, 0.0, 0.0, 0.15552440714560056, 0.0, 0.6591895668367961, 0.0, 0.8109519060470967, 0.0, 0.0, 0.0, 0.8805761640963191, 0.0, 0.0, 0.0, 0.0, 0.9430601507476991, 0.0, 0.0, 0.001, 0.0, 0.0, 0.0, 0.0, 0.001, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.001, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.1258569742346236, 0.0, 0.0, 0.0, 0.001, 0.0, 0.0, 0.0, 0.0, 0.001, 0.0, 0.001, 0.0, 0.0, 0.0, 0.0, 0.3253232400888777, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]

Component_2: Normalized State of Nodes for outdegree: [0.06643967566924305, 0.02658349066399505, 0.015785654675853068, 0.062366536846618914, 0.0937077284055685, 0.029621944171279103, 0.0008203369317684134, 0.0, 0.0, 0.006983612046846675, 0.06843670965905843, 0.007654449996778671, 0.0, 0.0, 0.03958449164588459, 0.04346439791244785, 0.0, 0.00201999715840849, 0.015389163806988083, 0.01347145764601326, 0.006338949470526362, 0.0, 0.012165555458811001, 0.0068515734476327595, 0.0013246030968618917, 0.0054384593444322055, 0.0001848643774239614 4, 0.005459381176368262, 0.0, 0.0057119200035762804, 0.0, 0.005534022478906958, 0.05795232540867972, 0.03363707524275727, 0.005501961639942215, 0.02163312421622191, 0.0, 0.005412734221145559, 0.0, 0.003702095226635353, 0.03791480854820876, 0.021681780999788243, 0.0026122980008977904, 0.003240442347168262, 0.0015430112911006286, 0.028375751602906406, 0.0005581289083801789, 0.0016275970849158746, 0.030423487556506906, 0.0064758982504139005, 0.0, 0.004208211484834739, 0.0039564906861230525, 0.0007375121624025244, 0.01724368808596066, 0.0, 0.001035436059151239, 0.012973587091958652, 0.0034785368100963307, 1.947760151102923e-05, 0.03023019996861371, 0.0010328003307785146, 0.002462266835616353, 0.01697708370422105, 0.0, 0.005729298280015612, 0.012748341624073964, 0.005766984137918769, 0.0029856576345796654, 0.005044905613330733, 0.0015232903054140559, 0.0007244531083887888, 0.0028115997824044174, 0.001280722524563881, 0.0020429235858241235, 0.0001325438113015510 3, 0.0039574580036280125, 0.0021994980961162177, 0.001165746590034552, 0.003598986529292678, 0.0, 0.006566284653139703, 0.004095246644216148, 0.0030107279289000735, 0.0008281793449568748, 0.0, 0.0024397905979945825, 0.0, 0.0026243918195129516, 0.0003493587477238452, 0.007148796726551789, 0.00047042917566608397, 0.0, 0.0027179712486862405, 0.003064190569899896, 0.005667928642332522, 0.006705128281218345, 0.0011772632467131428, 0.0029545393255919054, 0.00048057596178047786]

Component_3: Normalized State of Nodes for indegree: [0.2750538179442366, 0.05702293988977084, 0.17792785996693225, 0.01759678889831258, 0.0, 0.0, 0.0, 0.06292200997373343, 0.046181557547295006, 0.024194721738059866, 0.01028296843025719, 0.0, 0.03273841800304567, 0.042002103123824824, 0.0, 0.006475898250413901, 0.035398859723287594, 0.0, 0.03455714434035256, 0.06846759259571593, 0.0, 0.014607546977206823, 0.0, 0.0, 0.0, 0.0, 0.007577019526041205, 0.0, 1.747617173773696e-05, 0.0057669841379187695, 0.012937935288282226, 0.0, 0.0, 0.0, 0.002717971248686241, 0.0, 0.011520110077764285, 0.0, 0.01417233478112419, 0.0, 0.0, 0.0, 0.015389100271904916, 0.0, 0.0, 0.0, 0.0, 0.016481081153482895, 0.0, 0.0, 1.747617173773696e-05, 0.0, 0.0, 0.0, 0.0, 1.747617173773696e-05, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.747617173773696e-05, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.002199498096116218, 0.0, 0.0, 0.0, 1.747617173773696e-05, 0.0, 0.0, 0.0, 0.0, 1.747617173773696e-05, 0.0, 1.747617173773696e-05, 0.0, 0.0, 0.0, 0.0, 0.00568540481407026, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]

Component 4: Normalized Node Connection Matrix based on Path Frequency:

```
[[0.         0.02921231 0.01173709 ... 0.         0.         0.        ]
 [0.04042775 0.         0.00547731 ... 0.         0.         0.        ]
 [0.07642149 0.         0.         ... 0.         0.         0.        ]
 ...
 [0.         0.         0.         ... 0.         0.         0.        ]
 [0.         0.         0.00443401 ... 0.         0.         0.        ]
 [0.         0.         0.00443401 ... 0.         0.         0.        ]
```

Component5:Scale_free_evolving_matrix:]]

```
[[0.         0.02439024 0.02439024 ... 0.         0.         0.        ]
 [0.00909091 0.         0.02439024 ... 0.         0.         0.        ]
 [0.00909091 0.         0.         ... 0.         0.         0.        ]
 ...
 [0.         0.         0.         ... 0.         0.         0.        ]
 [0.         0.         0.00909091 ... 0.         0.         0.        ]
 [0.         0.         0.00909091 ... 0.         0.         0.        ]]
```

Probability of Next_Phase_Structure_of_Scale_Free_Network:

[[0. 0.326 0.3 ... 0.01 0.01 0.01]
[0.46 0. 0.463 ... 0.004 0.004 0.004] [0.568 0.009 0. ... 0.003 0.003 0.003]
...
[0.04 0.009 0.026 ... 0. 0.001 0.001]
[0.04 0.009 0.124 ... 0.001 0. 0.001]
[0.04 0.009 0.043 ... 0.001 0.001 0.]]

Next_Phase_Structure_of_Scale_Free_Network based on previously estimated probability (see Figure(5)):
[[0. 1. 1. ... 0. 0. 0.]
[0. 0. 0. ... 0. 0. 0.] [1. 0. 0. ... 0. 0. 0.]
...
[0. 0. 0. ... 0. 0. 0.]
[0. 0. 0. ... 0. 0. 0.]
[0. 0. 0. ... 0. 0. 0.]]

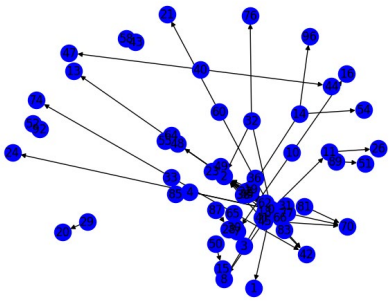


Figure 5: Next Phase Structure of Scale Free Network based on previously estimated probability Example of sequential 3 phases structure of dynamic scale free network:

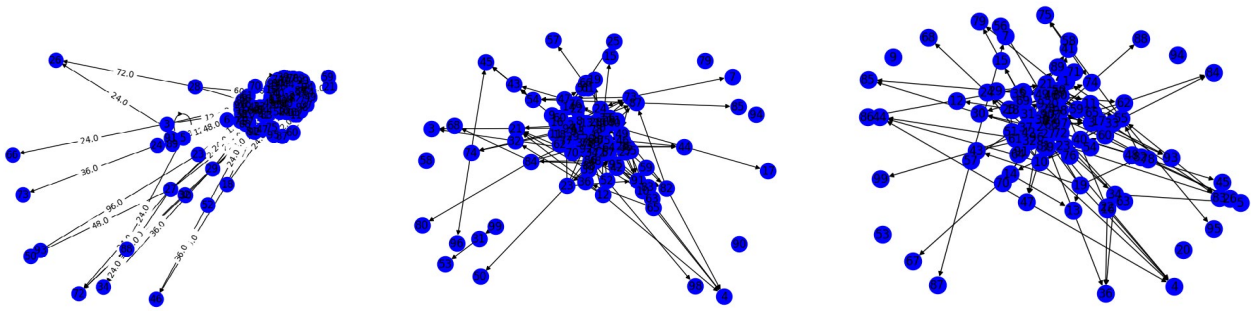


Figure 6: From left to right: phase_1, phase_2, phase_3

In Figure (vi), hubs are losing their degree because of coefficient value of evolving scale-free matrix are getting low

Synchronization of small world network:

Components needed for synchronization:

Here, components are used as synonyms “part of”, not components in a network.

Component 1: Small_world_probability_matrix:

[[0. 0.01 0.99 ... 0.001 0.01 0.01]
[0.01 0. 0.99 ... 0.001 0.001 0.01] [0.99 0.99 0. ... 0.001 0.001 0.001]
...
[0.001 0.001 0.001 ... 0. 0.99 0.99]
[0.01 0.001 0.001 ... 0.99 0. 0.99]
[0.01 0.01 0.001 ... 0.99 0.99 0.]]

State of nodes: [2.236597934763224, 1.4420062109400553, 2.6102507687491228, 0.444552598850808, 1.3863123154569323, 1.6917130886851472, 2.1043414049423665, 3.2506909130313573, 3.1744648267280304, 1.9742044434238704, 2.575638109693789, 2.6953093064400635, 3.2515080912317913, 3.132878124640869, 2.5124769150028547, 3.6184330360301646, 3.022089042104082, 3.39407489326182, 1.584884236244088, 2.3175584758816825, 3.094767607679564, 1.98, 3.4088773372891574, 3.506856983083226, 2.5198716538819275, 2.9699999999999998, 2.262579792386341, 2.9699999999999998, 3.96, 3.7019924549861196, 3.8137386437546814, 1.9971914167603384, 3.01009856124627, 1.8238387727827359, 1.1580959827032133, 1.4779950589645838, 2.5566438525771584, 4.941825254640231, 2.926410479216039, 3.134388696468041, 3.1681631674524287, 1.8479369830877892, 1.4786595273684078, 1.641828968525202, 1.8897875879659551, 1.7141124642564152, 1.98, 3.214035131814298, 4.197160132484978, 4.4403669391193255]

Component_2: Normalized State of Nodes: [0.01704630382488165, 0.010990297186182819, 0.019894111038754443, 0.0033881720752611914, 0.010565824352312916, 0.012893446267672084, 0.016038306385954604, 0.024775246405733913, 0.02419428681245515, 0.015046463305706895, 0.019630309533229615, 0.02054238744724242, 0.024781474555937915, 0.023877332411382776, 0.019148924435841447, 0.027578004943784828, 0.023032949819389595, 0.025868052069478766, 0.012079246698017859, 0.017663347219352178, 0.023586871868185286, 0.015090634328444362, 0.0259808693775514, 0.026727624431242707, 0.01920528367845681, 0.02263595149266654, 0.017244325396883756, 0.02263595149266654, 0.030181268656888725, 0.02821485577012907, 0.029066532978363254, 0.015221659269817169, 0.022941563979971692, 0.013900446461668178, 0.008826466157684152, 0.011264587360647524, 0.019485544185508644, 0.0376642817337612, 0.022303732543826706, 0.023888845283640704, 0.02414625851155083, 0.014084111754437627, 0.011269651628175259, 0.012513252825182229, 0.014403077499205613, 0.013064163836324354, 0.015090634328444362, 0.024495873178274268, 0.0319887923117454, 0.03384240089008258]

Component_3: Normalized Node Connection Matrix based on Path Frequency:

[[0. 0. 0.00498753 ... 0. 0. 0.]

[0. 0. 0.00696176 ... 0. 0. 0.]

[0.00394846 0.00467581 0. ... 0. 0. 0.]

...

[0. 0. 0. ... 0. 0.00187032 0.00852037]

[0. 0. 0. ... 0.00114298 0. 0.0104946]

[0. 0. 0. ... 0.00613051 0.00852037 0.]]

Component_4: Small_world_probability_matrix:

[[0. 0. 0.005 ... 0. 0. 0.]

[0. 0. 0.005 ... 0. 0. 0.]

[0.005 0.005 0. ... 0. 0. 0.]

...

[0. 0. 0. ... 0. 0.005 0.005]

[0. 0. 0. ... 0.005 0. 0.005]

[0. 0. 0. ... 0.005 0.005 0.]]

Probability of Next_Phase_Structure_of_Small_world_Network:

[[0. 0.008 0.5 ... 0.003 0.008 0.008]

[0.007 0. 0.499 ... 0.002 0.002 0.007]

[0.5 0.5 0. ... 0.004 0.004 0.004]

...

