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Maximum Ismail's Second Entropy(H_I^q)Formalism of Heavy-Tailed Queues with Hurst Exponent Heuristic Mean Queue Length Combined with Potential Applications of Hurst Exponent to Social Computing and Connected Health

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Abstract
The theory of Ismail's non-extensive maximum entropy solution (NME) is described in detail. It is used as an inductive inference technique for heavy-tailed queues with a non-robust mean queue length and a non-extensive "long-range" interaction. In our novel method, we substitute the non-robust mean queue length for the conventional Pollaczeck-Khinchin mean queue length. In other words, the new non-extensivity parameter q will be included in the resulting state probability function. Numerical portraits are provided to capture the influential effect of the derived formalism, $p_{q,I}(S_n)$, n=0,1,2,... on the stable $M/GE_{q,I}/1$ queue with heavy tails. More potentially, some applications of Hurst Exponent to social computing and connected health are provided. Conclusion with some challenging open problems and possible future research pathways are given.

Keywords: Queue, Noros Mean Queue Length, Stable M/G/1 Queue, Ismail's Second Entropy(H_I^q)

1. Introduction

The NME formalism, following the work of Rényi and Tsallis, is a mathematical technique used for inductive reasoning in physical systems with "long-range" interactions that exhibit non-extensive order. This approach builds upon Shannon's classical extensive maximum entropy (ME) formalism, which is used for analyzing "short-range" interactions in extensive systems[1-7]. The NME formalism provides a closed form expression to understand and reason about these complex physical systems. Stable steady state probabilities p(n) for $M / G / 1$ queue, where n = 0, 1, 2,... in accordance with the results of the first two EME applications to stable $M/G/1$ queues. The dependent functional of the EME mean constraints was maximized during its derivation.

Normalization,

$$\sum_{n=0}^{\infty} p(n) = 1$$

and Pollaczeck-Khinchin (P-K) mean queue length (MQL),

$$< \mathrm{n} > = \sum_{n=0}^{\infty} np(n) = L_q = \frac{\left(\frac{\rho}{2}(1-\rho+Ca^2+\rho Cs^2)\right)^{\frac{1}{2q-1}}}{(1-\rho)^{\frac{3-2q}{(2q-1)}}} \text{(Extended Heuristic Noros formula, [4])}$$

L_q serves as non-robust mean queue length(MQL)[8].

A major contribution of this paper is extending the definition of MQL to the unexplored case of non-robust mean queue length and Server

utilization (SU),
ρ = 1- p(0) [5].

For a stable M/G/1 queueing system, the proposed solution in [5-11] was shown to be stochastically exact when the service (S) times adhered to the GE distribution as indicated by figure 1.

$$F_s(t) = 1 - \tau_s e^{-\mu t \tau_s} \quad (1)$$
$$\tau_s = \frac{2}{1+C_S^2} \quad (2)$$

μ serves as service rate and C_s^2 serves as the service times' squared coefficient of variation (SCV).

There are six sections in the paper. Section II gives an overview of the long-range interactions(equivalently, Non-Extensive Maximum Entropy(NME)) for general physical systems as well as Shannon's Extensive Maximum Entropy (EME) functional. For the underlying queuing systems, Section III gives a brand-new precise expression for the NME state probability of H_I^q. New service time distributions of type GE corresponding to $p_{q,I}$ (n) are determined in Section IV. Section V presents an analysis of the outcomes of a typical numerical experiment and discusses the effects of different degrees of long-range interactions on the NME probabilistic representations of the stable M/G/1 queue. Section VI provides some HE applications to social computing and connected health. Finally, section VII gives concluding remarks combined with tough open problems and recommended directions for future research.

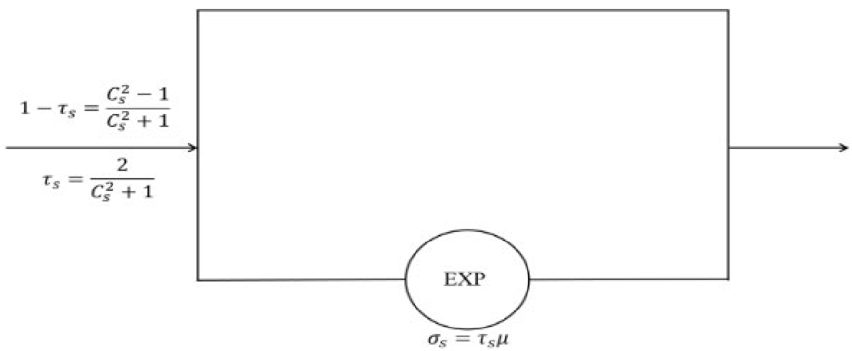

Figure 1: A Schematic Portrait of GE-Type Service Time Distribution with Parameters {1/μ, C_s^2>1}

2. Inductive ME Formalisms

Shannon's entropy that depicts expressing "short range" interactions $H_{1,S}$ (p)[3](equivalently, q→1), is defined by

$$H_{1,S}(p) = -c \sum_{S_n \in S} p_{1,S}(S_n) \log p_{1,S}(S_n) \quad (3)$$

Here c is a constant(c> 0) and $p_{1,S}$ (S_n),n=0,1,2,… represent short range interactions [13].

