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Green's Tensor of BIOT's Equations by Stationary Oscillations

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Abstract

Various mathematical models of deformable solids mechanics are used in the study of seismic processes in the earth's crust. The processes of waves propagation are most studied in elastic media. But these models do not take into account many real properties of the ambient array. These are, for example, the presence of groundwater, which complicates the construction and operation of surface and underground structures, affect the magnitude and distribution of stresses. Models, which take into account the water saturation form the earth's crust structures, the presence of gas bubbles, etc., are multi-component medium. A variety of multicomponent media, the complexity of the processes associated with their deformation, lead to a large difference in the methods of analysis and modelling used in the solution of wave problems. In this paper the processes of wave propagation in a two-component Biot medium under the action of periodic forces of various forms are considered. Using the Fourier transform of generalized functions, fundamental solutions are constructed - the Green's tensor of the Biot equations and its properties are studied. This tensor describes the process of propagation of harmonic waves of a fixed frequency in spaces of dimension N = 1,2,3 under the action of power sources concentrated at the origin of coordinates, described by a singular delta function. On its basis, generalized solutions of these equations are constructed under the action of various sources of periodic disturbances, which are described by both regular and singular generalized functions. For regularly acting forces, integral representations of solutions are given, which can be used to calculate the stress-strain state of a porous water-saturated medium.

Keywords: BIOT's Medium, Liquid Component, Solid Component, Stationary Oscilation, Fundamental Solution, Fourier Transformation, Regularization

1. Introduction

Various mathematical models of deformable solids mechanics are used in the study of seismic processes in the earth's crust. The processes of waves propagation are most studied in elastic media. But these models do not take into account many real properties of the ambient array. These are, for example, the presence of groundwater, which complicates the construction and operation of surface and underground structures, affect the magnitude and distribution of stresses. Models, which take into account the water saturation form the earth's crust structures, the presence of gas bubbles, etc., are multi-component medium. A variety of multicomponent media, the complexity of the processes associated with their deformation, lead to a large difference in the methods of analysis and modelling used in the solution of wave problems.

Porous medium saturated with liquid or gas, from the point of view of continuum mechanics, is essentially a two-phase continuous medium, one phase of which is particles of liquid (gas), other solid particles is its elastic skeleton. There are various mathematical models of such media, developed by various authors. The most famous of them are the models of M. Biot, V.N. Nikolaevsky, L.P. Horoshun [1-7]. However, the class of solved tasks to them is very limited and mainly associated with the construction and study of particular solutions of these equations based on the methods of full and partial separation of variables and theory of special functions in the works of Rakhmatullin, H. A., Saatov Ya. U., Filippov I. G., Artykov T. U. [6,7], Erzhanov Zh. S, Ataliev Sh.M., Alexeyeva L.A and the others [1-5] etc. In this regard, it is important to develop effective methods of solution of boundary value problems for such media with use of modern mathematical methods.

Periodic on time processes are very widespread in practice. By this cause here we consider the process of wave propagation in the Biot's medium, posed by the periodic forces of different types. Based on Fourier transformation of generalized functions we constructed fundamental solutions of oscillation equations of Biot's medium. It is Green's tensor, which describes the process of propagation of harmonic waves at a fixed frequency in the space-time of dimension N=1,2,3, under the action of concentrated at the coordinates origin. By use this tensor we construct generalized solutions of these equations for arbitrary sources of periodic disturbances, which can be described both regular and singular distributions. They can be used to calculate the stress-strain state of a porous water-saturated medium by seismic waves propagation.

2. Biot's Equations of a Two-Components Medium

The equations of motion of a homogeneous isotropic two-component M. Biot medium are described by the following system of second-order hyperbolic equations [1-3]:

$$\begin{gathered}(\lambda+\mu)\operatorname{grad}\operatorname{div}u_s+\mu\Delta u_s+Q\operatorname{grad}\operatorname{div}u_f+F^s(x,t)=\rho_{11}\ddot{u}_s+\rho_{12}\ddot{u}_f\\ Q\operatorname{grad}\operatorname{div}u_s+R\operatorname{grad}\operatorname{div}u_f+F^f(x,t)=\rho_{12}\ddot{u}_s+\rho_{22}\ddot{u}_f\end{gathered}\tag{1}$$

$(x,t)\in R^N\times[0,\infty)$. Here N is the dimension of the space. At a plane deformation N=2, the total spatial deformation corresponds to N=3, at N=1 the equations describe the dynamics of a porous liquid-saturated rod. We denote $u_s=u_{sj}(x,t)e_j$ a displacements vector of the elastic skeleton, $u_f=u_{fj}(x,t)e_j$ is the displacements vector of a liquid, $e_j\ (j=1,...,N)$ are the basic orts of the Lagrangian Cartesian coordinate system (everywhere by repeating indices there is summation from 1 to N).

