ISSN: 2996 - 2498

Research Article

Environmental Science and Climate Research

Lengthening of Day Curve Could Contain the Precusors of Earth-quakes and Impending Volcanic Eruptions

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Submitted: 2024, May 16; **Accepted:** 2024, Jun 05; **Published:** 2024, Jun 10

Citation: Sharma, B. K. (2024). Lengthening of Day Curve Could Contain the Precusors of Earth-quakes and Impending Volcanic Eruptions. *Env Sci Climate Res, 2*(1), 01-08.

Abstract

Study of Length of Day between 1890 to 2015 revealed that decrease in spin rate of Earth led to significant enhancement of Seismic Activity (SA) and visa-versa. Lengthening of Day powers the plate tectonic movement and SA which in turn leads to Earth-quakes and sudden volcanic eruptions. Earth's rotation rate variation (LOD curve) is a function of geodynamical processes (Moon's tidal drag on Earth) and seismic activities. Earth is tectonically asymmetric, tectonic equator is tilted about 30° with respect to the geographic equator and there is a global westerly drift of the lithosphere. Earth's rotation change is important force source of Tectonic movements of Earth's crust. 1% of energy released by de-spinning Earth is enough for Earth-quakes and sudden Volcanic eruptions. Earth's rotation rate variability can release 10^21Joules enough for triggering tectonic movements which in-turn may result in major Earth-quakes or sudden Volcanic eruptions. Earth's figure is subject to continuous pulsation. Long term observations of variation in Earth's rotation reveals enhancement in spin rate of Earth in Summer and significant decrease in Winter. The spin rate is directly related to the ellipsoidal shape of Earth. Because of high spin rate in Summer results in an enhanced ellipsoidal shape resulting in large surface area of the continents and a relaxed condition of tectonic Earth plates which implies less seismic activity. Just the opposite happens in Winter. In Winter there is low spin rate resulting in a spheroid shape of Earth which results in contracted surface area of the continents hence tighter packing of tectonic Earth plates resulting in higher seismic activities resulting in large Earthquakes and sudden volcanic eruptions. The Author has done the theoretical formulation of LOD curve for last 1.2Gy and this can be extended in future. If this is subtracted from the observed LOD curve the precursors of Earthquakes and sudden Volcanic eruptions are obtained. Thus Observed LOD curve will act as Early Warning and Forecasting System of Earthquakes and sudden volcanic eruptions.

Keywords: Plate Tectonics, Lithosphere, Length of Day Curve, Aphelion, Perihelion, Uniaxial Compression, Pulsating Geoids, Seismic Activity

1. Introduction

Earth's Rotation Rate (ω) variation is a function of geo-dynamical processes and seismic activities [1-8]. Earth's decelaration supplies energy to plate tectonics comparable to the computed budget dissipated by the deformation processes [9,11]. *the horizontal component of the tidal oscillation and torque would be able to shift the lithosphere relative to the mantle.* Earth's rotation and speed change are important force sources of Tectonic Movement of Earth's crust. They have two kinds of horizontal primary force:

1. Polar axial force directed toward the equatorial plane and is proportional to ω^2.
2. East West directed torque proportional to angular acceleration of Earth's rotation.

The statistical results of seismic belt strikes in the whole Earth, 1% of energy which is caused by speed change of Earth's rotation is enough for the Earth quake release [12,13]. Study of LODs between 1890 and 2015 revealed that decrease in ω within a given time lead to a significant Seismic Activity enhancement and visa versa [12].