Ismail's second entropy, namely H_I^q is defined by

$$H_I^q(p(n)) = -\varphi(q) \sum_{n=0}^{\infty} p(n) ln p(n), 1 > q > 0.5 \quad (4)$$

φ serves as a well-defined positive function, with q to be any real number. Notably, $\varphi(q)$→1, H_I^q of (2) reduces to the Shannon's formula(3).

3. The Non-Extensive Formalism of Stable *M*/*G*/1 Queueing System

3.1 The Shannonian Closed Form Expression

The authors of [5] have proved that EMS Shannon's maximum entropy measure given by [3],

$$H(p_{1,S}) = -\sum_{n=0}^{\infty} p_{1,S}(n) \ln(p_{1,S}) \quad (5)$$

Subject to,

Normalization, $\sum_{n=0}^{\infty} p_{1,S}(n) = 1$ (6)

SU, $p_{1,S}(0) = \sum_{n=0}^{\infty} h(n) p_{1,S}(n) = 1 - \rho = \lambda/\mu$ (7)

$$h(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \quad (8)$$

$$< n > = \quad L_q = \frac{\left(\frac{\rho}{2}(1-\rho+Ca^2+\rho Cs^2\ )\right)^{\frac{1}{2q-1}}}{(1-\rho)^{\frac{3-2q}{(2q-1)}}} \tag{9}$$

is given by

$$p_{1,S}(n) = \begin{cases} p_{1,S}(0), & n = 0 \\ p_{1,S}(0)\tau_s x^n & n > 0 \end{cases} \tag{10}$$

where $p_{1,S}(0) = 1 - \rho$, $\tau_s = 2/(1+ C^2_{s,1,S})$ and $x = \frac{<n>-\rho}{<n>}$.

B. $p_{q,I}$*'s NME formalism*

Theorem 1 The $p_{q,I}(n)$'s closed form expression under normalization (6) and SU (7) and the non-robust mean queue length, L_q subjective conditions are devised by

$$p_{q,I}(n) = \begin{cases} p(0), & n = 0 \\ p(0){\tau_s}^{\varphi(q)} x^n, & n > 0 \end{cases} \tag{11}$$

where $p_{q,R}(0)$satisfies:

$$p_{q,I}(0) = 1 - \rho \tag{12}$$

$p_{q,I}(0)$, τ_s and x satisfy

$$p_{q,I}(0) = 1 - \rho, \tau_{s,I} = \begin{cases} \frac{2}{(1+Cs^2\ )}, & Pollaczeck - Khinchin\ \ formula \\ \left(\frac{\rho^2(1-\rho)^{4-2q}}{\left[\left(\frac{\rho}{2}(2-\rho+\rho\ Cs^2)\right)^{\frac{1}{2q-1}} - \rho(1-\rho)^{\frac{3-2q}{(2q-1)}}\right]}\right)^{\frac{1}{\varphi(q)}} & Extended\ heuristic\ Noros\ formula \end{cases} \tag{13}$$

$$x = \begin{cases} \frac{\rho}{(\rho+(1-\rho)(\frac{2}{1+Cs^2})^{\frac{1}{q}})} & P - K \\ 1 - \frac{\rho(1-\rho)^{\frac{3-2q}{(2q-1)}}}{\left(\frac{\rho}{2}(2-\rho+\rho Cs^2)\right)^{\frac{1}{2q-1}}} & Extended\ heuristic\ Noros \end{cases} \tag{14}$$

$${\tau_s}^{\varphi(q)} = \frac{\rho(1-x)}{(1-\rho)x}, x = \frac{Mean\ Queue\ Length - \rho}{Mean\ Queue\ Length} \tag{15}$$

Proof By maximizing H_I^q subject to(7), (8) and (9), the Lagrangian reads as

$$\ell = [-(1 + lnp(n)) + (\alpha)(\varphi(q))^2(\textstyle\sum_{n=0}^{\infty} h(n))) - (\beta - \varphi(q))(\sum_{n=0}^{\infty} 1) + (\varphi(q))^2\gamma(\sum_{n=0}^{\infty} n)] = 0 \tag{16}$$

Hence, we obtain $p(n)$ in the form

$$p_{q,I}(n) = ae^{-\alpha\varphi(q)h(n)}\ e^{-\gamma n\varphi(q)}$$

$$a = e^{-\frac{\beta}{\varphi(q)}} \tag{17}$$

Clearly follows that $a = e^{-\alpha\varphi(q)}p_{q,I}(0)$ (18)

Hence , it follows that

$$p_{q,I}(n) = p(0)\tau_s^{\varphi(q)} x^n \quad (19)$$

By looking at the expression $\tau_s^{\varphi(q)} x^n$, we can see that

$$\tau_s = e^{-\alpha} \in (0,1) \quad (20)$$
$$x = e^{-\gamma\varphi(q)} \in (0,1) \quad (21)$$