[0.005 0.005 0.005 ... 0. 0.5 0.501]

[0.01 0.006 0.006 ... 0.501 0. 0.503]

[0.011 0.011 0.006 ... 0.502 0.503 0.]]

Next_Phase_Structure_of_Small_world_Network based on previously estimated Probability (see Figure(vii)):

[[0. 0. 0. ... 0. 0. 0.]

[0. 0. 1. ... 0. 0. 0.]

[1. 1. 0. ... 0. 0. 0.]

...

[0. 0. 0. ... 0. 0. 0.]

[0. 0. 0. ... 0. 0. 1.]

[0. 0. 0. ... 0. 0. 0.]]

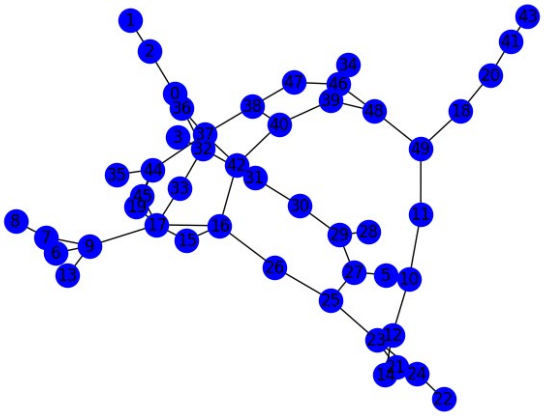


Figure 7: Next_Phase_Structure_of_Small_world_Network based on previously estimated Probability

For probabilistic failures in robustness, network algorithms based on shortest path distance might be considered. At this point, nodes with lowest measurement scores can be identified as the failure probable nodes in robustness. In this research, probabilistic failures in robustness are considered into four parts which are

1. Probabilistic Failures of Robustness in Betweenness of Static Scale-Free:

Probabilistic Failures in Robustness of Static Scale-free Network 1: {66: 0.0001546072974644403, 17: 0.0003092145949288806, 35: 0.0013399299113584827, 45: 0.0013399299113584827, 87: 0.0013399299113584827, 48: 0.0038651824366110078, 24: 0.004019789734075448, 6: 0.004741290455576169, 14: 0.006338899196042053, 5: 0.007627293341579055, 30: 0.010770975056689341, 2: 0.01963512677798392, 1: 0.04803133374561946, 0: 0.05292723149866007}

Probabilistic Failures in Robustness of Static Scale-free Network 2 : {19: 3.4357177214320065e-05, 41: 0.00010307153164296021, 6: 0.0001546072974644403, 13: 0.00041228612657184083, 37: 0.0010307153164296021, 70: 0.0010307153164296021, 17: 0.0011337868480725624, 10: 0.0012883941455370026, 32: 0.0013914656771799629, 16: 0.001443001443001443, 2: 0.020476877619734764, 0: 0.029272314986600698, 1: 0.04787672644815501}

Probabilistic Failures in Robustness of Static Scale-free Network 3 : {18: 5.1535765821480104e-05, 24: 0.00020614306328592042, 8: 0.0003092145949288806, 19: 0.0006699649556792414, 16: 0.001958359101216244, 36: 0.001958359101216244, 50: 0.001958359101216244, 85: 0.001958359101216244, 22: 0.0020614306328592042, 35: 0.0020614306328592042, 54: 0.0021645021645021645, 4: 0.0041228612657184084, 14: 0.012162440733869304, 6: 0.016336837765409194, 3: 0.021748093176664603, 12: 0.05411255411255411, 58: 0.0631828488971346, 0: 0.0714801071943929, 1: 0.07787054215625644, 2: 0.08663162234590806}

Probabilistic Failures in Robustness of Static Scale-free Network 4 : {93: 5.1535765821480104e-05, 23: 0.00010307153164296021, 69: 0.00020614306328592042, 2: 0.00041228612657184083, 28: 0.00092764378478664 18, 15: 0.0010307153164296021, 48: 0.0010307153164296021, 4: 0.0011337868480725624, 61: 0.0035044320758606472, 16: 0.010977118119975262, 0: 0.018759018759018756, 1: 0.0691609977324263}

Probabilistic Failures in Robustness of Static Scale-free Network 5 : {32: 5.1535765821480104e-05, 2: 0.00010307153164296021, 36: 0.00010307153164296021, 30: 0.0001546072974644403, 35: 0.00020614306328592042, 28: 0.0004122861265718408, 27: 0.0015460729746444 03, 49: 0.0015460729746444 03, 78: 0.0015460729746444 03, 41: 0.0017522160379303236, 3: 0.0091561877276163, 5: 0.043152614581186, 4: 0.051587301587301584, 1: 0.06173984745413316, 0: 0.06709956709956709}

Probabilistic Failures in Robustness of Static Scale-free Network 6 : {46: 3.4357177214320065e-05, 23: 0.00010307153164296021, 29: 0.0016491445062873633, 47: 0.0016491445062873633, 7: 0.0018037518037518038, 3: 0.004432075860647289, 84: 0.005978148835291692, 11: 0.0070088641517212944, 16: 0.007215007215007215, 13: 0.012093726379440665, 8: 0.01602762317048031, 6: 0.017333195904624475, 2: 0.052755445612588464, 0: 0.059558166701023846, 1: 0.06282209853638425}

Probabilistic Failures in Robustness of Static Scale-free Network 7 : {35: 0.00031265031265031263, 11: 0.000518793375936233, 44: 0.0007816257816257812, 29: 0.001958359101216244, 18: 0.0022710094138665565, 48: 0.002473716759431045, 5: 0.0032295746581460865, 46: 0.005875077303648732, 8: 0.0077303648732220155, 0: 0.008350511921940492, 10: 0.010832817975675119, 3: 0.05024050024050024, 1: 0.07194736480450767, 2: 0.07466501752216038}

Probabilistic Failures in Robustness of Static Scale-free Network 8 : {10: 0.00020614306328592042, 72: 0.0003092145949288806, 24: 0.0005153576582148011, 30: 0.0005153576582148011, 67: 0.0005153576582148011, 6: 0.0008245722531436817, 9: 0.0009276437847866418, 18: 0.0010307153164296021, 11: 0.0012626262626262625, 13: 0.0013399299113584827, 1: 0.0019325912183055034, 22: 0.0022675736961451248, 5: 0.0034700748986463276, 2: 0.0035216106644678077, 0: 0.006708238851095993, 3: 0.00990345633202776}

Probabilistic Failures in Robustness of Static Scale-free Network 9 : {25: 6.871435442864013e-05, 15: 0.0003092145949288806, 19: 0.0006699649556792414, 16: 0.0015460729746444 03, 4: 0.0016319659176802 03, 13: 0.0019583591012162 44, 48: 0.0021645021645021645, 28: 0.0027829313543599257, 18: 0.0055772005772005772, 7: 0.0077303648732220155, 6: 0.008589294303580016, 22: 0.0101010101010101, 1: 0.011475297189582902, 3: 0.026386312100597813, 2: 0.04519686662543806, 0: 0.07893561464990036}

Probabilistic Failures in Robustness of Static Scale-free Network 10 : {44: 0.00020614306328592042, 11: 0.0008245722531436817, 13: 0.0009276437847866418, 16: 0.0009276437847866418, 5: 0.004741290455576169, 0: 0.00943104514533086, 2: 0.026283240568954855, 1: 0.04396000824572253}

2. Probabilistic Failures of Robustness in Betweenness of Static Small World

Probabilistic Failures in Robustness of Static Small World Network 0 : {28: 0.0028344671201814054, 10: 0.0038606730571016283, 44: 0.004258786848072562, 20: 0.010742998498100538, 13: 0.012319134002807468, 43: 0.012702799373717738, 3: 0.01332070707070707, 14: 0.013443533979248263, 49: 0.013684402332361515, 21: 0.017992792355037254, 39: 0.018353542715787614, 46: 0.019042557499190153, 16: 0.025222401370360553, 1: 0.027042840945902178, 26: 0.02713428625418421, 18: 0.028409888486419104, 30: 0.029977741457333288, 42: 0.031061710398445096, 38: 0.03165087463556851, 40: 0.03191205053449951, 27: 0.035570905851518085, 35: 0.035673118453730696, 31: 0.038355463036585484, 37: 0.039299886621315186, 12: 0.04039756775726163, 17: 0.040667915795466816, 24: 0.04195370247666165, 45: 0.04361974948709641, 29: 0.04398899711399711, 5: 0.04485173037468956, 41: 0.046071226109491416, 8: 0.04646437038018671, 48: 0.04774764162519265, 34: 0.04907082241265916, 7: 0.05134410185430593, 4: 0.05135891838697961, 6: 0.05341856980887592, 36: 0.05546782204945469, 23: 0.062079738100146255, 2: 0.06297974276290602, 25: 0.06672188355861827, 11: 0.06946401576503616, 32: 0.06954717853952548, 33: 0.07497036693465264, 0: 0.07638891956493998, 19: 0.08349376908050377, 47: 0.08868612019122224, 9: 0.09210916870355645, 15: 0.1021799628942486}

Probabilistic Failures in Robustness of Static Small World Network 1 : {5: 0.006657285930245114, 23: 0.006802721088435374, 20: 0.007710118679506434, 42: 0.009347728990586135, 16: 0.011331794622610945, 49: 0.013393225255470152, 44: 0.013477891156462583, 32: 0.014692662779397469, 38: 0.014897149335924845, 25: 0.015725218658892127, 47: 0.015817338840298022, 45: 0.017144906303069 56, 2: 0.017327060694407627, 4: 0.019580830167564 86, 28: 0.020156803702722 07, 35: 0.022030588931099132, 6: 0.023883759853147613, 1: 0.023907441764584623, 37: 0.024311445357363723, 13: 0.029458886729294895, 31: 0.03149099105731759, 39: 0.03163338928645051, 41: 0.03227283157130096, 46: 0.032388173523377604, 0: 0.03298104956268221, 33:

0.0350291054372687, 19: 0.03866452425125894, 10: 0.03876004947433519, 21: 0.040524229172188356, 34: 0.041091337328582236, 3: 0.04403985800669473, 48: 0.044314746100460385, 22: 0.049520901434166736, 24: 0.05022059148334658, 43: 0.05222843105496167, 18: 0.05479632329122125, 8: 0.05641196954972463, 11: 0.057682448881428455, 26: 0.057792545228769705, 12: 0.059140236181052505, 30: 0.05937263794406651, 7: 0.06188626424595812, 29: 0.06278010302244996, 9: 0.06326368029174151, 27: 0.07204939335041376, 14: 0.07379473795290123, 15: 0.09498409753511795, 40: 0.10981986409282327, 36: 0.12283239955433825}

Probabilistic Failures in Robustness of Static Small World Network 2 : {8: 0.0017762660619803474, 32: 0.0030882194147500274, 15: 0.005829983508554937, 3: 0.006427154195011338, 4: 0.007354797979797979, 12: 0.009398283122772915, 23: 0.013393225255470152, 25: 0.01747485790853138, 43: 0.018579931972789118, 27: 0.019608438613540655, 0: 0.020126741172659536, 14: 0.02193923565097035, 17: 0.02543943443178137, 5: 0.02702664399092971, 18: 0.028781651549508697, 39: 0.029606377182907796, 9: 0.029818594104308385, 20: 0.03144795255764644, 13: 0.03176738227758636, 41: 0.03246918897429101, 24: 0.03256097171913498, 44: 0.03258965382689872, 31: 0.03565354713313896, 34: 0.03704685337338399, 47: 0.03766534391534392, 45: 0.03823732956386018, 35: 0.038608816542490015, 1: 0.0414378846776806, 37: 0.04453592042877757, 46: 0.045843149669680276, 30: 0.046972917955060804, 29: 0.05082570726703379, 28: 0.052247726291093644, 42: 0.05234855847100745, 36: 0.05317905120201037, 16: 0.05531536207556616, 49: 0.06348902656300615, 33: 0.06600268354094885, 2: 0.07043080219100627, 10: 0.07230894341863732, 38: 0.07596003769473157, 6: 0.07795069867773946, 11: 0.0838189658980475, 40: 0.08420585225432163, 7: 0.09066432669748997, 22: 0.09395031461357993, 26: 0.09440278636707206, 21: 0.10906587743322438}