In Earth-Moon system, Moon is in synchronous orbit around Earth, it is exerting a tidal drag on Earth and exerting braking torque around the Earth and in the process it is radially receding from Earth. In the process Length of Day is lengthening. Lengthening of day powers the plate tectonic movement [1]. Earth's Rotation Rate (ω) variation is a function of geo-dynamical processes and seismic activities [2-8]

There is a net rotation of the lirhosphere w.r.t. the mantle on the adopted mantle reference frame. Plate boundaries signatures are reviewed---- they are markedly asymmetric worldwide. Different assumptions about the depth of hotspot sources (below or within the aethenosphere characterized by variable viscosity which decouples the lithosphere from the deep mantle) predict different rates of net rotation of the lithosphere w.r.t. mantle. With fast (>1º/Ma) net rotation (shallow hot spot source) we observe all plates, albeit at different velocities, move westerly along a curve trajectory, with tectonic equator tilted about 30º w.r.t. the geographic equator --- this corresponds to global tectonic asymmetry. The conclusions are:

1.Tectonically asymmetric Earth,
2. Lies along the net rotation great circle.
3. Net rotation to global westward drift of the lithosphere.

Constraints on mantle convection.

Earth's rotation and speed change are important force sources of Tectonic Movement of Earth's crust [10]. They have two kinds of horizontal primary force:

1. Polar axial force directed toward the equatorial plane and is proportional to ω^2.
2. East West directed force proportional to angular acceleration of Earth's rotation.

The statistical results of seismic belt strikes in the whole Earth, 1% of energy which is caused by speed change of Earth's rotation, is enough for the Earth quake release [8].

Based on the time series of observational variations of LOD and seismic data in the world, the relation between the decadal fluctuation and seasonal variation in ω with seismic activity are studied. Overall correlation on temporal scale and regional discrepancy on spatial scale between global seismic activity and Earth's variable rotation, especially the seismic activity in Eurasian seismic zone (not including S.E.Asia) and lower California-eastern Alaska seismic zone correlating well with Earth's Variable Rotation.

2. Methods

According to the relation mentioned above, the observational data of Earth's rotation might provide a referential basis for monitoring global seismic activity.

$$F_C = centrifugal\ force = \frac{v^2}{R}$$

where v = tangential velocity of a point on equator = ωR *A*

When v increases then F_C increases. This leads to increased uni-axial compression along the spin axis. This leads to increased surface area and decrease in seismic activity [8]

$$T = observed\ length\ of\ day\ of\ Earth$$

$$P = astronomical\ day$$

$$\frac{T-P}{P} \sim 10^{-8}\ on\ an\ average.$$

These variations lead to occurrence of additional energy estimated at 10^{20} J

A hypothesis is made:

pulsating geoid due to rotation features, tidal forces and critical latitude in Solid Earth. This may explain the intensification of some geological processes in the solid Earth. These geological processes are seismic activities, hot-spot location and major ore deposits.

Rotaion Period of Earth = Function of geological processes.

On time scales shorter than several years mass movement of fluid envelopes- atmosphere, oceans and land hydrology- effect Earth's rotation period as given in the reference 9:

At a time scale of years- mantle and core are decoupled and only mantle moment of inertia comes into picture namely $C_m = 7.04\times10^{37}$ Kg-m^2 comes into picture.

At interdecadal interval, mantle and core are coupled and total $C = 8.037\times10^{37}$ Kg-m^2 has to be considered.

At smaller time scale:

$$\Delta LOD = \frac{86400s}{C_m \times \Omega} \times \Delta(angular\ momentum)$$

$$where\ \Omega(sidereal\ rotation\ rate) = \frac{2\pi}{86164.1s}$$

$$P_{day} = length\ of\ day$$

$$P_{rev} = orbital\ period\ about\ the\ Sun$$

$$P_{rot} = spin\ period\ of\ Earth$$

$$Therefore\ \frac{1}{P_{day}} = \frac{1}{P_{rev}} + \frac{1}{P_{rot}} \qquad B$$

$$P_{rev} \gg P_{rot}\ then\ P_{day} = P_{rot}$$

Non-dimensional variation of Earth^' angular rotation rate is as follow:

$$\nu = \frac{\omega - \Omega}{\Omega} = \frac{P_a - P_z}{P_a}$$

Where ω = angular rate of observed day,
And Ω = angular rate of standard day,
P_a = period of astronomical day (86400s)
P_z = period of observed day.
Spectrum is taken at 1 year, 0.5y, 28d,14d.