To conclude our proof, we have by (6),

$1 = p_{q,I}(0) + p_{q,I}(0)\tau_s^{\frac{1}{q}} \frac{x}{(1-x)}$, implying

$$\tau_s^{\varphi(q)} = \frac{\rho(1-x)}{(1-\rho)x} \quad (22)$$

Now, in this part of proof, we are now using the non-robust heuristic formula for the mean queue length as another further instructive development.
Resuming our proof, we are going obtain the value of x

$$L_q = \frac{\left(\frac{\rho}{2}(2-\rho+\rho Cs^2)\right)^{\frac{1}{2q-1}}}{(1-\rho)^{\frac{3-2q}{(2q-1)}}} = \sum_{n=0}^{\infty} n p_{q,I}(n) \qquad \text{(c,f., (9))}$$

Using the formula of the sum of Infinite Arithmetic-Geometric Progression

$$\sum_{k=0}^{\infty}(A + Bd)x^k = \frac{A}{1-x} + \frac{xd}{(1-x)^2} \quad (23)$$

By the proven formula (20), $\tau_s^{\varphi(q)} = \frac{\rho(1-x)}{(1-\rho)x}$, the RHS of equation (23) reduces to

$$L_q = \sum_{n=0}^{\infty} n p_{q,I}(0)\tau_s^{\varphi(q)} x^n = p_{q,I}(0)\frac{\rho(1-x)}{(1-\rho)x}\frac{x}{(1-x)^2} \quad (24)$$

Clearly it follows that

$$x = \frac{\rho}{\rho + p_{q,I}(0)\tau_s^{\varphi(q)}} = 1 - \frac{\rho(1-\rho)^{\frac{3-2q}{(2q-1)}}}{\left(\frac{\rho}{2}(2-\rho+\rho Cs^2)\right)^{\frac{1}{2q-1}}} \quad (25)$$

Using (25), we get

$$\tau_s^{\varphi(q)} = \frac{\rho(1-x)}{(1-\rho)x} \quad (26)$$

$$\Rightarrow \tau_s = \left(\frac{\rho^2(1-\rho)^{4-2q}}{\left[\left(\frac{\rho}{2}(2-\rho+\rho\, Cs^2)\right)^{\frac{1}{2q-1}} - \rho(1-\rho)^{\frac{3-2q}{(2q-1)}}\right]}\right)^{\frac{1}{\varphi(q)}} \quad (27)$$

It is to be noted that, for the choice of the well-defined positive function, $\varphi(q) = q$ implies by $q \to 1$, that $p_{q,I}$ (n) (c.f., (11)) reduces to the special case of $p_{1,S}(n)$ (c.f., (10).

4. Analytic H_I^q's NME Solution with Distinct $GE_{q,I}$ Type Service Time Distribution

Theorem 2.

The ME solution $p_{q,R}(n), n = 0,1,2,...$ is equivalent to the steady-state (or, equilibrium) for the undertaken queue with a H_I^q G-type service time distribution of the form GE-type, namely

$$f_{s,I}(t) = \left(1 - \tau_{s,I}{}^{\varphi(q)}\right)u_0(t) + \mu\tau_{s,I}^{\frac{2}{\varphi(q)}} e^{-\mu\tau_{s,I}^{\frac{1}{\varphi(q)}} t} \tag{28}$$

$$\tau_{s,I}^{\varphi(q)} = \frac{\rho(1-x)}{(1-\rho)x} = \frac{Mean\ Queue\ Length - \rho}{Mean\ Queue\ Length}$$

$$= 1 - \frac{\rho(1-\rho)^{\frac{3-2q}{(2q-1)}}}{\left(\frac{\rho}{2}(2-\rho+\rho Cs^2)\right)^{\frac{1}{2q-1}}}, q \in (0.5,1)$$

probability density function(*pdf*), where $u_0(t)$ is the unit impulse function

$$u_0(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases} \quad \text{such that} \int_{-\infty}^{\infty} u_0(t) = 1 \tag{29}$$

Proof. The Z-transform $Q(z)$ [13] of the ME solution $p_{q,I}(n), n = 0,1,2,...$ (or, Generating Function, $Q_I(z)$) is given by

$$Q_I(z) = \sum_{n=0}^{\infty} p(n)z^n, |z| < 1 \tag{30}$$

Hence, it holds by replacing $p_{q,R}(n)$ of (11) into (30),

$$Q_R(z) = p_{q,I}(0) + \frac{p_{q,I}(0)\tau_s{}^{\varphi(q)}xz}{(1-xz)} = \frac{p(0)(1-xz(1-\tau_s{}^{\varphi(q)}))}{1-xz} \tag{31}$$