Probabilistic Failures in Robustness of Static Small World Network 3 : {3: 0.002125850340136054, 31: 0.007873744735989634, 2: 0.010218253968253967, 16: 0.010807418205377388, 27: 0.010874568081200731, 32: 0.011237987258395421, 47: 0.011404006046863186, 29: 0.0141284688478566, 36: 0.015466067379332683, 41: 0.015745127416045786, 44: 0.017016925817946227, 14: 0.017973895907569377, 4: 0.01921937425763956, 48: 0.02031908001295757, 12: 0.02072940287226001, 7: 0.021880061548428895, 37: 0.022597789115646263, 30: 0.023634731670445955, 0: 0.02448709642587194, 1: 0.02520246193715582, 23: 0.025581403196199115, 38: 0.026323088759313242, 21: 0.026986151603498543, 46: 0.028698642155274807, 17: 0.03092707321023648, 40: 0.03214015765036174, 26: 0.03486428301479322, 42: 0.035947791815138755, 15: 0.03689362649821833, 10: 0.03759718172983479, 20: 0.03901374041680163, 6: 0.04163326044703595, 19: 0.0479119425547997, 9: 0.04858884029802396, 8: 0.0519470089623151, 5: 0.052694430946981966, 13: 0.057678706403196195, 33: 0.058003320375769364, 24: 0.06164662293488822, 18: 0.06195537738905085, 22: 0.06323729888780907, 43: 0.06436568675089084, 45: 0.06743568459129681, 25: 0.07240005129035744, 49: 0.07551830255911887, 28: 0.07926249865025375, 11: 0.08005884893639997, 39: 0.09778877820969654, 34: 0.10574789439585355, 35: 0.12400591188856494}

Probabilistic Failures in Robustness of Static Small World Network 4 : {0: 0.002130237015441097, 45: 0.0024659863945578234, 29: 0.0030470521541950115, 7: 0.0038295675413022347, 13: 0.004099854227405247, 48: 0.005739795918367346, 9: 0.006953555231616455, 38: 0.014152089407191447, 22: 0.01457557229240903, 44: 0.01528152025601005, 28: 0.015412783078599399, 30: 0.015542358718379123, 16: 0.016520894071914483, 8: 0.01753084170176007, 49: 0.017558174063276102, 20: 0.017709345643019108, 10: 0.021370532339920097, 42: 0.023135448262999285, 15: 0.024212760501025814, 32: 0.024428382464096746, 11: 0.02471250404923874, 46: 0.026160106899902814, 14: 0.028036591620775302, 24: 0.029112400732298704, 36: 0.02918289429769021, 37: 0.031772075713402244, 3: 0.03688221500721501, 17: 0.0403344671201814, 41: 0.04084905787711911, 2: 0.04087504049238743, 18: 0.04270732102364755, 26: 0.04555967228161107, 1: 0.04711163505806363, 35: 0.049514766223949895, 34: 0.05150969117805853, 23: 0.051732061872368, 43: 0.06086413822383209, 19: 0.06557472195227296, 47: 0.06679590486988445, 33: 0.06776603498542273, 21: 0.06798576753933896, 5: 0.07165839640329437, 4: 0.07243146356666764, 40: 0.07281285890979769, 12: 0.07702731886405355, 27: 0.07789824263038549, 6: 0.07830249163157328, 39: 0.08257324213956865, 25: 0.10536564012329318, 31: 0.10697683835438936}

Probabilistic Failures in Robustness of Static Small World Network 5 : {5: 0.0013463718820861676, 48: 0.005335884353741497, 19: 0.007532155108685721, 47: 0.007774538386783284, 17: 0.007802883057985098, 9: 0.007936507936507936, 25: 0.012315759637188207, 23: 0.01443452380952381, 27: 0.014597505668934238, 14: 0.01472098621333315, 7: 0.01732427861254392, 28: 0.018672420266808026, 36: 0.01947543669482445, 18: 0.02063351897535572, 32: 0.022862370056247604, 35: 0.023440316297459154, 40: 0.025867886837274593, 20: 0.02636877247846636, 49: 0.027627955944282478, 8: 0.027678571428571438, 15: 0.028189101148284823, 3: 0.031010705564276993, 1: 0.03198999838030451, 29: 0.032137826270479335, 39: 0.03338769129585455, 4: 0.03447039072039073, 10: 0.03550024710738995, 45: 0.037439997644079265, 31: 0.03858096976719426, 41: 0.04093749693239489, 0: 0.0419460641399417, 34: 0.04429458013641686, 16: 0.0447922896264733, 11: 0.04830259026687597, 38: 0.05391545812464181, 43: 0.05564200066751087, 24: 0.05735938758897944, 44: 0.058154838830859236, 30: 0.06374213466050199, 2: 0.06904798716023205, 6: 0.06971605963952902, 22:

0.07001252951508052, 21: 0.07268389679103962, 26: 0.07782333171363782, 33: 0.084040272369354, 13: 0.08484406749712871, 42: 0.09235851308555387, 46: 0.1176605282472629, 12: 0.1543316259515239}

Probabilistic Failures in Robustness of Static Small World Network 6 : {29: 0.0031496328690206236, 7: 0.0046997645211930925, 1: 0.005262323183241551, 38: 0.00941279289493575, 3: 0.01016156462585034, 20: 0.011520759097289706, 11: 0.012301051650541444, 27: 0.012749058858752736, 16: 0.013611801558230127, 4: 0.013807576116249583, 45: 0.015226487420364972, 44: 0.018349462800993414, 36: 0.021226446927977535, 8: 0.025051238444095587, 0: 0.02614456358078807, 10: 0.02668515819026024, 18: 0.028176238444095586, 9: 0.029563751630078162, 30: 0.02956507541966725, 19: 0.03175041842133678, 17: 0.03178411016421221, 41: 0.03319052451705512, 15: 0.03453517854028058, 46: 0.03531922537024578, 33: 0.035956227729186906, 42: 0.038944112859929186, 37: 0.04037755517347354, 23: 0.04206007522334053, 2: 0.04381616824218866, 40: 0.044786605118237774, 24: 0.04573301084780675, 25: 0.04844471241154916, 22: 0.05529884420190542, 34: 0.05595575531800021, 13: 0.057869782806007276, 12: 0.059012845619988465, 14: 0.05932549357294255, 43: 0.05998122720061494, 21: 0.06032252186588919, 5: 0.06348267898012794, 35: 0.06589562412266493, 39: 0.07248705801001719, 28: 0.0741739812530629, 47: 0.08523497949518358, 32: 0.0928926610049059, 48: 0.09460837018234976, 49: 0.09719670682680887, 26: 0.09878010311683781, 6: 0.1057513466314487}

Probabilistic Failures in Robustness of Static Small World Network 7 : {36: 0.002610980289551718, 40: 0.0029488581146744413, 35: 0.0030281557067271345, 9: 0.003622044055717525, 1: 0.010283041788143828, 39: 0.013033097854526424, 27: 0.013170097098668523, 31: 0.013339542166072776, 8: 0.01335607655760717, 34: 0.014400105280207317, 25: 0.016163548752834465, 18: 0.016344362836709776, 13: 0.01743737177410647, 5: 0.01833456367895144, 47: 0.019239957887917068, 37: 0.021323291221250405, 44: 0.022164468656815595, 10: 0.02216807663236235, 26: 0.022310293164885007, 14: 0.02395934564301911, 45: 0.024580228916963615, 49: 0.025841566785444338, 16: 0.027680388394674103, 4: 0.03000080984774862, 2: 0.030034216067379327, 17: 0.03397108843537416, 41: 0.03418772270813087, 23: 0.034817041896123536, 21: 0.035848715249225456, 29: 0.03708962522738034, 48: 0.03778007234639887, 3: 0.03842548466272956, 0: 0.03913083090379008, 7: 0.03934388315765866, 6: 0.04197329267482328, 43: 0.0422851878846777, 46: 0.06568843807364215, 11: 0.06657376778550247, 38: 0.06699276016878056, 19: 0.07052629201863898, 22: 0.07217087995149218, 32: 0.07751727675197062, 15: 0.07946361084116187, 30: 0.07954724319520239, 24: 0.08547652272142066, 12: 0.08707607585158604, 28: 0.09558926287242614, 33: 0.11442969587102239, 20: 0.1159384266527124}

Probabilistic Failures in Robustness of Static Small World Network 8 : {20: 0.0028628117913832196, 31: 0.004197097309342207, 32: 0.00500013497462477, 29: 0.009319322967282152, 12: 0.00958049886621315, 18: 0.011983722060252673, 48: 0.012430488068243168, 7: 0.015492387431162944, 21: 0.016643046107331822, 38: 0.017315373118944547, 36: 0.017632410106899896, 4: 0.020890022675736957, 44: 0.02092054534656575, 35: 0.02117089259946403, 40: 0.02122610949141561, 27: 0.021228471547349095, 1: 0.021775793650793653, 42: 0.02398364107547781, 5: 0.024153034229564843, 19: 0.025977891156462592, 30: 0.026272473275024296, 2: 0.026944646906381608, 37: 0.027921525753158403, 8: 0.03329857736745491, 47: 0.035266967537375696, 33: 0.037497975380628455, 9: 0.03930801577485251, 10: 0.0406955242414426, 0: 0.041134529208508804, 34: 0.04494826790745157, 22: 0.04556338408379224, 13: 0.04600981265522081, 26: 0.04608641075477811, 6: 0.047292409027102904, 24: 0.048506168340352006, 14: 0.05191124068675089, 15: 0.05247813411078717, 43: 0.05623374169292536, 23: 0.060472067614924775, 3: 0.06098524727351258, 39: 0.06315201946579498, 49: 0.06366504574412737, 46: 0.0720208952989565, 17: 0.0774401264834938, 25: 0.0777487582334521, 45: 0.08211966604823749, 11: 0.08498763141620284, 28: 0.0954206484180974, 41: 0.10000101230968576, 16: 0.10528916472794024}

Probabilistic Failures in Robustness of Static Small World Network 9 : {18: 0.006639429275983898, 19: 0.006802721088435374, 5: 0.006858766233766234, 37: 0.0068594898276771, 21: 0.00775944576964985, 46: 0.009014022275576898, 29: 0.012785808767951622, 10: 0.014201416497334866, 23: 0.014589355278130786, 7: 0.017226842035515506, 44: 0.01938539304610733, 40: 0.020418623798725843, 25: 0.021384346001692946, 32: 0.021440667231483558, 0: 0.021660886576852965, 36: 0.022401135778986923, 12: 0.02482864357864357, 16: 0.024990478625682707, 42: 0.02864270708283313, 49: 0.02918321188504261 6, 2: 0.03202455446603106, 8: 0.03217805201223569, 26: 0.03247870798891207, 11: 0.036048347910592804, 39: 0.036105767093162044, 28: 0.036389833711262275, 30: 0.03987473055600306, 45: 0.041822669238635625, 3: 0.04299857333320719, 17: 0.04313804345692099, 34: 0.043932174656964584, 38: 0.047962558039088644, 27: 0.048280708803667985, 24: 0.049386304360794143, 48: 0.04991299633406476, 43: 0.05050465393727699, 41: 0.054969677126039666, 9: 0.05627838478603785, 15: 0.05682342103785679, 31: 0.05839013274862613, 14: 0.06442054496901435, 47: 0.06677685726305173, 13: 0.0698650508599488, 4: 0.07377440042506068, 33: 0.0753635629798595, 1: 0.07878878324556599, 20: 0.08041399640664346, 6: 0.08137153851439567, 35: 0.09063179509232931, 22: 0.10562523016754712}

Probabilistic Failures in Robustness of dynamic Small World Network Phase 0 : {33: 0.0021056041464204726, 11: 0.005260973436993845, 23: 0.005260973436993845, 36: 0.006500377928949357, 5: 0.008323210236475542, 1: 0.009282879818594105, 28: 0.00974449303530936, 32: 0.01172254616132167, 17: 0.016591755749919015, 38: 0.019167071590540978, 40: 0.02095852229780801, 20: 0.021615848720440554, 18: 0.02295749649065976, 19: 0.023339137242198476, 29: 0.026216796242306443, 13: 0.027535498326314647, 24: 0.028365254832091565, 16: 0.02846243656192636, 8: 0.02901751970629521, 26: 0.029348882410106893, 37: 0.03143322805312601, 2: 0.032663184321347585, 48: 0.032772513767411734, 14: 0.033695740200842234, 27: 0.03378144908757154, 4: 0.03455417881438289, 22: 0.03694964096749811, 35: 0.038859869344563217, 6: 0.0417655355793111, 0: 0.04185394395853579, 43: 0.04369398553072022, 41: 0.04571793002915452, 42: 0.04597573156246626, 21: 0.046058066083576286, 31: 0.04702279721412373, 47: 0.04741861030126336, 15: 0.048065476190476186, 44: 0.05022709480617644, 3: 0.05523566569484936, 45: 0.05969927653601124, 25: 0.061044636108411626, 39: 0.06776468523917503, 34: 0.07539176384839652, 10: 0.07582739444984342, 49: 0.0769115781233128, 9: 0.07906813519058419, 30: 0.08298543623798724, 12: 0.08645158460209482, 46: 0.08913521757909511, 7: 0.10704871234207972}