These periods are related to orbital components of Earth motion in Sun-Earth-Moon system.

Earth is subject to continuous pulsation. This gives rise to a periodic character in ν.

Annual enhancement in SUMMER versus significant decrease in WINTER.

In winter slowing of rotation period is well established and

$$\nu = \frac{d\omega}{\omega} = 10^{-8}$$

Study of LODs between 1890 and 2015 revealed that decrease in ω within a given time lead to a significant Seismic Activity enhancement and visa versa. Earth's rotation rate variability can release energy of 10^{21}J enough for triggering earthquake processes.

3. Materials

Kinematics of Earth in its orbital path.

According to Kepler's first law, Earth is moving in an elliptical path with Sun on its one of the two foci.

The shortest orbital radius(perihelion point)=147×10^{6}Km.

The longest orbital radius(aphelion point)=152×10^{6}Km.

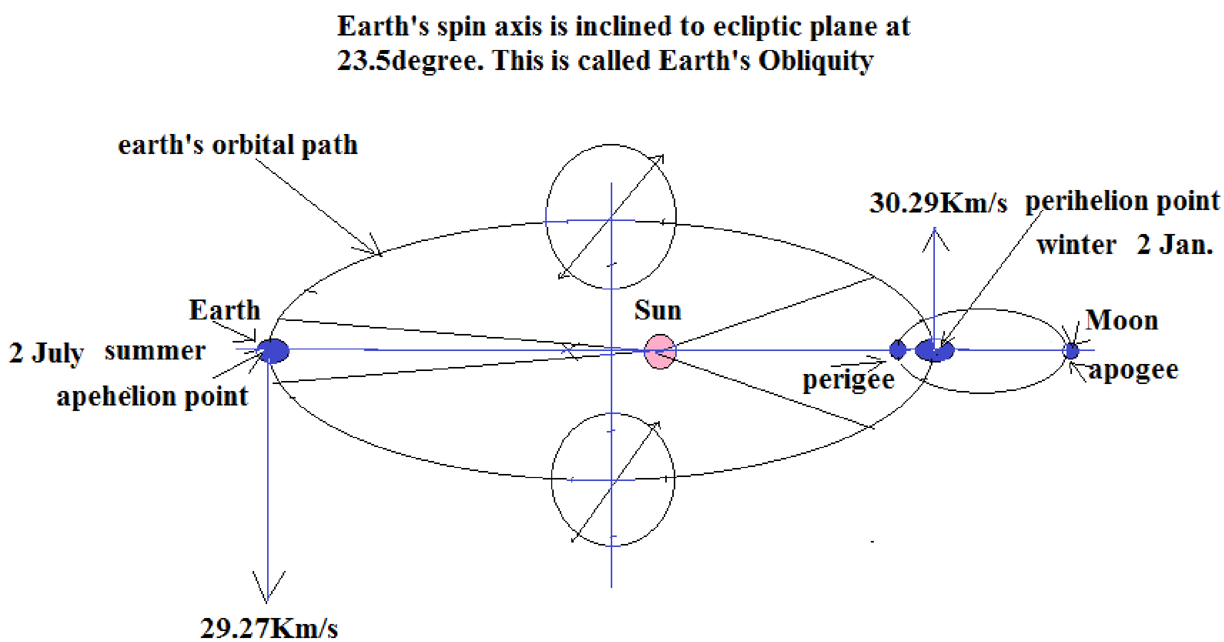

Figure 1: The Orbital Path of Earth Around Sun. [Credit: Author]

According to Kepler's Third Law:

$$a^3\Omega^2 = G(M+m) = 1.32653\times10^{20}\frac{m^3}{s^2} \qquad C$$

From (C) Author's calculation gives tangential velocity of 30.04Km/s (main reference gives 30.29Km/s) at perihelion and 29.5418Km/s(main reference gives 29.27Km/s) at aphelion.