Using the Pollaczek-Khinchine transform equation

$$Q_R(z) = \frac{p(0)(1-z)(F_s^*(\lambda-\lambda z)}{F_s^*(\lambda-\lambda z)-z} \tag{32}$$

where $F_s^*(\theta) = E\left[e^{-\theta s}\right] = \int_0^{\infty} e^{-\theta t} f_s(t)dt$ (33)

is the Laplace-Stieltjes transform of $f_{s,R}(t)$. It can be easily verified that $Q_I(0) = p_I(0)$and $Q_I(1) = 1$. We have

$$F_s^*(\lambda-\lambda z) = \frac{\mu(\tau_s)^{\varphi(q)} + (\lambda-\lambda z)(1-(\tau_s)^{\varphi(q)})}{\mu(\tau_s)^{\varphi(q)} + (\lambda-\lambda z)} \tag{34}$$

By $\theta = (\lambda - \lambda z)$, (34) rewrites to

$$F_s^*(\theta) = \frac{\mu\tau_s{}^{\varphi(q)} + \theta(1-\tau_s{}^{\varphi(q)})}{\mu(\tau_s)^{\varphi(q)} + \theta} \tag{35}$$

Now, we can obtain the service time cumulative di-stribuution function $F_s(t)$.

Corollary (2.1)

The service time cumulative function $F_s(t)$ is given according to the formula:

$$F_{s,I}(t) = \int_0^t f_{s,I}(x)\,dx \tag{36}$$

$$= \int_0^t \left(1 - \tau_{s,I}^{\varphi(q)}\right) u_0(t)\,dx + \mu\tau_{s,R}^{\frac{2}{\varphi(q)}} \int_0^t e^{-\mu\tau_{s,I}^{\frac{1}{\varphi(q)}} x}\,dx$$

$$= \left(1 - \tau_{s,I}^{\varphi(q)}\right) + \frac{\mu\tau_{s,I}^{\frac{2}{\varphi(q)}}}{\mu\tau_{s,I}^{\frac{\varphi(q)}{}}}\left(1 - e^{-\mu\tau_{s,I}{}^{\varphi(q)} t}\right) \tag{37}$$

The reader can easily check that if take the limiting case $\varphi(q) = q$ implies by $q \to 1$, in all the newly derived equations, reduces to the results obtained by the authors of [5].

Corollary 2.2
Based on $f_{s,I}(t)$ (c.f., (28), we have

$$E_I(S) = E_T(S) = \frac{1}{\mu} \quad (38)$$

$$E_I(S^2) = \frac{2}{\mu^2 \tau_{s,I}^{\varphi(q)}} \quad (39)$$

$$C_I s^2 = \frac{E_I(S^2)}{(E_I(S))^2} - 1 \ = \frac{(2-\tau_{s,I}^{\varphi(q)})}{\tau_{s,I}^{\varphi(q)}} \quad (40)$$

Proof.
We have

$$E_I(S) = \int_0^\infty t\, f_{s,I}(t)dt = \int_0^\infty t\, \mu\tau_{s,I}^{\frac{2}{\varphi(q)}} e^{-\mu\tau_{s,I}^{\frac{1}{\varphi(q)}} t}\, dt = \mu\tau_{s,I}^{\frac{2}{\varphi(q)}} \int_0^\infty t\, e^{-\mu\tau_{s,I}^{\frac{1}{\varphi(q)}} t} dt \quad (41)$$

Following the definition of gamma function,

$$\Gamma(m) = \int_0^\infty w^{m-1}\, e^{-w} dw \quad (42)$$

the substiution $\mathrm{w} = \mu\tau_{s,I}{}^{\frac{1}{\varphi(q)}} t$ will re-write $E_I(S)$ to be

$$E_I(S) = \frac{\mu\tau_{s,I}^{\frac{2}{\varphi(q)}}}{\mu^2\tau_{s,I}^{\frac{2}{\varphi(q)}}} = \frac{1}{\mu} \qquad \text{(c.f., (38))}$$

We can see that

$$E_I(S^2) = \int_0^\infty t^2\, f_{s,I}(t)dt = \int_0^\infty t^2 \mu\tau_{s,I}^{\frac{2}{\varphi(q)}}\, e^{-\mu\tau_{s,I}^{\frac{1}{\varphi(q)}} t} dt = \mu\tau_{s,I}^{\frac{2}{\varphi(q)}} \int_0^\infty t^2\, e^{-\mu\tau_{s,I}^{\frac{1}{\varphi(q)}} t} dt \quad (43)$$

the substiution $\mathrm{w} = \mu\tau_{s,I}^{\frac{1}{\varphi(q)}} t$ and (40) will re-write $E_I(S^2)$ to be

$$E_I(S^2) = \frac{2\mu\tau_{s,I}^{\frac{2}{\varphi(q)}}}{\mu^3\tau_{s,I}^{\frac{3}{\varphi(q)}}} = \frac{2}{\mu^2\tau_{s,I}^{\frac{1}{\varphi(q)}}}$$, as required.