Constants $\rho_{11},\rho_{12},\rho_{22}$ have the dimension of mass density and are associated with the density of the masses of the particles, composing a skeleton ρ_s and a fluid ρ_f, by relationships:

$$\rho_{11}=(1-m)\rho_s-\rho_{12},\quad \rho_{22}=m\rho_f-\rho_{12},$$

where m is a porosity of medium. The constant of the attached density ρ_{12} is related to the dispersion of the deviation of the micro-velocities of the fluid particles in the pores from the average velocity of the fluid flow and depends on the geometry of the pores. Elastic constants $\lambda,\ \mu$ are the Lame parameters of an isotropic elastic skeleton, and $Q,\ R$ characterize the interaction of the skeleton with a liquid on the basis of *Biot's law for stresses*:

$$\begin{gathered}\sigma_{ij}=(\lambda\partial_k u_k+Q\partial_k U_k)\delta_{ij}+\mu\left(\partial_i u_i+\partial_j u_i\right)\\ \sigma=-mp=R\partial_k U_k+Q\partial_k u_k\end{gathered}\tag{2}$$

Here $\sigma_{ij}(x,t)$ are the stress tensor in the skeleton, $p(x,t)$ is a pressure in the fluid. Further we use the notations for partial derivatives: $\partial_k=\dfrac{\partial}{\partial x_k},\ u_{j,k}=\partial_k u_j$, $\Delta=\partial_k\partial_k$ is Laplace operator. The external mass forces acting on the skeleton $F^s=F_j^s(x,t)e_j$ and on the liquid component $F^f=F_j^f(x,t)e_j$.

There are three sound speeds in this medium:

$$c_1^2=\frac{\alpha_1+\sqrt{\alpha_1{}^2-4\alpha_2\alpha_3}}{2\alpha_2},\quad c_2^2=\frac{\alpha_1-\sqrt{\alpha_1{}^2-4\alpha_2\alpha_3}}{2\alpha_2},\quad c_3=\sqrt{\frac{\rho_{22}\mu}{\alpha_2}}\tag{3}$$

where the next constants were introduced:

$$\alpha_1=(\lambda+2\mu)\rho_{22}+R\rho_{11}-2Q\rho_{12},\quad \alpha_2=\rho_{11}\rho_{22}-(\rho_{12})^2,\quad \alpha_3=(\lambda+2\mu)R-Q^2$$

The first two speeds C_1, C_2 ($C_1>C_2$) describe the velocity of propagation of two types of *dilatational waves*. The second slower dilatation wave is called *the repackaging wave*. A third velocity C_3 corresponds to *shear waves* and at $\rho_{12}=0$ coincides with velocity of transverse waves propagation in an elastic skeleton ($C_3<C_1$).

We introduce also two velocities of propagation of dilatational waves in corresponding elastic body and in an ideal compressible fluid:

$$c_s = \sqrt{\frac{\lambda + 2\mu}{\rho_{11}}}, \qquad c_f = \sqrt{\frac{R}{\rho_{22}}}$$

3. The Problems of Dynamics of Biot's Medium by Periodic Oscillations

Construction of motion equation solutions by periodic oscillations is very important for practice since existing power sources of disturbances are often periodic in time and therefore can be decomposed into a finite or infinite Fourier series in the form:

$$F^s(x,t) = \sum_n F_n^s(x) e^{-i\omega_n t}, \quad F^f(x,t) = \sum_n F_n^f(x) e^{-i\omega_n t} \tag{4}$$

where periods of oscillation of each harmonic $T_n = 2\pi / \omega_n$ are multiple to the general period T of oscillation. Therefore, it is enough to consider the case of stationary oscillations, when the acting forces are periodic on time with an oscillation frequency ω:

$$F^s(x,t) = F^s(x) e^{-i\omega t}, \quad F^f(x,t) = F^f(x) e^{-i\omega t} \tag{5}$$

The solution of the equations (1) can be represented in the similar form:

$$u_s(x,t) = u_s(x) e^{-i\omega t}, \quad u_f(x) = u_f(x) e^{-i\omega t} \tag{6}$$

where the complex amplitudes of the displacements $u_s(x)$, $u_f(x)$ must be determined. If the solution has been known for any frequency ω, then we get similar decomposition for the displacements of the medium:

$$u^s(x,t) = \sum_n u_n^s(x) e^{-i\omega_n t}, \quad u^f(x,t) = \sum_n u_n^f(x) e^{-i\omega_n t} \tag{7}$$

which give us the solution of problem for forces (4).

We get equations for complex amplitudes by stationary oscillations, substituting (6) into the system (1):

$$\begin{gathered}(\lambda + \mu)\,\mathrm{grad}\,\mathrm{div}\,u^s + \mu\Delta u^s + Q\,\mathrm{grad}\,\mathrm{div}\,u^f + F^s(x) = -\rho_{11}\omega^2 u^s - \rho_{12}\omega^2 u^f \\ Q\,\mathrm{grad}\,\mathrm{div} u^s + R\,\mathrm{grad}\,\mathrm{div}\,u^f + F^f(x) = -\rho_{12}\omega^2 u^s - \rho_{22}\omega^2 u^f\end{gathered} \tag{8}$$

To construct the solutions of this system we define Green tensor of it.