Earth_Sun system has orbital angular momentum + spin angular momentum of Earth + spin angular momentum of Sun.

Along the orbital path of Earth orbital angular momentum goes from a maximum in WINTER to a minimum in SUMMER.

Since (Jorb+Jspin) of Earth has to remain constant hence since Jorb is at maximum hence spin angular momentum of Earth is at minimum in WINTER to a maximum in SUMMER since Jorb is at minimum. Therefore Earth experiences the largest reduction in spin rate in Winter hence largest seismic activity in Winter. Reasons are explained below.

Earth-Moon constitute a gravitationally bound tidally interacting system which has a constant angular momentum. This is further coupled with Sun. Hence we must consider Orbital Angular Momentum of Earth around Sun (J_{orb_E}), spin angular momentum of Earth around its spin axis(J_{spin_E}), Orbital Angular Momentum of Moon around Earth (J_{orb_M}), spin angular momentum of Moon around its spin axis(J_{spin_M}), These four components added together is maintained constant.

$$J_T = J_{Orb\_E} + J_{Spin\_E} + J_{orb\_M} + J_{spin\_M} \qquad D$$

$$J_T = Ma^2 \times \Omega_{Earth} + C \times \omega_{Earth} + m^* a_M^2 \times \Omega_{Moon} + 0.4mR_M^2 \times \omega_{Moon} \qquad E$$

$$\Omega_{Earth} = \frac{\frac{2\pi}{31.5569088 \times 10^6} radians}{s}, \omega_{Earth} = \frac{2\pi}{86400}\frac{radians}{s},$$

$$\Omega_{Moon} = \omega_{Moon} = \frac{2\pi}{27.33 \times 86400}\frac{radians}{s}$$

The orbital angular velocity of Moon and spin angular velocity Moon are synchronized because Moon is tidally locked with Earth. Hence we see the same face of Moon. The other side of the Moon is never visible.
From (E) total angular momentum comes to be:

$$J_T = 2.66019 \times 10^{40} \frac{Kg - m^2}{s} \quad assuming\ D = 86400s\ (mean\ day\ length)$$

$$and\ a = 1.49598 \times 10^{11} m (mean\ distance\ of\ Earth\ from\ Sun).$$

Highest tides are caused by strongest tidal drag on Earth which is in Winter(2 January) when Earth is at perihelion point hence we have largest diurnal day(largest LOD) and lowest tides are caused by weak tidal drag on Earth which is in Sumer (2 July) when Earth is at apehelion point hence we have shortest diurnal day(shortest LOD).

Rotation rate or spin rate of Earth that is ω determines the uniaxial compression.

In winter we have largest drag hence largest reduction in ω i.e. largest Δω. Therefore reduced ω causes reduced compression hence decrease in surface area and increase in Seismic Activity (SA).

In summer just the opposite happens. At aphelion, tidal drag is less hence less reduction in ω therefore enhanced uniaxial compression leading to increased surface area leading to decreased SA.

Earth’s figure is subject to continuous pulsation. From summer to winter:

$$\frac{d\omega}{\omega} \sim 10^{-8}$$

Maximal SA was shown to occur in Winter month (2 January) for the whole globe.

Long term observation of variation in Earth’s rotation rate reveals a clear annual enhancement of the spin rate in June to August versus significant decrease in Winter [11].

The slowing of rotation or spin in winter time can be considered a well established phenomena confirmed by observations.

The maximal SA was shown to occur during winter month (2 Jan) for N. Hemisphere as well as for S. Hemisphere.

The spectrum for the ν time series (for 1962 to 2016) was calculated in Ref [9]. The specific periods of this spectrum, namely, 1 year, 0.5 year, 28 days and about 14 days, represent well known periods related to orbital components of the Earth's motion in the Sun-Earth-Moon system.

In Ref. [9], a model was proposed of a pulsating geoid. Increase of the angular rate of the Earth's rotation leads to increase of the equatorial radius and to enhancement of the planet's compression. Decrease of the angular rate, to the contrary, leads to decrease of the equatorial radius and to a reduction of the compression. Thus, the Earth's figure is subject to continuous pulsation. As shown above, these processes should exhibit a periodic character.