5. Experimental Setup with Interpretations and Applications of Hurst Exponent to Engineering

Portraying the Norosian Lagrange multiplier $\tau_{s,I,R,Noros}$ vs (q,ρ) of $q \in (0.5,1)$, $\rho = 0.5$, $\mu = 3$, $C_s^2 = 4$ and $\varphi(q) = q$ (c.f., figure 2), with ρ to serve as the underlying server utilization of $M/G/1$ queue. Clearly, the progressive increase of q implies the increase of $\tau_{s,q,I,Noros}$, because of long range interactions. As a representative of long-range interactions. As q approaches 1, we reach the Shannonian case, with

$$\tau_{s,q,I,Noros} = \frac{2}{1 + \mathrm{C}_{s,1,S}^2} \quad (43)$$

Also,

$$\tau_{s,I} = \left(\frac{\rho^2(1-\rho)^{4-2q}}{\left[\left(\frac{\rho}{2}(2-\rho+\rho\, Cs^2) \right)^{\frac{1}{2q-1}} - \rho(1-\rho)^{\frac{3-2q}{(2q-1)}} \right]} \right)^{\frac{1}{\varphi(q)}} \quad \text{(c.f., (27))}$$

Implies

$$\tau_{s,I} = \left(\frac{2^{(2q-4)}}{\left[(2.5)^{\frac{1}{2q-1}} - 2^{\left(\frac{4-4q}{2q-1}\right)} \right]} \right)^{\frac{1}{q}} \tag{44}$$

In figure 2, MSR stands for Mean Service Rate

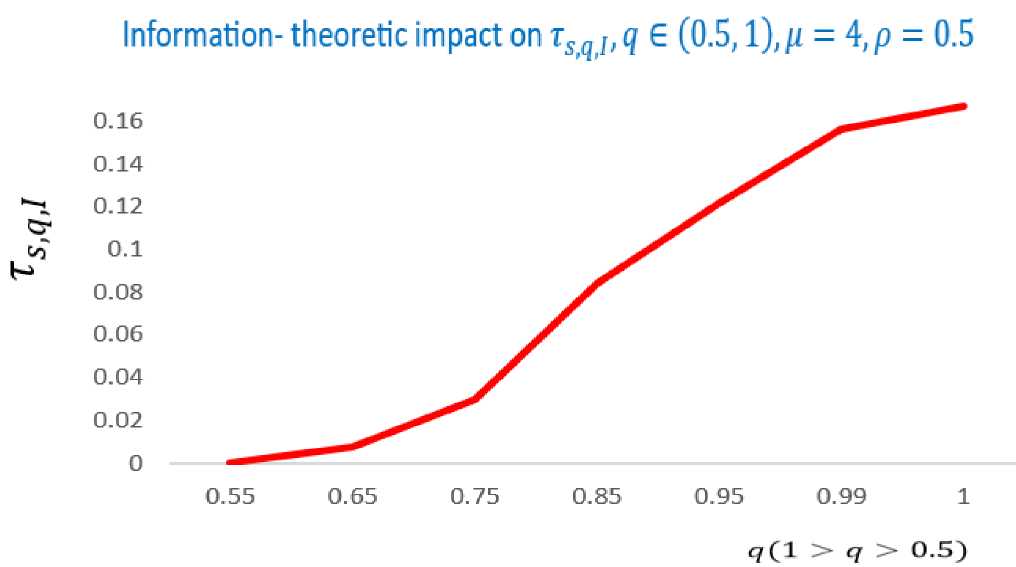

Figure 2: Information theoretic impact on Norosian Lagrange multiplier $\tau_{s,I,R,Noros}$.

The MQL heuristic expression (c.f., (9)) is generated [14] from the formula:

$$L_H = \frac{\rho^{\frac{1}{2(1-H)}}}{2^{\frac{1}{2(1-H)}}} \left(\frac{[1-\rho+Ca^2+\rho Cs^2\,]^{\frac{1}{2(1-H)}}}{(1-\rho)^{\frac{H}{(1-H)}}} \right) \tag{45}$$

By taking the Hurst parameter, $H = 1.5 - q, 1 > q > 0.5$.

It should be noted that the heuristic formula (44) explicitly accounts for the detrimental combined effects on queueing system performance of traffic burstiness (by the SCVs Ca^2, Cs^2) and self-similarity (via parameter H). When $Ca^2 = Cs^2 = 1$, the expression (44) reduces correctly to the Noros formula. Additionally, equation (44) produces the MQL expression for the stable $GE/GE/1$ queue for $H = 0.5$(i.e., $q \rightarrow 1$)14–17].

6. Applications of Hurst Exponent to Social Computing and Connected Health

6.1 Applications of Hurst Exponent to Social Computing

Sentiment analysis is a useful tool for summarizing a story arc's storyline and for capturing the attitudes, emotions, and moods. among other representations in literature [18]. Using Kazuo Ishiguro's "Never Let Me Go" as an example, the authors suggest applying fractal analysis and nonlinear adaptive filtering to examine the narrative coherence and dynamic progression of a novel. They show that these techniques can extract a story arc that reflects the novel's tragic trend and that the time-varying Hurst exponent reflects the plot's dynamic progression. These results point to the possible applications of multifractal theory in large-scale literary analysis and computational narratology.