4. Green Tensor of Biot's Equations by Stationary Oscillations

Let's construct fundamental solutions of the system (1) in the form:

$$\begin{pmatrix} F^s \\ F^f \end{pmatrix} = \begin{pmatrix} \delta_k^{[j]} e_k \\ \delta_{k+N}^{[j]} e_k \end{pmatrix} \delta(x) e^{-i\omega t}, \quad k = 1,\ldots,N, j = 1,\ldots,2N \tag{9}$$

where δ_k^j is Kronecker symbol. They describe the motion of Biot medium at the action of sources of stationary oscillations, concentrated in the point x=0. The upper index of this tensor fixes the current concentrated force and its direction. The lower index corresponds to component of the movement of the skeleton and fluid, respectively $K = 1, \ldots, N$ and $K = N + 1, \ldots, 2N$.

Their complex amplitudes $U_m^j(x,\omega)$ $(j, m = 1,\ldots,2N)$ satisfy to the next system of equation:

$$(\lambda+\mu)U^k_{j,ji}+\mu U^k_{i,jj}+\omega^2\rho_{11}U^k_i+QU^{k+N}_{j,ji}-\omega^2\rho_{12}U^{k+N}_i+\delta(x)\delta^k_j=0\frac{1}{2}$$
$$QU^k_{j,ji}+\rho_{12}\omega^2U^k_{i\ ,tt}+RU^{k+N}_{j,ji}+\rho_{22}\omega^2U^{k+N}_i+\delta(x)\delta^k_{j+N}=0 \quad (10)$$
$$j=1,...,2N,\quad k=1,...,2N$$

Since fundamental solutions are not unique, we'll construct such, which tend to zero at infinity:

$$U^j_i(x,\omega)\to 0 \quad \text{at} \quad \|x\|\to\infty \quad (11)$$

and satisfy to radiation condition of type of Somerfield radiation conditions [10].
Matrix of such fundamental equations is names *Green tensor* of Eq. (8).

5. Construction of Fourie Transformant of Green's Tensor

To construct $U^j_m(x,\omega)$ we use the Fourier transformation. For this the next basic function were introduced:

$$f_{0k}(\xi,\omega)=\frac{1}{c_k{}^2\|\xi\|^2-\omega^2},\quad f_{jk}(\xi,\omega)=\frac{f_{(j-1)k}(\xi,\omega)}{-i\omega},\quad j=1,2; \quad (12)$$

where $\xi=(\xi_1,...,\xi_N)$ are Fourier variables. The next theorem was proved [1,2].

Theorem 1. *Components of Fourier transform of fundamental solutions have the form*

$$j=\overline{1,N},\quad k=\overline{1,N},$$
$$\overline{U}^k_j=(-i\xi_j)(-i\xi_k)[\beta_1f_{21}+\beta_2f_{22}+\beta_3f_{23}]+\frac{1}{\alpha_2}\left(\rho_{12}\delta^k_{j+N}-\rho_{22}\delta^k_j\right)f_{03}$$
$$\overline{U}^k_{j+N}=(-i\xi_j)(-i\xi_k)[\gamma_1f_{21}+\gamma_2f_{22}+\gamma_3f_{23}]-\frac{\mu}{\alpha_2}\delta^k_{j+N}\|\xi\|^2f_{23}-\frac{1}{\alpha_2}\left(\rho_{11}\delta^k_{j+N}+\rho_{12}\delta^k_j\right)f_{03}$$
$$j=1,...,N\quad k=N+1,...,2N$$
$$\overline{U}^k_j=(-i\xi_j)(-i\xi_{k-N})[\eta_1f_{21}+\eta_2f_{22}+\eta_3f_{23}]+\frac{1}{\alpha_2}\left(\rho_{12}\delta^k_{j+N}-\rho_{22}\delta^k_j\right)f_{03}$$
$$\overline{U}^k_{j+N}=(-i\xi_j)(-i\xi_{k-N})[\varsigma_1f_{21}+\varsigma_2f_{22}+\varsigma_3f_{23}]-\frac{\mu}{\alpha_2}\delta^k_{j+N}\|\xi\|^2f_{23}-\frac{1}{\alpha_2}\left(\rho_{11}\delta^k_{j+N}+\rho_{12}\delta^k_j\right)f_{03}$$

where the next constants have been introduced:

$$d_1=(\lambda+\mu)\rho_{22}-Q\rho_{12},\quad d_1=Q\rho_{22}-R\rho_{12},$$
$$q_1=Q\rho_{12}-(\lambda+\mu)\rho_{12},\quad q_2=\rho_{11}R-Q\rho_{12},$$
$$\beta_j=(-1)^{1+j}\frac{D_1\rho_{22}c_j^2}{\alpha_2\upsilon_{3j}}\left(d_1\upsilon_{fj}+\frac{d_2}{\rho_{22}}b_{fj}\right),\quad j=1,2,$$
$$\beta_3=-\frac{\rho_{22}c_3^2}{\alpha_2\left(\rho_{11}\rho_{22}-\rho_{12}^2\right)\upsilon_{31}\upsilon_{32}}\left(d_1\upsilon_{f3}+\frac{d_2}{\rho_{22}}b_{f3}\right),$$
$$\gamma_j=(-1)^{1+j}\frac{D_1c_j^2}{\alpha_2\upsilon_{3j}}\left(q_1b_{sj}+q_2b_{fj}\right),\quad j=1,2,\quad \gamma_3=\frac{D_1c_3^2\upsilon_{12}}{\alpha_2\upsilon_{31}\upsilon_{32}}\left(q_1b_{s3}+q_2b_{f3}\right)$$