Probabilistic Failures in Robustness of dynamic Small World Network Phase 1 : {9: 0.005881519274376417, 19: 0.013888888888888888, 40: 0.022817460317460323, 44: 0.03316326530612245, 43: 0.03330498866213152, 6: 0.03741496598639456, 8: 0.03741496598639456, 26: 0.03741496598639456, 37: 0.04265873015873016, 39: 0.045068027210884355, 15: 0.046414399092970515, 23: 0.049036281179138316, 49: 0.04988662131519273, 41: 0.050028344671201805, 34: 0.05144557823129251, 3: 0.05406746031746031, 4: 0.05442176870748299, 16: 0.06441326530612244, 18: 0.06738945578231292, 25: 0.0679563492063492, 48: 0.06831065759637188, 1: 0.07490079365079366, 14: 0.0794359410430839, 7: 0.11096938775510204, 42: 0.13151927437641722, 32: 0.15192743764172334, 46: 0.15348639455782312, 0: 0.1585175736961451, 30: 0.16808390022675737, 21: 0.19834183673469388, 35: 0.2232142857142857, 22: 0.25864512471655327, 12: 0.27706916099773243, 20: 0.35785147392290245} Probabilistic Failures in Robustness of dynamic Small World Network Phase 2 : {9: 0.015221088435374147, 13: 0.016439909297052156, 15: 0.016921768707482996, 14: 0.02585034013605443, 10: 0.028684807256235827, 2: 0.03251133786848072, 18: 0.0391156462585034, 25: 0.0391156462585034, 28: 0.0391156462585034, 32: 0.0391156462585034, 36: 0.0391156462585034, 4: 0.044472789115646255, 12: 0.04767573696145123, 7: 0.07327097505668935, 35: 0.07375283446712018, 41: 0.07653061224489796, 48: 0.07738095238095238, 34: 0.08622448979591837, 6: 0.10856009070294784, 0: 0.11224489795918367, 20: 0.11394557823129252, 42: 0.12227891156462585, 33: 0.1244897959183673, 43: 0.146797052154195, 29: 0.1525510204081632, 45: 0.16587301587301587, 21: 0.17859977324263035, 1: 0.18197278911564624, 24: 0.2273809523809523, 39: 0.23982426303854876, 37: 0.3395975056689342, 22: 0.3557823129251701, 46: 0.3695861678004535}

Probabilistic Failures in Robustness of dynamic Small World Network Phase 3 : {10: 0.002551020408163265, 34: 0.008064058956916097, 47: 0.011890589569160996, 27: 0.019274376417233556, 9: 0.020691609977324263, 18: 0.025368480725623584, 38: 0.026403061224489793, 26: 0.027140022675736966, 16: 0.02806122448979592, 15: 0.032454648526077094, 1: 0.034438775510204085, 5: 0.036564625850340135, 17: 0.036564625850340135, 11: 0.0391156462585034, 39: 0.03982426303854875, 14: 0.042120181405895685, 22: 0.0423611111111111, 44: 0.04251700680272108, 42: 0.04889455782312925, 2: 0.04988662131519274, 35: 0.050779478458049886, 0: 0.0536422902494331, 36: 0.054138321995464846, 21: 0.05426587301587301, 31: 0.05555555555555555, 6: 0.05984977324263039, 8: 0.06361961451247165, 33: 0.06458333333333334, 37: 0.0665107709750567, 40: 0.06695011337868481, 12: 0.08860544217687073, 20: 0.10209750566893423, 41: 0.10371315192743764, 49: 0.11748866213151929, 7: 0.11933106575963717, 48: 0.13241213151927436, 32: 0.15821995464852606, 29: 0.16203231292517004, 45: 0.16621315192743763, 46: 0.17443310657596367, 4: 0.18239795918367346, 30: 0.2974773242630385}

Probabilistic Failures in Robustness of dynamic Small World Network Phase 4 : {40: 0.013251133786848071, 41: 0.013251133786848071, 9: 0.017573696145124718, 14: 0.02161281179138323, 42: 0.021683673469387755, 34: 0.0233843537414966, 15: 0.0243764172335601, 36: 0.0274234693877551, 25: 0.029195011337868483, 43: 0.030895691609977318, 8: 0.0391156462585034, 29: 0.0391156462585034, 23: 0.04109977324263039, 4: 0.04159580498866211, 16: 0.045422335600907006, 2: 0.055839002267573684, 21: 0.05590986394557823, 6: 0.0570436507936508, 32: 0.06292517006802721, 22: 0.06490929705215419, 12: 0.07043650793650794, 17: 0.07405045351473924, 10: 0.07497165532879818, 27: 0.07653061224489796, 20: 0.08347505668934242, 45: 0.09828514739229025, 3: 0.10126133786848075, 44: 0.11663832199546487, 19: 0.12188208616780045, 0: 0.15554138321995462, 35: 0.15979308390022678, 47: 0.161281179138322, 39: 0.1677295918367347, 37: 0.17928004535147393, 18: 0.20337301587301587, 5: 0.21867913832199548, 7: 0.22356859410430835, 48: 0.24539399092970518, 46: 0.292375283446712}

Probabilistic Failures in Robustness of dynamic Small World Network Phase 5 : {30: 0.005810657596371882, 13: 0.009141156462585032, 14: 0.009141156462585032, 41: 0.017857142857142856, 46: 0.02283163265306122, 4:

0.027281746031746032, 48: 0.027338435374149664, 22: 0.027437641723356002, 2: 0.03312074829931972, 1: 0.033460884353741505, 7: 0.036564625850340135, 29: 0.036564625850340135, 18: 0.038321995464852605, 12: 0.04027777777777779, 45: 0.04299886621315194, 9: 0.04828514739229025, 19: 0.05861678004535147, 0: 0.061507936507936505, 47: 0.06257086167800453, 35: 0.06607142857142857, 20: 0.06829648526077099, 33: 0.06849489795918366, 3: 0.07101757369614513, 16: 0.07209467120181404, 21: 0.07867063492063492, 8: 0.0790107709750567, 10: 0.0802721088435374, 34: 0.08358843537414967, 37: 0.0885345804988662, 38: 0.09410430839002268, 6: 0.10873015873015875, 25: 0.11291099773242627, 43: 0.1370464852607709, 40: 0.1386054421768707, 15: 0.14750566893424036, 39: 0.16142290249433106, 42: 0.21299603174603168}

For probabilistic failures in stability, network algorithms based on shortest path distance might also be considered. For this, nodes with not very high measurement scores can be identified as the failure probable nodes in stability. Here, the first 75% of descending order of centralities based on shortest path distance have been considered as probabilistic failures in stability. In this research, probabilistic failures in stability are considered into four parts which are

5. Probabilistic Failures in stability of static scale-free

Probabilistic Failures in Stability of Static Scale-Free Networks 0 : {1: 0.053442589156874866, 2: 0.03483817769532055, 0: 0.023036487322201607, 25: 0.008091115233972376, 49: 0.0037621109049680475, 11: 0.002473716759431045, 15: 0.0017522160379303236, 9: 0.0010307153164296021, 18: 0.0009276437847866418, 30: 0.0009276437847866418, 57: 0.0009276437847866418, 51: 0.00010307153164296021}

Probabilistic Failures in Stability of Static Scale-Free Networks 1 : {0: 0.03644437573009001, 1: 0.027923795780938638, 2: 0.012712155569298425, 4: 0.0077303648732220155, 3: 0.005918023775166634, 37: 0.0041228612657184084, 21: 0.0020278609221466364 3, 24: 0.0013742870885728026, 25: 0.0011337868480725624, 20: 0.0004380540094825809, 18: 0.00020614306328592042}

Probabilistic Failures in Stability of Static Scale-Free Networks 2 : {1: 0.07647564076135505, 0: 0.03505462791177077, 2: 0.017285095856524428, 4: 0.0064350992922421486, 10: 0.0052051123479694905, 17: 0.0032982890125747267, 19: 0.003229574658146087, 38: 0.0031952174809317664, 3: 0.0018552875695732837, 16: 0.0018552875695732837, 56: 0.0017522160379303236, 44: 0.0016491445062873633, 45: 0.0016491445062873633, 22: 0.001398337112622827, 24: 0.0012712155569298425, 23: 0.0006424792139077853, 20: 0.0006012506012506012, 27: 0.00041228612657184083, 12: 0.00020614306328592042}

Probabilistic Failures in Stability of Static Scale-Free Networks 3 : {0: 0.07403112760255619, 1: 0.051948051948051945, 2: 0.020038236102521 76, 10: 0.008391740534597675, 35: 0.006802721088435374, 32: 0.004638218923933206, 8: 0.00460386174671889, 14: 0.003890950319521748, 11: 0.0038136466707895276, 17: 0.003401360544217687, 19: 0.0003092145949288806, 3: 0.0001546072974644403, 69: 0.00010307153164296021}

Probabilistic Failures in Stability of Static Scale-Free Networks 4 : {0: 0.05952380952380952, 1: 0.038342609771181195, 11: 0.018295196866625438, 2: 0.010410224695938981, 5: 0.008537758537758539, 8: 0.0048443619872191295, 18: 0.002508073936645365, 22: 0.002061430632859 2042, 29: 0.0013399299113584827, 23: 0.0012368583797155224, 48: 0.0012368583797155224, 13: 0.0011337868480725624, 3: 0.0010478939050367622, 9: 0.0005668934240362812, 39: 0.0003607503607503607 5, 21: 0.0002748574177145606}

Probabilistic Failures in Stability of Static Scale-Free Networks 5 : {1: 0.0682161753590325, 2: 0.03396550539407682, 15: 0.021438878581735724, 14: 0.018140589569160998, 0: 0.016718202432488144, 9: 0.012320483749055177, 63: 0.0045351473922902496, 26: 0.003401360544217687, 35: 0.0031127602556173983, 16: 0.0027004741290455572, 72: 0.002473716759431045, 44: 0.0023706452277880846, 27: 0.0011337868480725624, 21: 0.0007730364873222015, 23: 0.0002576782910740053, 19: 0.00010307153164296021, 8: 5.1535765821480104e-05}

Probabilistic Failures in Stability of Static Scale-Free Networks 6 : {2: 0.044990723562152134, 0: 0.010358688930117502, 9: 0.008245722531436817, 1: 0.002473716759431045, 3: 0.0012368583797155224, 8: 0.0008245722531436817, 18: 0.0008245722531436817, 4: 0.0007215007215007215, 44: 0.0007215007215007215, 69: 0.0007215007215007215, 15: 0.00010307153164296021}

Probabilistic Failures in Stability of Static Scale-Free Networks 7 : {2: 0.07767298838727409, 0: 0.040610183467326325, 4: 0.02971036899608328, 1: 0.02868824297395726, 9: 0.01021267092695664, 10: 0.0017006802721088435, 11: 0.0016749123891981034, 8: 0.0016491445062873633, 19: 0.0016491445062873633, 20: 0.0016491445062873633, 90: 0.0016491445062873633, 23: 0.001546072974644403, 46: 0.001546072974644403}

Probabilistic Failures in Stability of Static Scale-Free Networks 8 : {0: 0.12299869442726587, 3: 0.023912595341166767, 12: 0.023603380746237888, 8: 0.020923520923520924, 2: 0.020511234796949083, 6: 0.01333058475915 6189, 1: 0.012677798392084105, 4: 0.00946540232254518, 37: 0.009070294784580499, 14: 0.007266542980828694, 27: 0.005926613069470212, 16: 0.004019789734075448, 21: 0.0032467532467532465, 31: 0.0018552875695732837, 63: 0.0018552875695732837, 10: 0.00010307153164296021, 42: 0.00010307153164296021}

Probabilistic Failures in Stability of Static Scale-Free Networks 9 : {1: 0.06498660070088641, 0: 0.0605201676630248, 3: 0.023414416271559126, 2: 0.013811585240156668, 37: 0.007146292860578574, 26: 0.003092145949288806, 20: 0.0017006802721088435, 67: 0.0016491445062873633, 23: 0.0006699649556792414, 30: 0.0001546072974644403, 14: 0.00013742870885728026, 43: 0.00010307153164296021, 75: 0.00010307153164296021}