The Syuzhet sentiment dictionary was utilized by the writers [18] to extract sentiment time series from the book "Never Let Me Go." To find story arcs, they first normalize the emotion ratings and then use a nonlinear adaptive filtering technique. The ensuing plot arcs demonstrate that "Never Let Me Go" is a tragedy since there is a peak of positive sentiment in the middle of the book, contrasted with negative sentiment at the beginning and end. In addition, the authors argue that, in addition to simply categorizing the literary genre—which figure 3(c,f., [18]) visualizes—story arcs can offer insights into the narrative structure and the dynamic growth of attitudes.

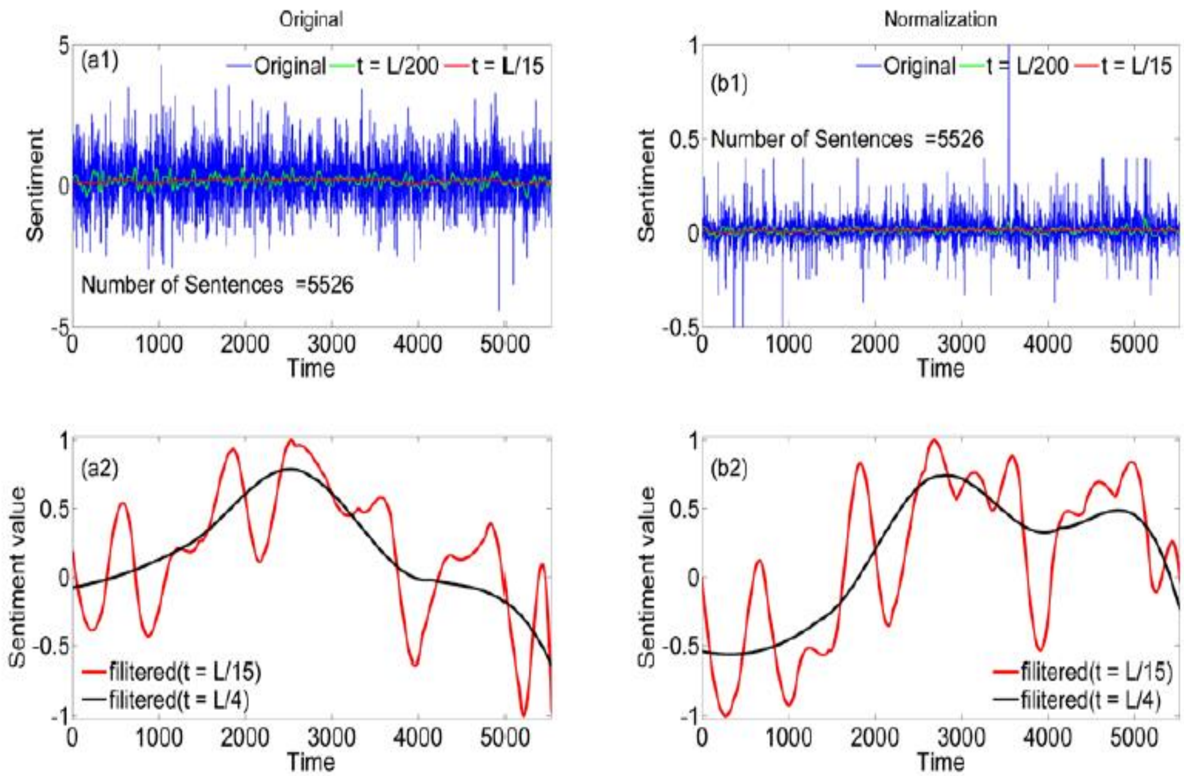

Figure 3: The sentiment time series of "Never Let Me Go" refers to the analysis of the emotional tone or sentiment expressed throughout the novel. By extracting sentiment scores at the sentence level and applying nonlinear adaptive filtering, the story arcs of the novel are identified and depicted. The sentiment time series reveals that "Never Let Me Go" follows a tragic narrative structure, with negative sentiments at the beginning and end, and a peak of positive sentiment in the middle.

A popular field of research in online social networks is influence maximization, which focuses on finding a subset of nodes that can start a chain reaction of adoptions that would maximize the propagation of influence. [19] proposed an approach to evaluate the potential influence of a node by combining its past activity pattern with its connections. The authors of [19] put out a unique Hurst-based Influence Maximisation (HBIM)model to examine the dissemination of seed nodes. For early adopter identification, the suggested technique performs better than other current algorithms.

OSNs, or online social networks[20], have grown significantly in popularity recently because of their applications in a variety of real-world fields, including social awareness campaigns, recommendation systems, and marketing. The model selects a minimal number of seed nodes, and if $H > 0.5$, it only activates the inactive successor of each seed node. This process is repeated until no further activations are feasible. The average and predicted influence spreads of the suggested model are much higher than those of existing Influence Maximisation methods.