$$\eta_j = (-1)^{1+j} \frac{D_1 c_j^2}{\alpha_2 \upsilon_{3j}} \left(d_1 b_{fj} + d_2 d_{sj} \right), \quad j = 1,2, \quad \eta_3 = -\frac{c_3^2 \upsilon_{12}}{\alpha_2 \upsilon_{31} \upsilon_{32}} \left(d_1 b_{f3} + d_2 d_{s3} \right),$$

$$\zeta_j = (-1)^{1+j} \frac{D_1 c_j^2}{\alpha_2 \upsilon_{3j}} \left(q_1 b_{fj} + q_2 d_{sj} \right), \quad j = 1,2, \quad \zeta_3 = -\frac{c_3^2 \upsilon_{12}}{\alpha_2 \upsilon_{31} \upsilon_{32}} \left(q_1 b_{f3} + q_2 d_{s3} \right),$$

This form is very convenient for constructing originals of Green tensor.

6. Green Tensor and Generalized Solutions

The originals of basic function:

$$\Phi_{0m}(x,\omega) = F^{-1}\left[f_{0m}(\xi,\omega) \right],$$

are fundamental solution of Helmholtz equation:

$$\left(\Delta + k_m^2 \right) \Phi_{0m} + c_m^{-2} \delta(x) = 0, \quad k_m = \frac{\omega}{c_m} \tag{13}$$

Fundamental solutions of Helmholtz equation which satisfy to Sommerfeld conditions of radiation, are well known [10]:

for $N = 3$

$$\Phi_{0m} = \frac{1}{4\pi r c^2} e^{i k_m r}, \quad k_m = \frac{\omega}{c_m};$$

for $N = 2$

$$\Phi_{0m} = \frac{i}{4c^2} H_0^{(1)}\left(k_m r \right),$$

where $H_j^{(1)}\left(k_m r \right)$ is cylindrical Hankel function of the first kind;

for $N = 1$ $\hat{O}_{0m} = \frac{\sin k_m |x|}{2 k_m c_m^2}$.

These functions (subject to factor $e^{-i\omega t}$)) describe harmonic waves which move from x=0 to infinity and decay at infinity. Last property is true only for N=2,3. In the case N=1 all fundamental solutions of Eq. (16) don't decay at infinity.

Using theorem 1 the next theorem was proved.

Theorem 2. *The components of Green tensor of Biot's equations at stationary oscillations with frequency ω, satisfying the conditions of radiation, have the form*:

$$for \quad j = \overline{1,N}, \quad k = \overline{1,N},$$

$$U_j^k = -\omega^{-2} \sum_{m=1}^{3} \beta_m \frac{\partial^2 \Phi_{0m}}{\partial x_j \partial x_k} + \frac{1}{\alpha_2} \left(\rho_{12} \delta_{j+N}^k - \rho_{22} \delta_j^k \right) \Phi_{03},$$

$$U_{j+N}^k = -\omega^{-2} \sum_{m=1}^{3} \gamma_m \frac{\partial^2 \Phi_{0m}}{\partial x_j \partial x_k} + \frac{\mu}{\alpha_2 \omega^2} (c_3^{-2} \delta(x) + k_3^2 \Phi_{0m}) \delta_{j+N}^k - \frac{1}{\alpha_2} \left(\rho_{11} \delta_{j+N}^k + \rho_{12} \delta_j^k \right) \Phi_{03};$$

$$for \quad j = 1,\dots,N \quad k = N+1,\dots,2N$$

$$U_j^k = -\omega^{-2} \sum_{m=1}^{3} \eta_m \frac{\partial^2 \Phi_{0m}}{\partial x_j \partial x_k} + \frac{1}{\alpha_2} \left(\rho_{12} \delta_{j+N}^k - \rho_{22} \delta_j^k \right) \Phi_{03}$$

$$U_{j+N}^{k} = -\omega^{-2}\sum_{m=1}^{3}\varsigma_m \frac{\partial^2\Phi_{0m}}{\partial x_j \partial x_k} + \frac{\mu}{\alpha_2\omega^2}(c_3^{-2}\delta(x) + k_3^2\Phi_{0m})\delta_{j+N}^{k} - \frac{1}{\alpha_2}\left(\rho_{11}\delta_{j+N}^{k} + \rho_{12}\delta_j^k\right)\Phi_{03}$$

Under the action of arbitrary mass forces with frequency ω in the Biot's medium, the solution for complex amplitudes has the form of a tensor-functional convolution:

$$u_j(x,t) = U_j^k(x,\omega) * F_k(x)e^{-i\omega t},\ j,k = \overline{1,2N} \tag{14}$$