We should note that long-term observations of variations in the Earth's rotation rate reveal a clear annual enhancement of the rate in June-August versus its general significant decrease in winter [13]. The difference between the maximum value of ν in August and its average level in February-April approaches about 1.2×10^{-8}. The slowing of rotation in winter time can be considered a well-established phenomenon, confirmed by observations. The value of the dimensionless variation of the angular rotation rate approaches $\nu = d\omega/\omega = 10^{-8}$.

Figure 2 shows an example of the global SA distribution over the months of a year for the above 12 seismically active regions of the Earth. A clearly expressed maximum is also seen in the month of December. Earthquakes with MS(Surface Wave magnitude) >7.5 were used for this diagram. The recently performed investigation of long-period variations of the Earth's rotation rate on the basis of LOD observations during the period of time between 1890 and 2015 from the data in Refs [4,19,15]. revealed that decrease of the Earth's rotation rate within a certain time interval leads to a significant SA enhancement for this interval, while an increase of the angular rate leads to an SA weakening. Thus, the Earth's seismic activity manifests clear inhomogeneity both in space and time. Periods of seismic activity (SA) enhancement alternate with periods of its decay. In case of strong earthquakes the cycles of SA enhancement and decay last for several dozens of years (according to different estimates for about 30, 55 and 70 years). Figure 2 shows global SA distribution over the months of a year for 12 seismically active regions. A clearly expressed maxima is seen in December as shown in Figure 2.