6.2 Applications of Hurst Exponent to Connect Health

Electroencephalographic (EEG) signals can be used by the authors [21] to automatically detect epileptic seizures using machine learning techniques. The method utilizes non-linear features, From the EEG signals, metrics like the Hurst exponent and logarithmic Higuchi fractal dimension (HFD) are obtained.

The framework(see figure 4) involves segmenting the EEG data, extracting features such as the Hurst Exponent and Logarithmic HFD, and classifying the data using SVM and KNN classifiers with 10-fold cross-validation.

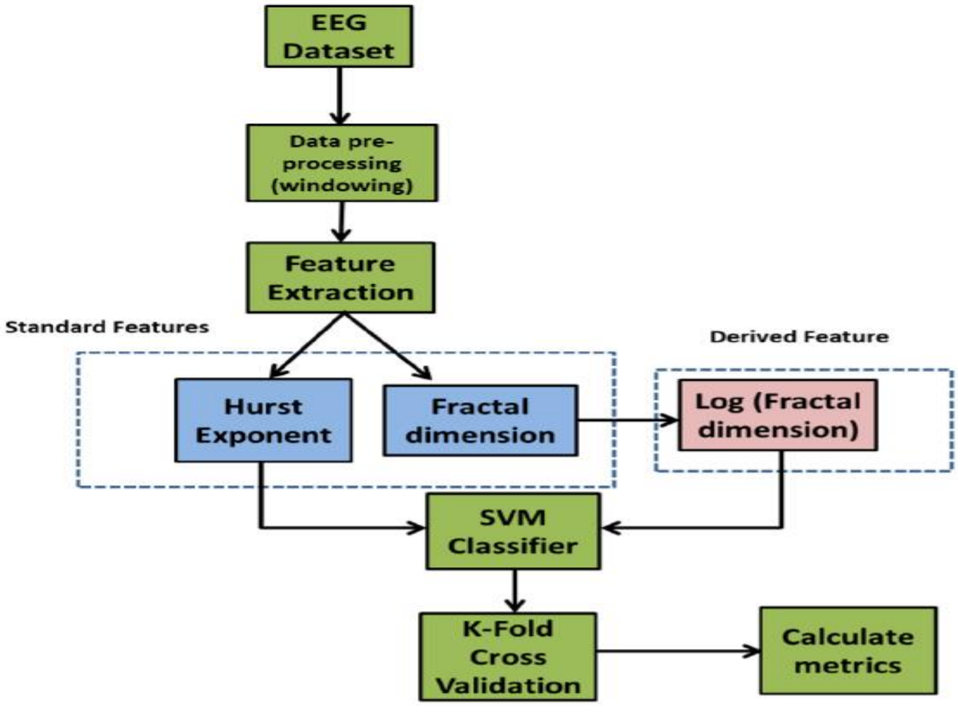

Figure 4: A proposed framework for detecting epileptic seizures using EEG (electroencephalogram) data (c.f., [21]).

The authors of [22] undertook a study to evaluate the use of head movements as non-verbal cues in assessing the performance of resuscitation teams during a simulated trauma scenario. They aimed to track these head movements as a means of understanding and improving team dynamics and communication in emergency situations. The Hurst exponent (*H*) is used in this study to quantify the decrease in head movement complexity and the increase in attention on a specific task during simulated trauma resuscitation. The motion data gathered during the scenarios was used to compute the H values, which show how persistent the data is over time. H values ranging from 0.8 to 1 indicate considerable persistence (subjects primarily focusing on a particular scene) in the results, suggesting a change in the direction of focus after online training and TeamSTEPPS, as visualized by figures 5 and 6(c.f., [22]).