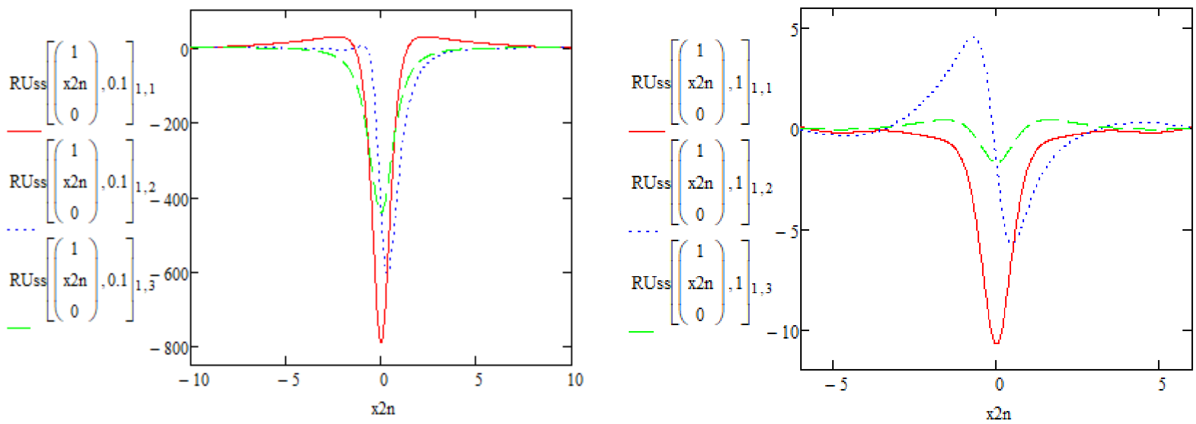
Note that the constructed solutions can be used to solve non-stationary problems, to study non-stationary processes in the Biot environment. For this purpose, the inverse Fourier transform in time should be used. Such solutions were constructed by the authors in [12].

7. Dynamics of Biot's Medium by Action of Concentrated Periodic Mass Forces

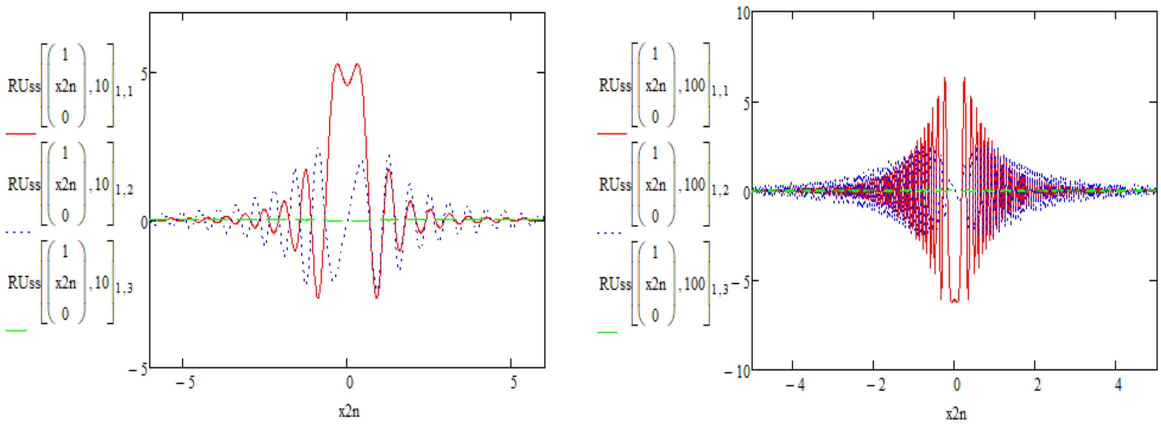
Here are some results of the computer implementation in the Mathcad system of the Green tensor, which describe the dynamics of a two-component Biot medium under the action of concentrated sources of stationary oscillations of various frequencies (from 1 to 100).

Figures 1.1-1.4 show the displacements of the solid component under the action of a force along the X2 axis, and Figures 1.5-1.8 show the displacements of the liquid component under the action of a force along the X2 axis.

The following designations are used on the graphs: R e U displacement at the beginning of the oscillation period at $t = nT$, Im U - displacement after a quarter of the periodat at $t = nT + \frac{T}{4}$, $n = 0, 1, 2, \ldots$, $T = \frac{2\pi}{\omega}$ is oscillation period. $\lambda = 1,\ \mu = 1,\ Q = 1,\ R = 2,\ \rho_{11} = 1,\ \rho_{22} = 3,\ \rho_{22} = 0.02.$