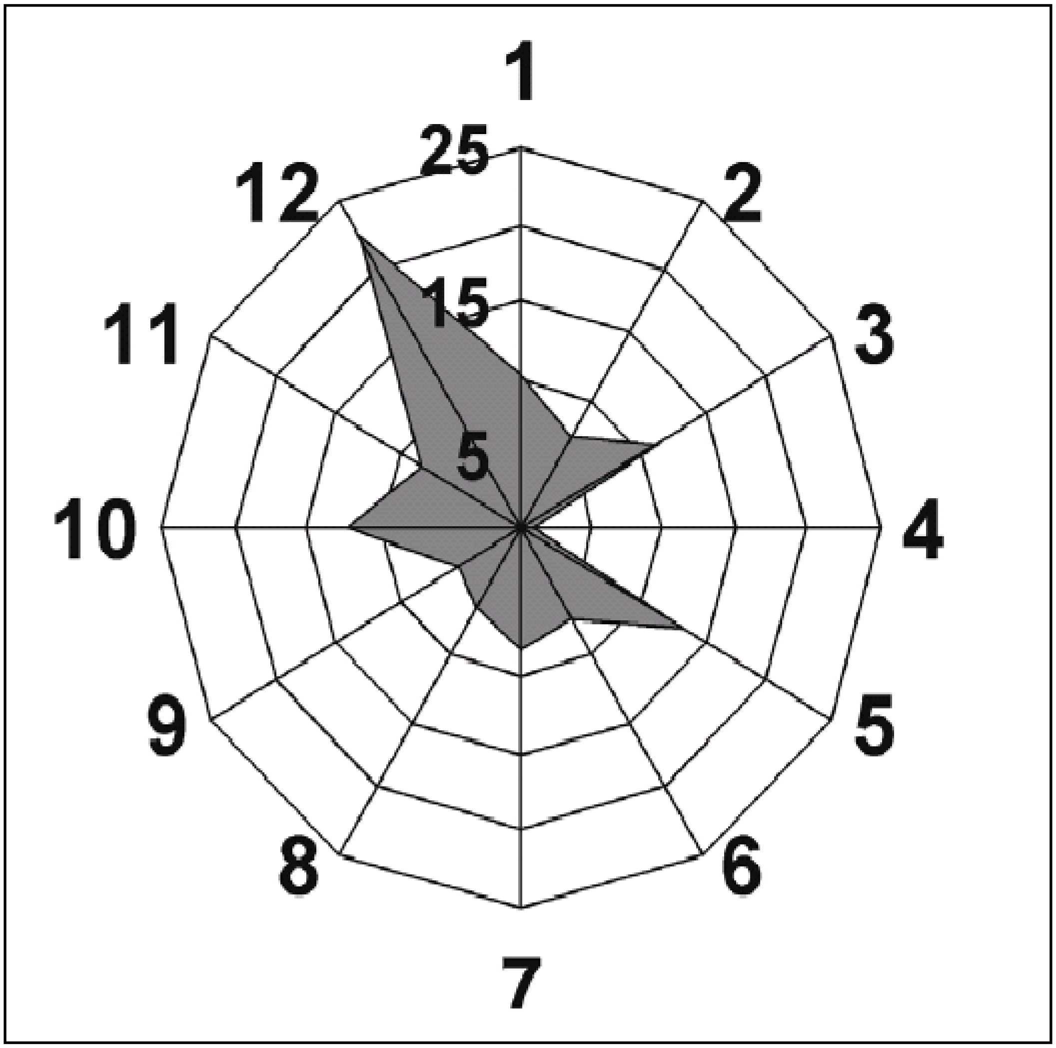

Figure 2: General Monthly Distribution of Seismic Activity for 11Pacific Subregions. The months (from 1st to 12th) of a year are indicated along the perimeter of the circular diagram while the vertical axis represents the total number of SA maxima within all the regions considered. [Credit: Reference 12]

3.1. Analysis of the Relationship Between the Compression (Ellipticity) of a Celestial Body and its Angular Rotation Rate and other Parameters

The formulae for the ellipticity of a planet first obtained by Huygens and Newton have the following forms respectively [11,13,15]:

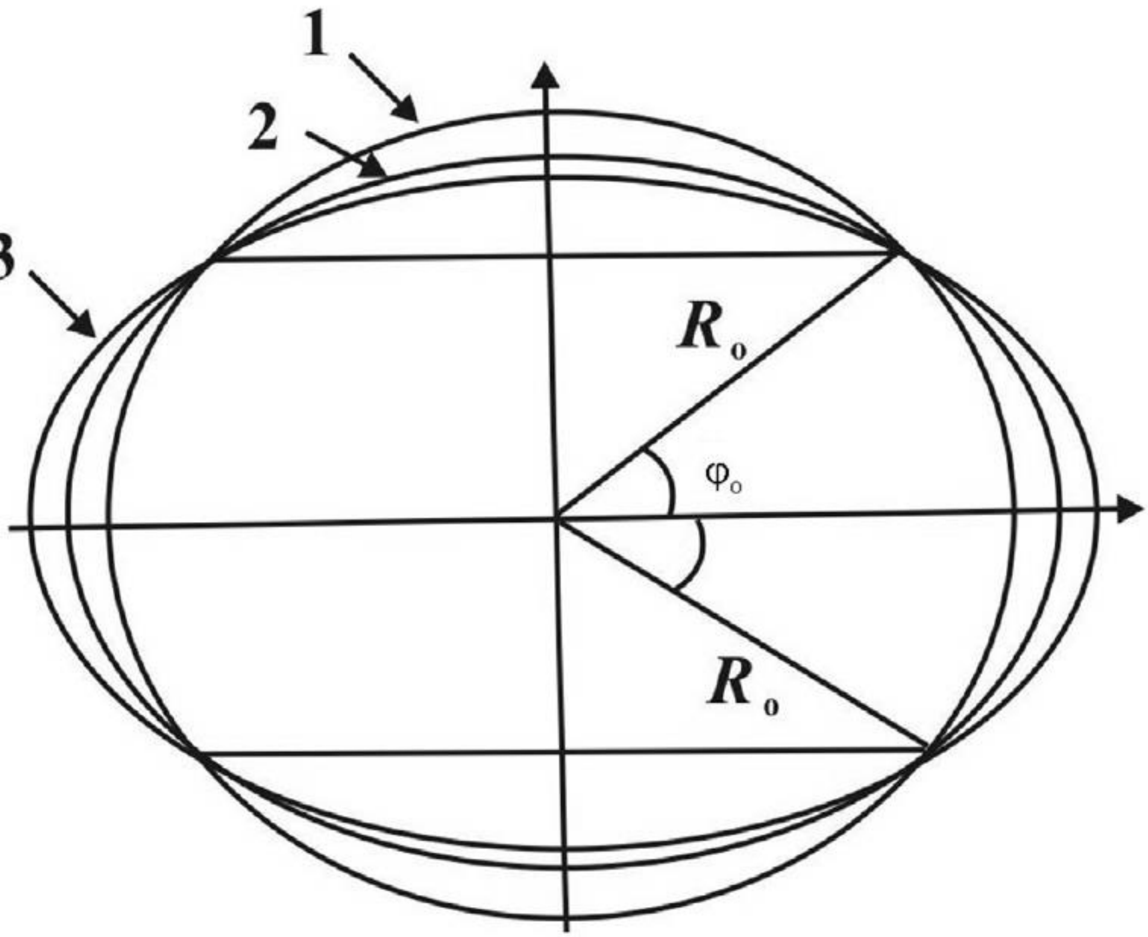

Figure 3: Dependence of pulsations of the surface area of an ellipsoid on the sign of the angular rate variation dω: (1) dω < 0; (2) dω=0; (3) dω > 0. [Credit: Reference 15]

$$\varepsilon_H = \frac{1}{2\omega^2 R^3} \times \frac{1}{GM} \qquad F$$

$$\varepsilon_N = \frac{5}{4\omega^2 R^3} \times \frac{1}{GM} \qquad G$$

M = mass of the planet, R = radius of the Planet; ω= spin angular velocity.

$$\varepsilon = \frac{\omega^2 R_0^3}{GM} = \frac{\omega^2 R_0}{g_0} \qquad H$$

(H) can be represented as the ratio of the centrifugal force at the equator of a rotating sphere of radius R0 to the gravitational force (g_0) at a motionless sphere of radius R0 [14,17]. The value R0 of the radius vector of an ellipsoid is defined as a parameter characterizing latitudes, for which the ellipsoid's radius vector is equal to the radius vector of a sphere of equal volume.

At those indicated latitudes ± φ_0 (φ_0 = 35º 50' 52''), the special feature of which was noted by the French mathematician A. Veronnet, Variations of the angular rotation rate of a planet are very small, while the surface curvature is independent of the ellipticity and coincides with that for a sphere (Figure 3) [17].

Expressions (F,G) and (H) allow to obtain an estimate for the variation of the planet's ellipticity related to changes in the angular rotation velocity.

$$\frac{d\varepsilon}{\varepsilon} = 2\frac{d\omega}{\omega} \qquad I$$

The equation for the radius vector of ellipsoid can be represented as [16].

$$r = R_0\left(1 + \varepsilon\left(\frac{1}{3} - Sin^2\varphi\right)\right) \qquad J$$

where φ is the geocentric latitude, ε = (R-H)/R is the geometric compression of an ellipsoid with semi-axis R and H, R0 is the radius of a sphere of the same volume as the ellipsoid. Then, the variation of the radius vector of the ellipsoid can be represented by the expression.

$$\frac{dr}{r} = \left(\frac{1}{3} - Sin^2\varphi\right)d\varepsilon \qquad K$$

Setting from observations ε = 0.0033, dω/ω = 10^{-3}, dε= 10^{-10} , r = 6.37 × 10^9 mm it is possible to obtain an estimate for possible variation of the radius vector of the ellipsoid, dr = 1 ~ 10 mm, that is achieved in case of natural variations of the Earth's angular rotation rate.

It follows from the geometric properties of an ellipsoid of rotation that variations of the angular rotation rate of a planet lead to characteristic changes in the area of its various surface parts.