6. Probabilistic Failures in stability of static small world

Probabilistic Failures in Stability of Static Small World Networks 0 : {15: 0.1021799628942486, 9: 0.09210916870355645, 47: 0.08868612019122224, 19: 0.08349376908050377, 0: 0.07638891956493998, 33: 0.07497036693465264, 32: 0.06954717853952548, 11: 0.06946401576503616, 25: 0.06672188355861827, 2: 0.06297974276290602, 23: 0.062079738100146255, 36: 0.05546782204945469, 6: 0.05341856980887592, 4: 0.05135891838697961, 7: 0.05134410185430593, 34: 0.04907082241265916, 48: 0.04774764162519265, 8: 0.04646437038018671, 41: 0.046071226109491416, 5: 0.04485173037468956, 29: 0.04398899711399711, 45: 0.04361974948709641, 24: 0.04195370247666165, 17: 0.040667915795466816, 12: 0.04039756775726163, 37: 0.039299886621315186, 31: 0.038355463036585484, 35: 0.035673118453730696, 27: 0.035570905851518085, 40: 0.03191205053449951, 38: 0.03165087463556851, 42: 0.031061710398445096, 30: 0.029977741457333288, 18: 0.028409888486419104, 26: 0.02713428625418421, 1: 0.027042840945902178, 16: 0.025222401370360553, 46: 0.019042557499190153}

Probabilistic Failures in Stability of Static Small World Networks 1 : {36: 0.12283239955433825, 40: 0.10981986409282327, 15: 0.09498409753511795, 14: 0.07379473795290123, 27: 0.07204939335041376, 9: 0.06326368029174151, 29: 0.06278010302244996, 7: 0.06188626424595812, 30: 0.05937263794406651, 12: 0.059140236181052505, 26: 0.057792545228769705, 11: 0.057682448881428455, 8: 0.05641196954972463, 18: 0.05479632329122125, 43: 0.05222843105496167, 24: 0.05022059148334658, 22: 0.049520901434166736, 48: 0.044314746100460385, 3: 0.04403985800669473, 34: 0.041091337328582236, 21: 0.040524229172188356, 10: 0.03876004947433519, 19: 0.03866452425125894, 33: 0.0350291054372687, 0: 0.03298104956268221, 46: 0.032388173523377604, 41: 0.03227283157130096, 39: 0.03163338928645051, 31: 0.03149099105731759, 13: 0.029458886729294895, 37: 0.024311445357363723, 1: 0.023907441764584623, 6: 0.023883759853147613, 35: 0.022030588931099132, 28: 0.02015680370272207, 4: 0.01958083016756486, 2: 0.017327060694407627, 45: 0.01714490630306956}

Probabilistic Failures in Stability of Static Small World Networks 2 : {21: 0.10906587743322438, 26: 0.09440278636707206, 22: 0.09395031461357993, 7: 0.09066432669748997, 40: 0.08420585225432163, 11: 0.0838189658980475, 6: 0.07795069867773946, 38: 0.07596003769473157, 10: 0.07230894341863732, 2: 0.07043080219100627, 33: 0.06600268354094885, 49: 0.06348902656300615, 16: 0.05531536207556616, 36: 0.05317905120201037, 42: 0.05234855847100745, 28: 0.052247726291093644, 29: 0.05082570726703379, 30: 0.046972917955060804, 46: 0.045843149669680276, 37: 0.04453592042877757, 1: 0.0414378846776806, 35: 0.038608816542490015, 45: 0.03823732956386018, 47: 0.03766534391534392, 34: 0.03704685337338399, 31: 0.03565354713313896, 44: 0.03258965382689872, 24: 0.03256097171913498, 41: 0.03246918897429101, 13: 0.03176738227758636, 20: 0.03144795255764644, 9: 0.029818594104308385, 39: 0.029606377182907796, 18: 0.028781651549508697, 5: 0.02702664399092971, 17: 0.02543943443178137, 14: 0.02193923565097035, 0: 0.020126741172659536}

Probabilistic Failures in Stability of Static Small World Networks 3 : {35: 0.12400591188856494, 34: 0.10574789439585355, 39: 0.09778877820969654, 11: 0.08005884893639997, 28: 0.07926249865025375, 49: 0.07551830255911887, 25: 0.07240005129035744, 45: 0.06743568459129681, 43: 0.06436568675089084, 22: 0.06323729888780907, 18: 0.06195537738905085, 24: 0.06164662293488822, 33: 0.058003320375769364, 13: 0.057678706403196195, 5: 0.05269443094698196 6, 8: 0.0519470089623151, 9: 0.04858884029802396, 19: 0.0479119425547997, 6: 0.04163326044703595, 20: 0.03901374041680163, 10: 0.03759718172983479, 15: 0.03689362649821833, 42: 0.035947791815138755, 26: 0.03486428301479322, 40: 0.03214015765036174, 17: 0.03092707321023648, 46: 0.028698642155274807, 21: 0.026986151603498543, 38: 0.026323088759313242, 23: 0.025581403196199115, 1: 0.02520246193715582, 0: 0.02448709642587194, 30: 0.023634731670445955, 37: 0.022597789115646263, 7: 0.021880061548428895, 12: 0.02072940287226001, 48: 0.02031908001295757, 4: 0.01921937425763956}

Probabilistic Failures in Stability of Static Small World Networks 4 : {31: 0.10697683835438936, 25: 0.10536564012329318, 39: 0.08257324213956865, 6: 0.07830249163157328, 27: 0.07789824263038549, 12: 0.07702731886405355, 40: 0.07281285890979769, 4: 0.07243146356666764, 5: 0.07165839640329437, 21: 0.06798576753933896, 33: 0.06776603498542273, 47: 0.06679590486988445, 19: 0.06557472195227296, 43: 0.06086413822383209, 23: 0.051732061872368, 34: 0.05150969117805853, 35: 0.049514766223949895, 1: 0.04711163505806363, 26: 0.04555967228161107, 18: 0.04270732102364755, 2: 0.04087504049238743, 41: 0.04084905787711911, 17: 0.0403344671201814, 3: 0.03688221500721501, 37: 0.031772075713402244, 36: 0.02918289429769021, 24: 0.029112400732298704, 14: 0.028036591620775302, 46: 0.026160106899902814, 11: 0.02471250404923874, 32: 0.024428382464096746, 15: 0.024212760501025814, 42: 0.023135448262999285, 10: 0.021370532339920097, 20: 0.017709345643019108, 49: 0.017558174063276102, 8: 0.01753084170176007, 16: 0.016520894071914483}

Probabilistic Failures in Stability of Static Small World Networks 5 : {12: 0.1543316259515239, 46: 0.1176605282472629, 42: 0.09235851308555387, 13: 0.08484406749712871, 33: 0.084040272369354, 26: 0.07782333171363782, 21: 0.07268389679103962, 22: 0.07001252951508052, 6: 0.06971605963952902, 2: 0.06904798716023205, 30: 0.06374213466050199, 44: 0.058154838830859236, 24: 0.05735938758897944, 43: 0.05564200066751087, 38: 0.05391545812464181, 11: 0.04830259026687597, 16: 0.0447922896264733, 34: 0.04429458013641686, 0: 0.0419460641399417, 41: 0.04093749693239489, 31: 0.03858096976719426, 45: 0.037439997644079265, 10: 0.03550024710738995, 4: 0.03447039072039073, 39: 0.03338769129585455, 29: 0.032137826270479335, 1: 0.03198999838030451, 3: 0.031010705564276993, 15: 0.028189101148284823, 8: 0.027678571428571438, 49: 0.027627955944282478, 20: 0.02636877247846636, 40: 0.025867886837274593, 35: 0.023440316297459154, 32: 0.022862370056247604, 18: 0.02063351897535572, 36: 0.01947543669482445, 28: 0.018672420266808026}

Probabilistic Failures in Stability of Static Small World Networks 6 : {6: 0.1057513466314487, 26: 0.09878010311683781, 49: 0.09719670682680887, 48: 0.09460837018234976, 32: 0.0928926610049059, 47: 0.08523497949518358, 28: 0.0741739812530629, 39: 0.07248705801001719, 35: 0.06589562412266493, 5: 0.06348267898012794, 21: 0.06032252186588919, 43: 0.05998122720061494, 14: 0.05932549357294255, 12: 0.059012845619988465, 13: 0.057869782806007276, 34: 0.05595575531800021, 22: 0.05529884420190542, 25: 0.04844471241154916, 24: 0.04573301084780675, 40: 0.044786605118237774, 2: 0.04381616824218866, 23: 0.04206007522334053, 37: 0.04037755517347354, 42: 0.038944112859929186, 33: 0.035956227729186906, 46: 0.03531922537024578, 15: 0.03453517854028058, 41: 0.03319052451705512, 17: 0.03178411016421221, 19: 0.03175041842133678, 30: 0.02956507541966725, 9: 0.029563751630078162, 18: 0.028176238444095586, 10: 0.02668515819026024, 0: 0.02614456358078807, 8: 0.025051238444095587, 36: 0.021226446927977535, 44: 0.018349462800993414}

Probabilistic Failures in Stability of Static Small World Networks 7 : {20: 0.1159384266527124, 33: 0.11442969587102239, 28: 0.09558926287242614, 12: 0.08707607585158604, 24: 0.08547652272142066, 30: 0.07954724319520239, 15: 0.07946361084116187, 32: 0.07751727675197062, 22: 0.07217087995149218, 19: 0.07052629201863898, 38: 0.06699276016878056, 11: 0.06657376778550247, 46: 0.06568843807364215, 43: 0.0422851878846777, 6: 0.04197329267482328, 7: 0.03934388315765866, 0: 0.03913083090379008, 3: 0.03842548466272956, 48: 0.03778007234639887, 29: 0.03708962522738034, 21: 0.035848715249225456, 23: 0.034817041896123536, 41: 0.03418772270813087, 17: 0.03397108843537416, 2: 0.03003421606737932 7, 4: 0.03000080984774862, 16: 0.027680388394674103, 49: 0.025841566785444338, 45: 0.024580228916963615, 14: 0.02395934564301911, 26: 0.022310293164885007, 10: 0.02216807663236235, 44: 0.022164468656815595, 37: 0.021323291221250405, 47: 0.019239957887917068, 5: 0.01833456367895144, 13: 0.01743737177410647, 18: 0.016344362836709776}

Probabilistic Failures in Stability of Static Small World Networks 8 : {16: 0.10528916472794024, 41: 0.10000101230968576, 28: 0.0954206484180974, 11: 0.08498763141620284, 45: 0.08211966604823749, 25: 0.0777487582334521, 17: 0.0774401264834938, 46: 0.0720208952989565, 49: 0.06366504574412737, 39: 0.06315201946579498, 3: 0.06098524727351258, 23: 0.060472067614924775, 43: 0.05623374169292536, 15: 0.05247813411078717, 14: 0.05191124068675089, 24: 0.048506168340352006, 6: 0.047292409027102904, 26: 0.04608641075477811, 13: 0.04600981265522081, 22: 0.04556338408379224, 34: 0.04494826790745157, 0: 0.04113452920850880 4, 10: 0.0406955242414426, 9: 0.03930801577485251, 33: 0.037497975380628455, 47: 0.035266967537375696, 8: 0.03329857736745491, 37: 0.027921525753158403, 2: 0.026944646906381608, 30: 0.026272473275024296, 19: 0.025977891156462592, 5: 0.024153034229564843, 42: 0.02398364107547781, 1: 0.021775793650793653, 27: 0.021228471547349095, 40: 0.02122610949141561, 35: 0.02117089259946403, 44: 0.02092054534656575}

Probabilistic Failures in Stability of Static Small World Networks 9 : {22: 0.10562523016754712, 35: 0.09063179509232931, 6: 0.08137153851439567, 20: 0.08041399640664346, 1: 0.07878878324556599, 33:

0.0753635629798595, 4: 0.07377440042506068, 13: 0.0698650508599488, 47: 0.06677685726305173, 14: 0.06442054496901435, 31: 0.05839013274862613, 15: 0.05682342103785679, 9: 0.05627838478603785, 41: 0.054969677126039666, 43: 0.05050465393727699, 48: 0.04991299633406476, 24: 0.049386304360794143, 27: 0.048280708803667985, 38: 0.047962558039088644, 34: 0.043932174656964584, 17: 0.04313804345692099, 3: 0.04299857333320719, 45: 0.041822669238635625, 30: 0.03987473055600306, 28: 0.036389833711262275, 39: 0.036105767093162044, 11: 0.036048347910592804, 26: 0.03247870798891207, 8: 0.03217805201223569, 2: 0.03202455446603106, 49: 0.029183211885042616, 42: 0.02864270708283313, 16: 0.024990478625682707, 12: 0.02482864357864357, 36: 0.022401135778986923, 0: 0.021660886576852965, 32: 0.021440667231483558, 25: 0.021384346001692946}