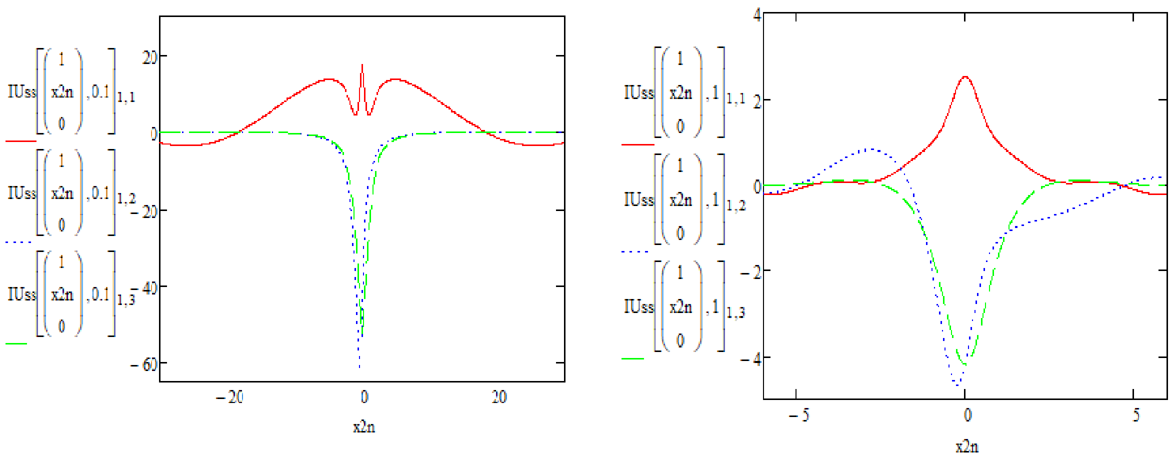


a) b)

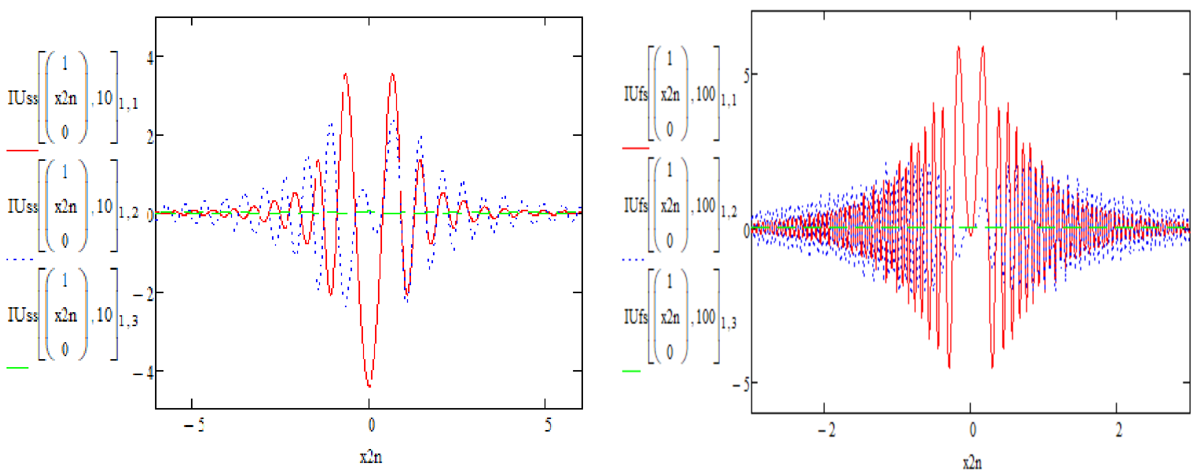


c) d)

Figure 1.1: Displacement of a solid component under the action of a force F^s along the X2 axis at $t = nT$ and a) $\omega = 0.1$, b) $\omega = 1$, c) $\omega = 10$, d) $\omega = 100$



a) b)



c) d)

Figure 1.2: Displacement of a solid component under the action of a force F^s along the X2 axis at $t = nT + \frac{T}{4}$ and a) $\omega = 0.1$, b) $\omega = 1$, c) $\omega = 10$, d) $\omega = 100$