Decrease of the surface area in the equatorial part between the latitudes ±φ0 is accompanied by increase of the surface areas in the polar parts outside these latitudes, and vice versa.

In Figure 3 a graphic illustration is shown of the described scheme. Let the initial state of an ellipsoid corresponds to curve 2, when there are no variations of the angular rate (dω = 0). Then, curve 1 corresponds to a drop of the angular rate (dω < 0), while curve 3 corresponds to increase of the rate (dω > 0). At the latitudes of φ0 the surface element area does not change, in the first approximation. We shall now estimate the variation of kinetic energy of the Earth's rotation, using the expression:

$$E = \frac{1}{2} I \times \omega^2 \qquad L$$

By partial derivative method:

$$dE = \frac{\delta E}{\delta \omega} \times d\omega + \frac{\delta E}{\delta I} \times dI$$

Therefore:

$$\frac{dE}{E} = 2\frac{d\omega}{\omega} + \frac{dI}{I} \qquad M$$

Energy released due to spin rate change:

$$dE = 2E\frac{d\omega}{\omega} \qquad N$$

$$E = 0.5I\omega^2 = 0.5 \times 8.03 \times 10^{37} \times (\frac{2\pi}{86400})^2 = 2 \times 10^{29} J$$

$$dE = 2 \times 2 \times 10^{29} J \times 10^{-8} \sim 4 \times 10^{21} \qquad O$$

We note that the additional energy due to the Earth's rotation variability is comparable to the total amount of energy (10^21 J), released yearly as a result of earthquakes. This conclusion is consistent with the hypothesis of the contribution made by the Earth's rotation variability into the earthquake preparation processes.

4. Discussions

Theoretical Length of Day curve has been determined [19]. This theoretical curve does not take into account of Plate Tectonic movements. Theoretical LOD curve accounts for the tidal drag of Moon on Earth and the resulting lengthening of day. So if Theoretical LOD curve is subtracted from the observed LOD curve the difference between the two curves gives the features corresponding to plate tectonic movements and this contains the precursors of major Earthquakes and Sudden Volcanic Eruptions. So the Observed LOD Curve could be developed as Early Warning Forecasting System (EWFS) for major Earthquakes and Sudden Volcanic Eruptions.

5. Conclusions

1. Analysis of the observations made by the International Earth Rotation and Reference System Service, IERS, revealed regularities in the natural variations of the Earth's angular rotation rate.
2. Changes in the ellipticity of the Earth's figure, caused naturally by variations of its angular rotation rate, are manifested in the high-precision GPS measurements and justify further research for improvement of space systems and technologies.
3. In this work, for the first time comparison has been performed of the observable evidence on the geodetic ellipsoid dynamics and theoretical estimates obtained from studies of the pulsation model of the Earth's shape due to variations of its rotation rate.
4. The new ideas concerning the possible nature of earthquake occurrence open up alternative approaches to the mitigation catastrophic events risks.

Acknowledgment

This continued research has been made possible under Emeritus Fellowship Scheme sponsored by University Grants Commission, India. The Grant number is EMERITUS/2012-13-GEN-855/. I acknowledge the infrastructural help given to me by the Director, IIT, Patna, in completing this paper. I acknowledge the cooperation of the staff members of IIT, Patna, who have helped me by material and moral support. I also acknowledge the moral support given by the Director, NIT, Patna, and my colleagues of NIT, Patna, in continuing my research in superannuation period. I also acknowledge the role of my D.Sc. supervisor Prof. Bhola Ishwar who mentored me through this new field of Astronomy and Astrophysics. Last of all I acknowledge the inspiration provided by George Gamow and his book 'The story of Moon' which laid the groundwork of this paper while I was 18 years old just introduced to General Mechanics in 2-year Physics course in my first 2 years of 5-year B.Tech. integrated course at IIT, Karagpur, in period 1962-67. This paper has been almost 50 years in making.

Conflict of Interests

There is no conflict of interest financial or otherwise with anybody.

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