7. Probabilistic Failures in stability of dynamic scale-free

Probabilistic Failures in Stability of dynamic Scale-free Networks Phases:

Probabilistic Failures in Stability of dynamic Scale-free Network Phase 0 : {2: 0.07010582010582012, 0: 0.034133855562426985, 3: 0.019669483955198238, 1: 0.016869374012231157, 33: 0.014807943379371953, 24: 0.0055658627087198514}
Probabilistic Failures in Stability of dynamic Scale-free Network Phase 1 : {2: 0.013553906411049267, 23: 0.007266542980828694, 54: 0.003968253968253968, 33: 0.003452896310039167, 3: 0.002989074417645846, 32: 0.002473716759431045}
Probabilistic Failures in Stability of dynamic Scale-free Network Phase 2 : {2: 0.0020614306328592042, 24: 0.0011337868480725624, 78: 0.0011337868480725624, 1: 0.0009276437847866418, 41: 0.0009276437847866418, 61: 0.0008245722531436817}
Probabilistic Failures in Stability of dynamic Scale-free Network Phase 3 : {2: 0.008829794544080258, 3: 0.006527863670720814, 22: 0.005359719645433932, 32: 0.003916718202432488, 0: 0.0034700748986463267, 23: 0.0032982890125747267}
Probabilistic Failures in Stability of dynamic Scale-free Network Phase 4 : {2: 0.04866694152408438, 3: 0.03483817769532055, 10: 0.030646602075173505, 4: 0.028585171442314303, 22: 0.026197347625919052, 1: 0.011767333195904624}
Probabilistic Failures in Stability of dynamic Scale-free Network Phase 5 : {2: 0.006184291898577612, 9: 0.005256648113790971, 24: 0.004225932797361369, 10: 0.0018552875695732837, 1: 0.001443001443001443, 4: 0.0013399299113584827}
Probabilistic Failures in Stability of dynamic Scale-free Network Phase 6 : {2: 0.023070844499415925, 10: 0.019428983714698, 4: 0.018810554524840237, 3: 0.011320689892118464, 24: 0.007060399917542775, 33: 0.004225932797361369}
Probabilistic Failures in Stability of dynamic Scale-free Network Phase 7 : {0: 0.028550814265099977, 2: 0.023191094619666047, 68: 0.014739229024943309, 48: 0.013090084518655946, 27: 0.011956297670583383, 33: 0.008761080189651618}
Probabilistic Failures in Stability of dynamic Scale-free Network Phase 8 : {2: 0.023088023088023088, 0: 0.010616367759224902, 13: 0.009379509379509378, 24: 0.009276437847866418, 3: 0.004019789734075448, 4: 0.004019789734075448}
Probabilistic Failures in Stability of dynamic Scale-free Network Phase 9 : {10: 0.013605442176870748, 95: 0.007318078746650174, 2: 0.006802721088435374, 3: 0.005359719645433931, 33: 0.004741290455576169, 5: 0.003401360544217687}
Probabilistic Failures in Stability of dynamic Scale-free Network Phase 10 : {2: 0.04504225932797361, 0: 0.03452896310039167, 1: 0.03050917336631622, 4: 0.02329416615130901, 10: 0.020923520923520924, 3: 0.01948051948051948}

8. Probabilistic Failures in stability of dynamic small world

Probabilistic Failures in Stability of Dynamic Small World Networks phase 0 : {7: 0.10704871234207972, 46: 0.08913521757909511, 12: 0.08645158460209482, 30: 0.08298543623798724, 9: 0.07906813519058419, 49: 0.0769115781233128, 10: 0.07582739444984342, 34: 0.07539176384839652, 39: 0.06776468523917503, 25: 0.061044636108411626, 45: 0.05969927653601124, 3: 0.05523566569484936, 44: 0.05022709480617644, 15: 0.048065476190476186, 47: 0.04741861030126336, 31: 0.04702279721412373, 21: 0.046058066083576286, 42: 0.04597573156246626, 41: 0.04571793002915452, 43: 0.04369398553072022, 0: 0.04185394395853579, 6: 0.0417655355793111, 35: 0.038859869344563217, 22: 0.03694964096749811, 4: 0.03455417881438289, 27: 0.03378144908757154, 14: 0.033695740200842234, 48: 0.032772513767411734, 2: 0.032663184321347585, 37: 0.03143322805312601, 26: 0.029348882410106893, 8: 0.02901751970629521, 16: 0.02846243656192636, 24: 0.028365254832091565, 13: 0.027535498326314647, 29: 0.026216796242306443, 19: 0.023339137242198476, 18: 0.02295749649065976}

Probabilistic Failures in Stability of Dynamic Small World Networks phase 1 : {20: 0.35785147392290245, 12: 0.27706916099773243, 22: 0.25864512471655327, 35: 0.2232142857142857, 21: 0.19834183673469388, 30: 0.16808390022675737, 0: 0.1585175736961451, 46: 0.15348639455782312, 32: 0.15192743764172334, 42: 0.13151927437641722, 7: 0.11096938775510204, 14: 0.0794359410430839, 1: 0.07490079365079366, 48: 0.06831065759637188, 25: 0.0679563492063492, 18: 0.06738945578231292, 16: 0.06441326530612244, 4: 0.05442176870748299, 3: 0.05406746031746031, 34: 0.05144557823129251, 41: 0.050028344671201805, 49: 0.04988662131519273, 23: 0.049036281179138316, 15: 0.046414399092970515, 39: 0.045068027210884355, 37: 0.04265873015873016, 6: 0.03741496598639456, 8: 0.03741496598639456, 26: 0.03741496598639456, 43: 0.03330498866213152, 44: 0.03316326530612245, 40: 0.022817460317460323, 19: 0.013888888888888888, 9: 0.005881519274376417}

Probabilistic Failures in Stability of Dynamic Small World Networks phase 2 : {46: 0.3695861678004535, 22: 0.3557823129251701, 37: 0.3395975056689342, 39: 0.23982426303854876, 24: 0.2273809523809523, 1: 0.18197278911564624, 21: 0.17859977324263035, 45: 0.16587301587301587, 29: 0.1525510204081632, 43: 0.146797052154195, 33: 0.1244897959183673, 42: 0.12227891156462585, 20: 0.11394557823129252, 0: 0.11224489795918367, 6: 0.10856009070294784, 34: 0.08622448979591837, 48: 0.07738095238095238, 41: 0.07653061224489796, 35: 0.07375283446712018, 7: 0.07327097505668935, 12: 0.04767573696145123, 4: 0.044472789115646255, 18: 0.0391156462585034, 25: 0.0391156462585034, 28: 0.0391156462585034, 32: 0.0391156462585034, 36: 0.0391156462585034, 2: 0.03251133786848072, 10: 0.028684807256235827, 14: 0.02585034013605443, 15: 0.016921768707482996, 13: 0.016439909297052156, 9: 0.015221088435374147}

Probabilistic Failures in Stability of Dynamic Small World Networks phase 3 : {30: 0.2974773242630385, 4: 0.18239795918367346, 46: 0.17443310657596367, 45: 0.16621315192743763, 29: 0.16203231292517004, 32: 0.15821995464852606, 48: 0.13241213151927436, 7: 0.11933106575963717, 49: 0.11748866213151929, 41: 0.10371315192743764, 20: 0.10209750566893423, 12: 0.08860544217687073, 40: 0.06695011337868481, 37: 0.0665107709750567, 33: 0.06458333333333334, 8: 0.06361961451247165, 6: 0.05984977324263039, 31: 0.05555555555555555, 21: 0.05426587301587301, 36: 0.054138321995464846, 0: 0.0536422902494331, 35: 0.050779478458049886, 2: 0.04988662131519274, 42: 0.04889455782312925, 44: 0.04251700680272108, 22: 0.0423611111111111, 14: 0.042120181405895685, 39: 0.03982426303854875, 11: 0.0391156462585034, 5: 0.036564625850340135, 17: 0.036564625850340135, 1: 0.034438775510204085, 15: 0.032454648526077094, 16: 0.02806122448979592, 26: 0.027140022675736966, 38: 0.026403061224489793, 18: 0.025368480725623584, 9: 0.020691609977324263}

Probabilistic Failures in Stability of Dynamic Small World Networks phase 4 : {46: 0.292375283446712, 48: 0.24539399092970518, 7: 0.22356859410430835, 5: 0.21867913832199548, 18: 0.20337301587301587, 37: 0.17928004535147393, 39: 0.1677295918367347, 47: 0.161281179138322, 35: 0.15979308390022678, 0: 0.15554138321995462, 19: 0.12188208616780045, 44: 0.11663832199546487, 3: 0.10126133786848075, 45: 0.09828514739229025, 20: 0.08347505668934242, 27: 0.07653061224489796, 10: 0.07497165532879818, 17: 0.07405045351473924, 12: 0.07043650793650794, 22: 0.06490929705215419, 32: 0.06292517006802721, 6: 0.0570436507936508, 21: 0.05590986394557823, 2: 0.055839002267573684, 16: 0.045422335600907006, 4: 0.04159580498866211, 23: 0.04109977324263039, 8: 0.0391156462585034, 29: 0.0391156462585034, 43: 0.030895691609977318, 25: 0.029195011337868483, 36: 0.0274234693877551, 15: 0.0243764172335601, 34: 0.0233843537414966, 42: 0.021683673469387755, 14: 0.02161281179138323, 9: 0.017573696145124718, 40: 0.013251133786848071, 41: 0.013251133786848071}

Probabilistic Failures in Stability of Dynamic Small World Networks phase 5 : {42: 0.21299603174603168, 39: 0.16142290249433106, 15: 0.14750566893424036, 40: 0.1386054421768707, 43: 0.1370464852607709, 25: 0.11291099773242627, 6: 0.10873015873015875, 38: 0.09410430839002268, 37: 0.0885345804988662, 34: 0.08358843537414967, 10: 0.0802721088435374, 8: 0.0790107709750567, 21: 0.07867063492063492, 16: 0.07209467120181404, 3: 0.07101757369614513, 33: 0.06849489795918366, 20: 0.06829648526077099, 35: 0.06607142857142857, 47: 0.06257086167800453, 0: 0.061507936507936505, 19: 0.05861678004535147, 9: 0.04828514739229025, 45: 0.04299886621315194, 12: 0.04027777777777779, 18: 0.038321995464852605, 7: 0.036564625850340135, 29: 0.036564625850340135, 1: 0.033460884353741505, 2: 0.03312074829931972, 22: 0.027437641723356002, 48: 0.027338435374149664, 4: 0.027281746031746032, 46: 0.022831632653061224, 41: 0.017857142857142856, 13: 0.009141156462585032, 14: 0.009141156462585032, 30: 0.005810657596371882}

For probabilistic failures in sustainability, network algorithms based on shortest path distance might be considered also. At this point, nodes with very high measurement scores can be identified as the failure probable nodes in sustainability. Here, the first 25% of descending order of centralities based on shortest path distance have been considered as probabilistic failures in sustainability. In this research, probabilistic failures in sustainability are considered into four parts which are

9. Probabilistic Failures in sustainability of static scale-free

Probabilistic Failures in Sustainability of Static Scale-Free Networks 0 : {1: 0.053442589156874866, 2: 0.03483817769532055, 0: 0.023036487322201607, 25: 0.008091115233972376, 49: 0.0037621109049680475, 11: 0.002473716759431045, 15: 0.0017522160379303236, 9: 0.0010307153164296021, 18: 0.0009276437847866418, 30: 0.0009276437847866418, 57: 0.0009276437847866418, 51: 0.0010307153164296021}

Probabilistic Failures in Sustainability of Static Scale-Free Networks 1 : {0: 0.03644437573009001, 1: 0.027923795780938638, 2: 0.012712155569298425, 4: 0.0077303648732220155, 3: 0.005918023775166634, 37: 0.0041228612657184084, 21: 0.0020786092214663643, 24: 0.0013742870885728026, 25: 0.0011337868480725624, 20: 0.0004380540094825809, 18: 0.00020614306328592042}