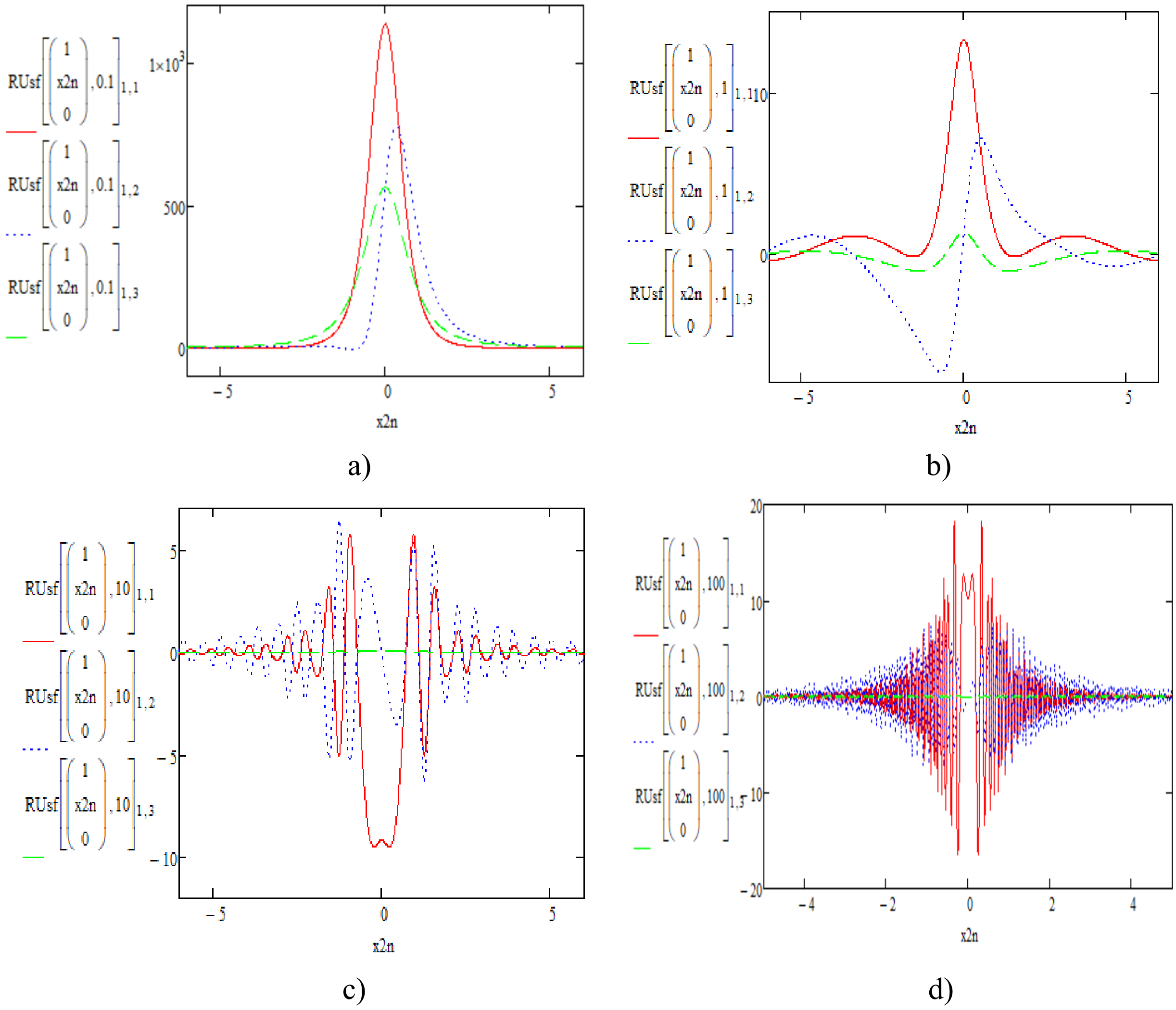
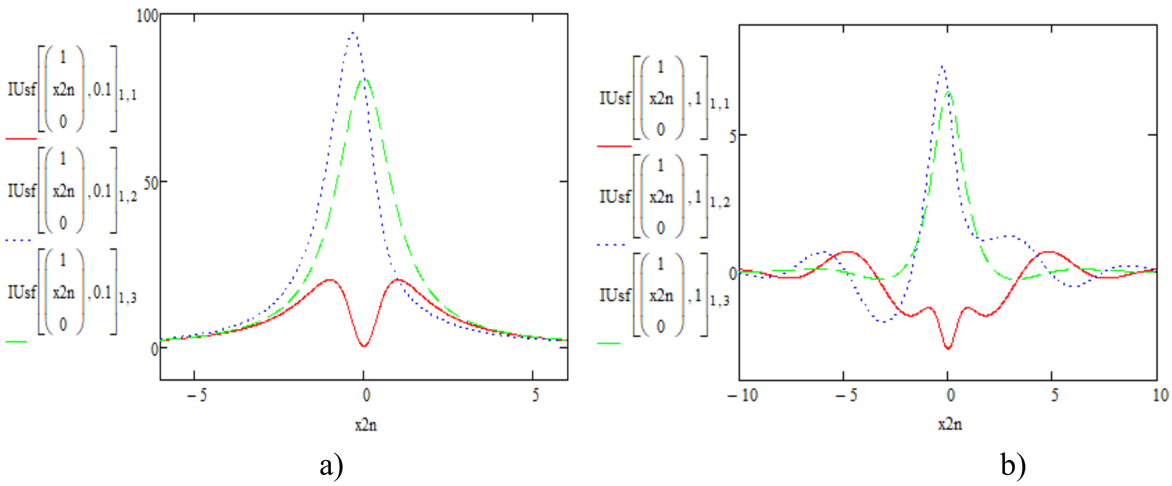
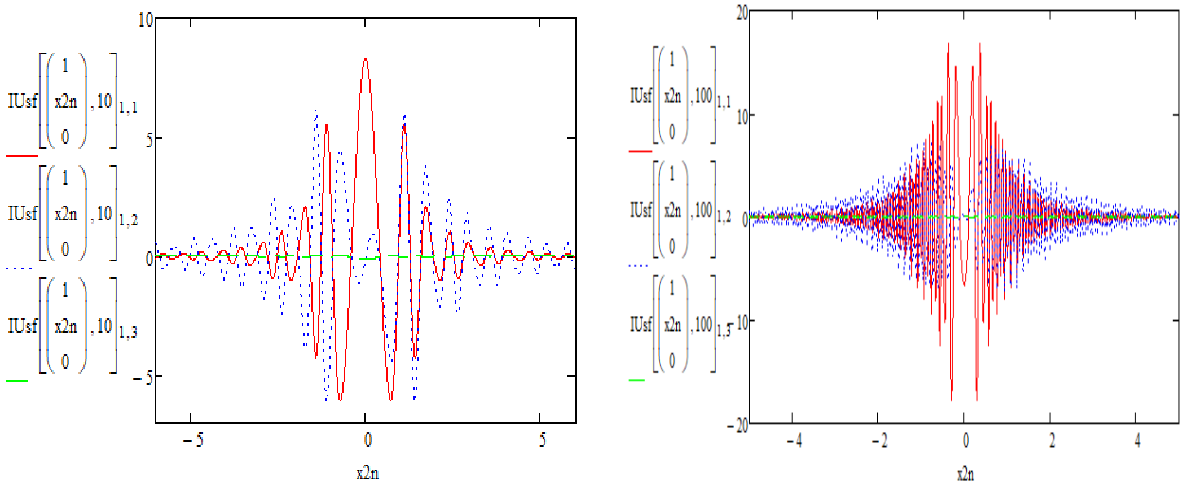