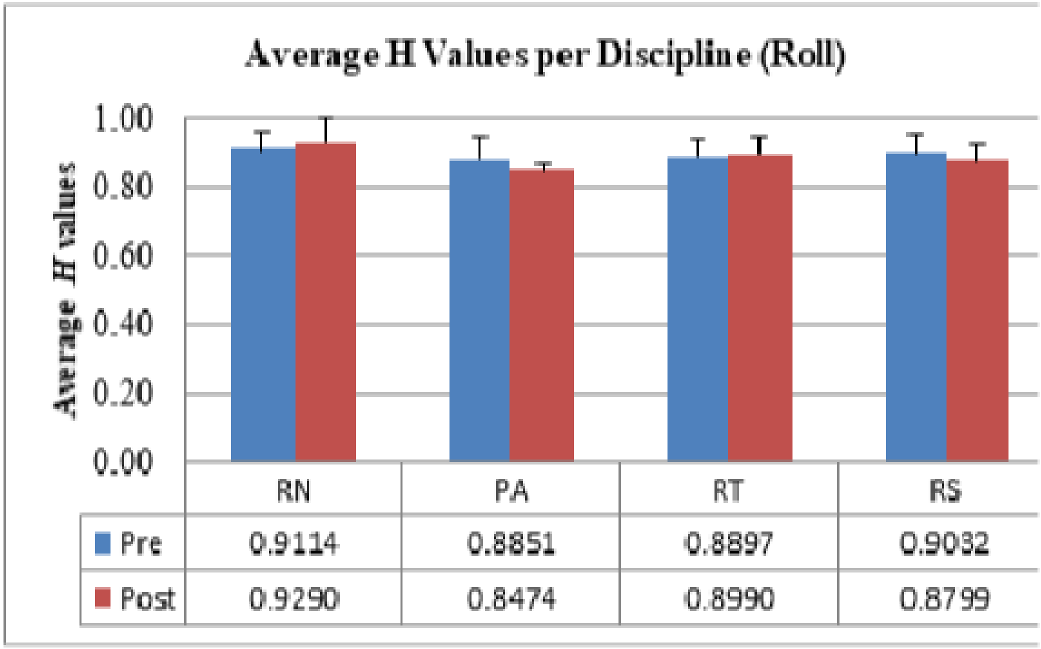

Figure 5: The clustered column charts that display the *H* estimates of roll head motions, which involve looking up and down. The *H* estimates, calculated using the Hurst exponent algorithm, provide a measure of the persistence of data over time.

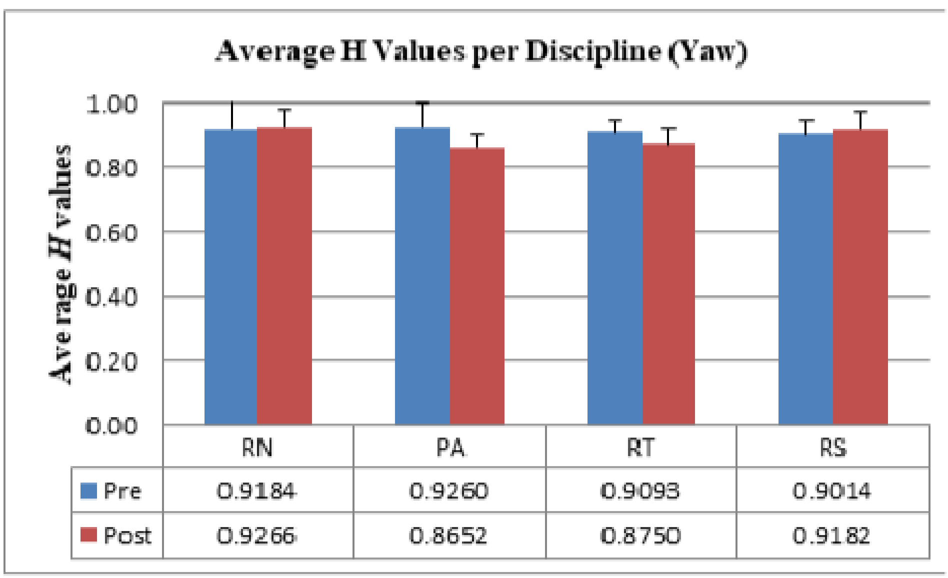

Figure 6: The clustered column charts that display the *H* estimates of yaw head motions, which represent the movement of the head when looking left and right.

7. Conclusions, Open Problems and Future Work

$p_{q,I}$ (n),n=0,1,2,…was derived as a novel representation of H_I^q's in NME format as a method of inductive inference. To illustrate the effect of H_I^q's NME state probability $p_{q,I}$ (n) , n=0,1,2,…, numerical portraits were given. Additionally, it is formally demonstrated for the first time that there are, in fact, underlying q-dependent families of service time CDF, $F_{s,q,I}$ of the $GE_{q,I}$- type, that makes the NME solution of H_I^q stochastically exact. Some of The Hurst Exponent possible roles to advance social computing and connected health are highlighted.

Some emerging sophisticated open problems are as follows:

- **Open Problem One**

Is it analytically possible to unlock the challenge of determining the threshold for $p_{q,I}$ (n) and $F_{s,I}$ (t) (c,f., (11) and (37)) respectively, with respect to the involved parameters?

- **Open Problem Two**

A very tough open problem can be formulated as, if given both $E_I(S^2)$ and $C_I s^2$ (c.f., (39) and (40)) respectively. Can we find a closed for expression that to determine the thresholds with respect to the involved parameters? The question is still open.

- **Open Problem Three**

Can we extend the undertaken analysis of this paper to the EME phase for $q \notin (0.5,1)$) and L_q(c.f., (9)). What will the resulting solutions look like?

- **Open Problem Four**

Is it mathematically feasible to replace all the provided applications pf Hurst Exponent of section VI by L_H (c.f.,(44)) for $q \notin (0.5,1)$), $H = 1 - q$. If this challenging open problem is unlocked, are the obtained results interpretable in real-life environments?

Next phase research involves solving the provided open problems and using other higher order entropies with various selections of constraints towards a revolution information-theoretic queueing theory.

Refrences

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