Probabilistic Failures in Sustainability of Static Scale-Free Networks 2 : {1: 0.07647564076135505, 0: 0.03505462791177077, 2: 0.017285095856524428, 4: 0.0064350992922421486, 10: 0.0052051123479694905, 17: 0.0032982890125747267, 19: 0.003229574658146087, 38: 0.0031952174809317664, 3: 0.0018552875695732837, 16: 0.0018552875695732837, 56: 0.0017522160379303236, 44: 0.0016491445062873633, 45: 0.0016491445062873633, 22: 0.001398337112622827, 24: 0.0012712155569298425, 23: 0.0006424792139077853, 20: 0.0006012506012506012, 27: 0.00041228612657184083, 12: 0.00020614306328592042}

Probabilistic Failures in Sustainability of Static Scale-Free Networks 3 : {0: 0.07403112760255619, 1: 0.051948051948051945, 2: 0.020038823610252176, 10: 0.008391740534597675, 35: 0.006802721088435374, 32: 0.004638218923933206, 8: 0.00460386174671889, 14: 0.003890950319521748, 11: 0.0038136466707895276, 17: 0.003401360544217687, 19: 0.0003092145949288806, 3: 0.0001546072974644403, 69: 0.0010307153164296021}

Probabilistic Failures in Sustainability of Static Scale-Free Networks 4 : {0: 0.05952380952380952, 1: 0.038342609771181195, 11: 0.018295196866625438, 2: 0.010410224695938981, 5: 0.008537758537758539, 8: 0.0048443619872191295, 18: 0.002508073936645365, 22: 0.0020614306328592042, 29: 0.0013399299113584827, 23: 0.0012368583797155224, 48: 0.0012368583797155224, 13: 0.001137868480725624, 3: 0.0010478939050367622, 9: 0.0005668934240362812, 39: 0.00036075036075036075, 21: 0.0002748574177145606}

Probabilistic Failures in Sustainability of Static Scale-Free Networks 5 : {1: 0.0682161753590325, 2: 0.03396550539407682, 15: 0.021438878581735724, 14: 0.018140589569160998, 0: 0.016718202432488144, 9: 0.012320483749055177, 63: 0.0045351473922902496, 26: 0.003401360544217687, 35: 0.0031127602556173983, 16: 0.0027004741290455572, 72: 0.002473716759431045, 44: 0.0023706452277880846, 27: 0.0011337868480725624, 21: 0.0007730364873222015, 23: 0.00025767882910740053, 19: 0.00010307153164296021, 8: 5.1535765821480104e-05}

Probabilistic Failures in Sustainability of Static Scale-Free Networks 6 : {2: 0.044990723562152134, 0: 0.010358688930117502, 9: 0.008245722531436817, 1: 0.002473716759431045, 3: 0.0012368583797155224, 8: 0.0008245722531436817, 18: 0.0008245722531436817, 4: 0.0007215007215007215, 44: 0.0007215007215007215, 69: 0.0007215007215007215, 15: 0.00010307153164296021}

Probabilistic Failures in Sustainability of Static Scale-Free Networks 7 : {2: 0.07767298838727409, 0: 0.040610183467326325, 4: 0.02971036899608328, 1: 0.02868824297395726, 9: 0.01021267092695664, 10: 0.0017006802721088435, 11: 0.0016749123891981034, 8: 0.0016491445062873633, 19: 0.0016491445062873633, 20: 0.0016491445062873633, 90: 0.0016491445062873633, 23: 0.001546072974644403, 46: 0.001546072974644403}

Probabilistic Failures in Sustainability of Static Scale-Free Networks 8 : {0: 0.12299869442726587, 3: 0.023912595341166767, 12: 0.023603380746237888, 8: 0.020923520923520924, 2: 0.020511234796949083, 6: 0.013330584759156189, 1: 0.012677798392084105, 4: 0.00946540232254518, 37: 0.009070294784580499, 14: 0.007266542980828694, 27: 0.005926613069470212, 16: 0.004019789734075448, 21: 0.0032467532467532465, 31: 0.0018552875695732837, 63: 0.0018552875695732837, 10: 0.00010307153164296021, 42: 0.00010307153164296021}

Probabilistic Failures in Sustainability of Static Scale-Free Networks 9 : {1: 0.06498660070088641, 0: 0.0605201676630248, 3: 0.023414416271559126, 2: 0.013811585240156668, 37: 0.007146292860578574, 26: 0.003092145949288806, 20: 0.0017006802721088435, 67: 0.0016491445062873633, 23: 0.0006699649556792414, 30: 0.0001546072974644403, 14: 0.0001374287088572802, 43: 0.00010307153164296021, 75: 0.00010307153164296021}

10. Probabilistic Failures in sustainability of static small world

Probabilistic Failures in Sustainability of Static Small World Networks 0 : {15: 0.1021799628942486, 9: 0.09210916870355645, 47: 0.08868612019122224, 19: 0.08349376908050377, 0: 0.07638891956493998, 33: 0.07497036693465264, 32: 0.06954717853952548, 11: 0.06946401576503616, 25: 0.06672188355861827, 2: 0.06297974276290602, 23: 0.062079738100146255, 36: 0.05546782204945469}

Probabilistic Failures in Sustainability of Static Small World Networks 1 : {36: 0.12283239955433825, 40: 0.10981986409282327, 15: 0.09498409753511795, 14: 0.07379473795290123, 27: 0.07204939335041376, 9: 0.06326368029174151, 29: 0.06278010302244996, 7: 0.06188626424595812, 30: 0.05937263794406651, 12: 0.059140236181052505, 26: 0.057792545228769705, 11: 0.057682448881428455}

Probabilistic Failures in Sustainability of Static Small World Networks 2 : {21: 0.10906587743322438, 26: 0.09440278636707206, 22: 0.09395031461357993, 7: 0.09066432669748997, 40: 0.08420585225432163, 11: 0.0838189658980475, 6: 0.07795069867773946, 38: 0.07596003769473157, 10: 0.07230894341863732, 2: 0.07043080219100627, 33: 0.06600268354094885, 49: 0.06348902656300615}

Probabilistic Failures in Sustainability of Static Small World Networks 3 : {35: 0.12400591188856494, 34: 0.10574789439585355, 39: 0.09778877820969654, 11: 0.08005884893639997, 28: 0.07926249865025375, 49: 0.07551830255911887, 25: 0.07240005129035744, 45: 0.06743568459129681, 43: 0.06436568675089084, 22: 0.06323729888780907, 18: 0.06195537738905085, 24: 0.06164662293488822}

Probabilistic Failures in Sustainability of Static Small World Networks 4 : {31: 0.10697683835438936, 25: 0.10536564012329318, 39: 0.08257324213956865, 6: 0.07830249163157328, 27: 0.07789824263038549, 12: 0.07702731886405355, 40: 0.07281285890979769, 4: 0.07243146356666764, 5: 0.07165839640329437, 21: 0.06798576753933896, 33: 0.06776603498542273, 47: 0.06679590486988445}

Probabilistic Failures in Sustainability of Static Small World Networks 5 : {12: 0.1543316259515239, 46: 0.1176605282472629, 42: 0.09235851308555387, 13: 0.08484406749712871, 33: 0.084040272369354, 26: 0.07782333171363782, 21: 0.07268389679103962, 22: 0.07001252951508052, 6: 0.06971605963952902, 2: 0.06904798716023205, 30: 0.06374213466050199, 44: 0.058154838830859236} Probabilistic Failures in Sustainability of Static Small World Networks 6 : {6: 0.1057513466314487, 26: 0.09878010311683781, 49: 0.09719670682680887, 48: 0.09460837018234976, 32: 0.0928926610049059, 47: 0.08523497949518358, 28: 0.0741739812530629, 39: 0.07248705801001719, 35: 0.06589562412266493, 5: 0.06348267898012794, 21: 0.06032252186588919, 43: 0.05998122720061494}

Probabilistic Failures in Sustainability of Static Small World Networks 7 : {20: 0.1159384266527124, 33: 0.11442969587102239, 28: 0.09558926287242614, 12: 0.08707607585158604, 24: 0.08547652272142066, 30: 0.07954724319520239, 15: 0.07946361084116187, 32: 0.07751727675197062, 22: 0.07217087995149218, 19: 0.07052629201863898, 38: 0.06699276016878056, 11: 0.06657376778550247}

Probabilistic Failures in Sustainability of Static Small World Networks 8 : {16: 0.10528916472794024, 41: 0.10000101230968576, 28: 0.0954206484180974, 11: 0.08498763141620284, 45: 0.08211966604823749, 25: 0.0777487582334521, 17: 0.0774401264834938, 46: 0.0720208952989565, 49: 0.06366504574412737, 39: 0.06315201946579498, 3: 0.06098524727351258, 23: 0.060472067614924775}

Probabilistic Failures in Sustainability of Static Small World Networks 9 : {22: 0.10562523016754712, 35: 0.09063179509232931, 6: 0.08137153851439567, 20: 0.08041399640664346, 1: 0.07878878324556599, 33: 0.0753635629798595, 4: 0.07377440042506068, 13: 0.0698650508599488, 47: 0.06677685726305173, 14: 0.06442054496901435, 31: 0.05839013274862613, 15: 0.05682342103785679}

11. Probabilistic Failures in sustainability of dynamic scale-free

Probabilistic Failures in Sustainability of dynamic Scale-free Network Phase 0 : {2: 0.06901927437641722, 0: 0.056901927437641726, 1: 0.0381235827664399, 3: 0.015306122448979591, 11: 0.008361678004535146, 7: 0.003826530612244898, 32: 0.003826530612244898, 39: 0.0004251700680272, 26: 0.0002125850340136054}

Probabilistic Failures in Sustainability of dynamic Scale-free Network Phase 1 : {4: 0.13180272108843538, 1: 0.1254251700680272, 0: 0.11181972789115646, 2: 0.045068027210884355, 9: 0.04039115646258503, 19: 0.03954081632653061, 12: 0.02848639455782313, 21: 0.022959183673469385, 10: 0.012329931972789115, 11: 0.011904761904761904, 20: 0.007227891156462584, 18: 0.0055272108843537416}

Probabilistic Failures in Sustainability of dynamic Scale-free Network Phase 2 : {2: 0.10671768707482993, 1: 0.08319160997732426, 4: 0.052295918367346934, 9: 0.05102040816326531, 0: 0.04223356009070294, 12: 0.025510204081632654, 10: 0.01225907029478458, 24: 0.008503401360544217, 11: 0.00467687074829932, 40: 0.004251700680272108, 43: 0.004251700680272108, 31: 0.003401360544217687}

Probabilistic Failures in Sustainability of dynamic Scale-free Network Phase 3 : {4: 0.157312925170068, 2: 0.11309523809523807, 1: 0.1067176870748299, 9: 0.07482993197278912, 0: 0.06604308390022674, 3: 0.042658730158730146, 21: 0.02040816326530612, 23: 0.02040816326530612, 13: 0.017006802721088433, 22: 0.006944444444444445, 24: 0.005952380952380952, 39: 0.005952380952380952}

Probabilistic Failures in Sustainability of dynamic Scale-free Network Phase 4 : {4: 0.0858843537414966, 0: 0.06561791383219955, 2: 0.06547619047619048, 1: 0.045422335600907034, 25: 0.02040816326530612, 9: 0.018565759637188206, 10: 0.007653061224489796, 43: 0.00467687074829932, 12: 0.004251700680272108, 18: 0.003826530612244898, 45: 0.003826530612244898, 46: 0.002976190476190476}

Probabilistic Failures in Sustainability of dynamic Scale-free Network Phase 5 : {0: 0.07908163265306122, 2: 0.07397959183673469, 1: 0.052721088435374146, 4: 0.04251700680272109, 18: 0.006802721088435374, 9: 0.006377551020408163, 12: 0.003826530612244898, 39: 0.003401360544217687, 46: 0.003401360544217687, 47: 0.003401360544217687, 21: 0.002976190476190476, 17: 0.002125850340136054}

Probabilistic Failures in Sustainability of dynamic Scale-free Network Phase 6 : {1: 0.08577806122448979, 2: 0.04850481859410429, 0: 0.03571428571428571, 4: 0.01488095238095238, 18: 0.0077947845804988624, 20: 0.003401360544217687, 26: 0.003401360544217687, 10: 0.0031887755102040817, 7: 0.002976190476190476, 45: 0.002976190476190476, 46: 0.0025864512471655324, 12: 0.001062925170068027}

Probabilistic Failures in Sustainability of dynamic Scale-free Network Phase 7 : {2: 0.11862244897959183, 4: 0.0778061224489796, 1: 0.053784013605442174, 0: 0.05208333333333333, 18: 0.03231292517006803, 23: 0.027210884353741496, 15: 0.02040816326530612, 35: 0.008503401360544217, 10: 0.007653061224489796, 7: 0.00510204081632653, 24: 0.00467687074829932, 45: 0.003401360544217687}

Probabilistic Failures in Sustainability of dynamic Scale-free Network Phase 8 : {0: 0.13010204081632654, 2: 0.07674319727891156, 32: 0.04889455782312925, 1: 0.041666666666666664, 18: 0.04102891156462585, 4: 0.02593537414965986, 41: 0.008928571428571428, 31: 0.008503401360544217, 26: 0.00467687074829932, 7: 0.004251700680272108, 33: 0.004251700680272108, 48: 0.004251700680272108}

Probabilistic Failures in Sustainability of dynamic Scale-free Network Phase 9 : {2: 0.11494776482021381, 0: 0.07284580498866214, 1: 0.066756762228701, 4: 0.06475238905085845, 18: 0.04619169096209914, 12: 0.01913265306122449, 10: 0.013888888888888888, 30: 0.00992974570780693, 16: 0.009652372853903468, 5: 0.008442662779397474, 11: 0.008219954648526076, 39: 0.0073503806284418525}

Probabilistic Failures in sustainability of dynamic small world

Probabilistic Failures in Sustainability of Dynamic Small World Networks phase 0 : {48: 0.13198457579835135, 2: 0.12185776713837937, 32: 0.08855802408098325, 22: 0.0818111357652174, 47: 0.07995963078871242, 20: 0.07934854497354499, 39: 0.06742005868281378, 38: 0.06522909252501088, 44: 0.06017843645394664, 40: 0.0596465845190335, 5: 0.057475569592916526, 4: 0.056634521940644 4}

Probabilistic Failures in Sustainability of Dynamic Small World Networks phase 1 : {48: 0.078656462585034, 45: 0.0744047619047619, 2: 0.05385487528344669, 5: 0.047193877551020405, 0: 0.04209183673469387, 29: 0.03996598639455782, 18: 0.038407029478458046, 27: 0.03784013605442176, 26: 0.03628117913832199, 38: 0.03443877551020408, 7: 0.031037414965986394, 33: 0.029761904761904 76}

Probabilistic Failures in Sustainability of Dynamic Small World Networks phase 2 : {41: 0.20443594104308385, 16: 0.17410714285714282, 5: 0.17339852607709752, 7: 0.15787981859410424, 47: 0.15398242630385486, 42: 0.14936224489795918, 44: 0.1457766439909297, 6: 0.128047052154195, 34: 0.1276218820861678, 33: 0.12244897959183673, 48: 0.12093253968253968, 1: 0.1203231292517007}

Probabilistic Failures in Sustainability of Dynamic Small World Networks phase 3 : {5: 0.23204365079365077, 38: 0.22881235827664392, 22: 0.15158730158730158, 7: 0.13270975056689338, 36: 0.1293650793650793, 29: 0.12760770975056687, 42: 0.114328231292517, 40: 0.10918367346938776, 8: 0.10503117913832202, 24: 0.09693877551020408, 25: 0.08628117913832198, 14: 0.08416950113378684}

Probabilistic Failures in Sustainability of Dynamic Small World Networks phase 4 : {0: 0.17361111111111113, 36: 0.157312925170068, 2: 0.14980158730158732, 22: 0.14257369614512472, 32: 0.10983560090702948, 3: 0.07426303854875281, 8: 0.07256235827664398, 29: 0.07029478458049887, 34: 0.06292517006802721, 4: 0.05725623582766439, 18: 0.050028344671201805, 26: 0.049744897959183666}

Probabilistic Failures in Sustainability of Dynamic Small World Networks phase 5 : {32: 0.3120748299319728, 30: 0.2852891156462585, 5: 0.2274659863945578, 7: 0.18707482993197277, 31: 0.18112244897959184, 36: 0.16879251700680273, 23: 0.125, 22: 0.11777210884353741, 19: 0.09141156462585033, 21: 0.0820578231292517, 24: 0.07908163265306122, 2: 0.07610544217687075}

Randomly generated dynamic scale free networks:

Degree of a hub (node number 0) in 10 phases of dynamic scale free networks:

[38, 33, 21, 10, 19, 22, 17, 21, 17, 15]

Degree of a hub (node number 3) in 10 phases of dynamic scale free networks:

[32, 19, 16, 17, 17, 15, 20, 17, 19, 20]

Degree of a hub (node number 1) in 10 phases of dynamic scale free networks:

[17, 16, 14, 15, 13, 23, 17, 22, 20, 26]

Degree of a hub (node number 6) in 10 phases of dynamic scale free networks:

[16, 14, 14, 16, 14, 24, 17, 14, 18, 17]

Degree of a hub (node number 7) in 10 phases of dynamic scale free networks:

[9, 11, 13, 12, 10, 12, 10, 12, 12, 2]

Degree of a hub (node number 19) in 10 phases of dynamic scale free networks:

[9, 3, 5, 3, 4, 6, 1, 4, 4, 6]

Degree of a hub (node number 5) in 10 phases of dynamic scale free networks:

[8, 6, 2, 5, 3, 6, 5, 6, 9, 5]

Degree of a hub (node number 8) in 10 phases of dynamic scale free networks:

[8, 3, 2, 4, 6, 2, 3, 1, 5, 3]

Degree of a hub (node number 29) in 10 phases of dynamic scale free networks:

[8, 5, 4, 3, 7, 3, 5, 4, 1, 3]

Degree of a hub (node number 2) in 10 phases of dynamic scale free networks:

[7, 4, 2, 4, 3, 4, 4, 8, 2, 4]

Defense Mechanisms:

Here, four defense mechanisms have been introduced. For defense mechanisms, the air defense system's network securing strategies [5] have been followed where seven defense mechanisms as network securing strategies have been applied to small world networks. In this research paper, the first four defense mechanisms from air defense system's network securing strategies [5] have been applied to scale-free networks. Please, see [5] for elaborated descriptive defense mechanisms.

Defense mechanisms in phase 1 of dynamic scale-free network:

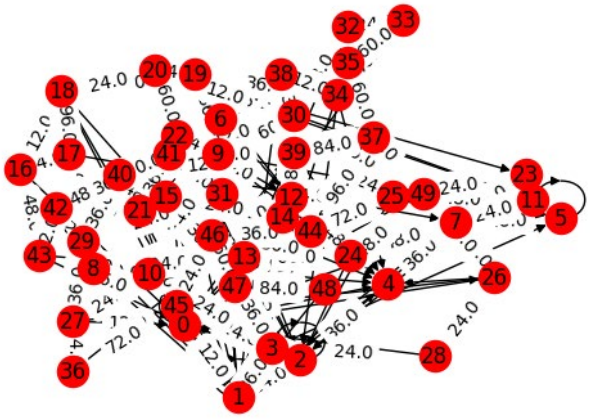


Figure 8: Unprotected nodes because of not placing any defense mechanism in any node

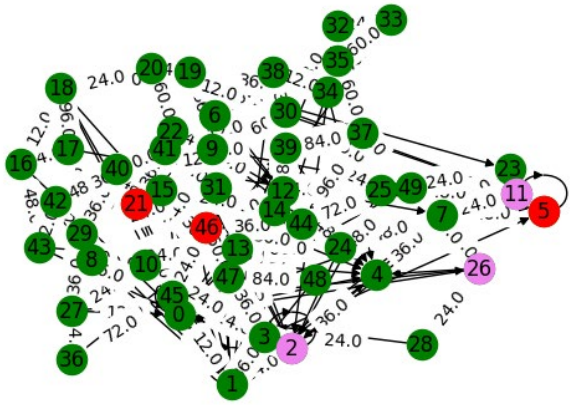


Figure 9: Placing defense mechanism 1 in places of node number 2, number 26 and number 11 and unprotected nodes for probabilistic failures are node number 5, node number 21, and node number 46

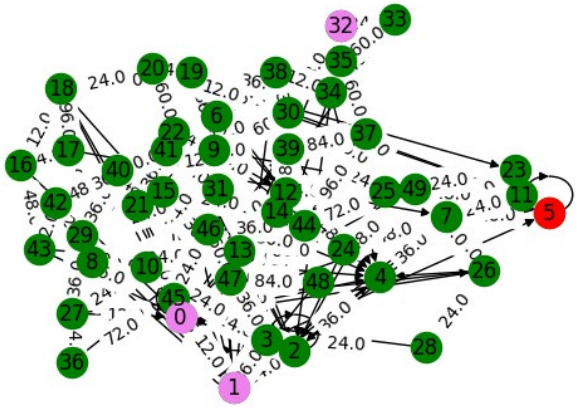


Figure 10: Placing defense mechanism 2 in places of node number 0, number 1 and number 32. Unprotected nodes for probabilistic failures is node number 5

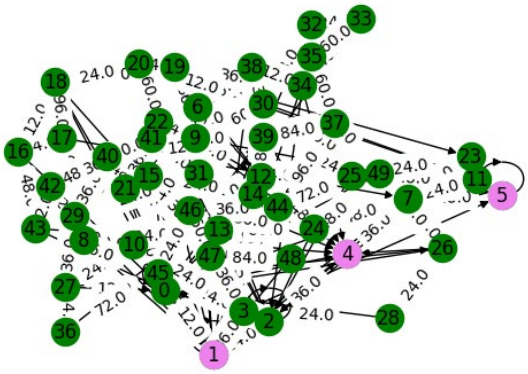


Figure 11: Placing defense mechanism 3 in places of node number 1, node number 4 and node number 5 and no unprotected Nodes for probabilistic failure

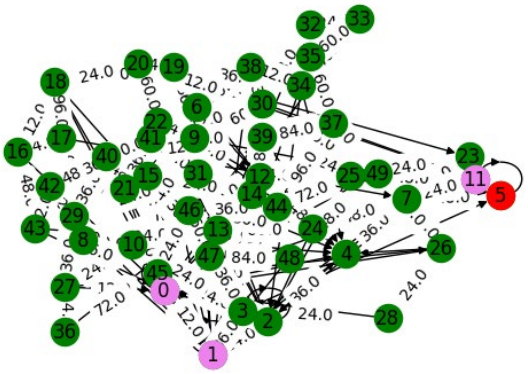


Figure 12: Placing defense mechanism 4 in places of node number 0, node number 1 and node number 11. Unprotected nodes for probabilistic failures is node number 5

5. Discussion and Conclusion

From the experiment, in histograms (Figure (1)), network measurement algorithm network_algorithm_1 (based on fraction of nodes that it is connected to) has few high centrality values which implies scale-free network has few hubs. On the other hand, in histograms (Figure (2)), network_algorithm_1 has no high centrality values which implies there is no hub that exists in the small world network. In histograms Figure(1), network_algorithm_2 (based on a ranking of nodes in graph which is based on the structure of links) has few centrality values between 0.4 to 0.6 which implies, in scale-free network, important nodes with centrality values from 0.4 to 0.6 exists which are hubs, meanwhile, in histograms Figure(2), in small world network, there is no centralities values from 0.4 to 0.6 exists which implies there is no node exists with high degree (hub) in the small world network. By observing the correlation result of static scale-free network, we can say, nodes which have high degrees (hubs) are usually close to the most other nodes and having tendency to be in the shortest path of any two nodes in network and also having tendency to connect with important nodes and these nodes are also most important nodes in scale-free networks. For static small world network, network_algorithm_1 and network_algorithm_3; network_algorithm_1 and network_algorithm_6; network_algorithm_3 and network_algorithm_6; network_algorithm_4 and network_algorithm_6 are highly correlated which implies nodes with high degree are having tendency to be in the shortest path of any two nodes and also these nodes are important nodes; and nodes which are close to most other nodes are also important nodes. From the experiment, we can also say that a few states of nodes in dynamic scale-free networks are very high, meanwhile, most of the states of nodes in dynamic small world networks are approximately similar to each other. In dynamic scale-free networks, hubs have the tendency of being hubs in every phase. But in dynamic small world networks, all nodes have a tendency to connect with nearest nodes. Robustness in any network depends on most of the nodes including nodes with low degree. Stability in dynamic scale-free networks is based on hubs, on the other hand, stability in dynamic small world networks is based on nodes with high betweenness centrality [15]. Sustainability in a dynamic scale-free network depends on hubs with very high degree, meanwhile, sustainability in a dynamic small world network is based on nodes with very high betweenness centrality. In defense mechanisms, in the case of the same number of nodes, short defense ranges work on a scale-free network but defense ranges need to be long for the same number of nodes in a small world network (see [5]).

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