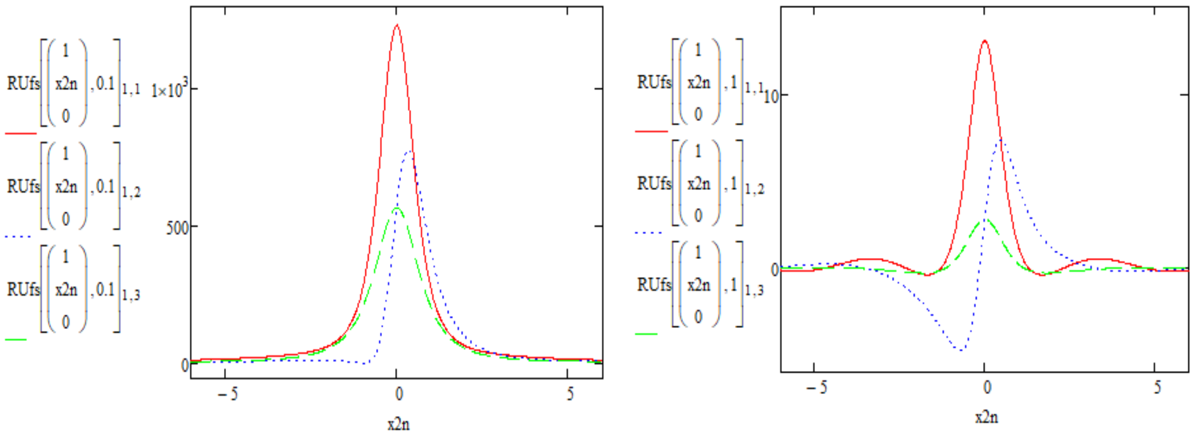
Figure 1.3: Displacement of a solid component under the action of a force F^s along the X2 axis at $t = nT$ and a) $\omega = 0.1$, b) $\omega = 1$, c) $\omega = 10$, d) $\omega = 100$



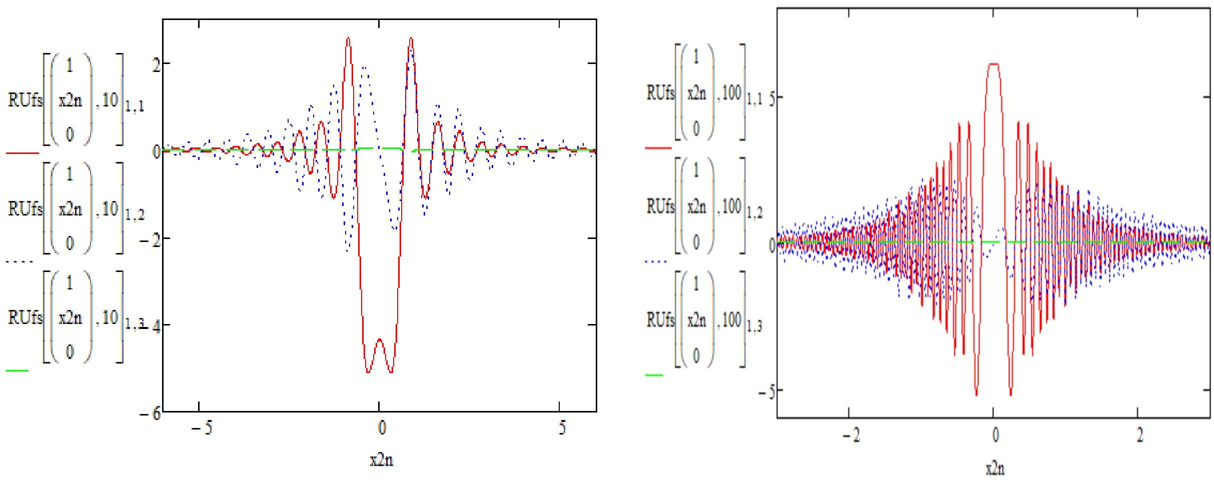


c) d)

Figure 1.4: Displacement of a solid component under the action of a force F^s along the X2 axis at $t = nT + \dfrac{T}{4}$ and a) $\omega = 0.1$, b) $\omega = 1$, c) $\omega = 10$, d) $\omega = 100$



a) b)



c) d)

Figure 1.5: Displacement of fluid under the action of force F^f along the X2 axis at $t = nT$ and a) $\omega = 0.1$, b) $\omega = 1$, c) $\omega = 10$, d) $\omega = 100$

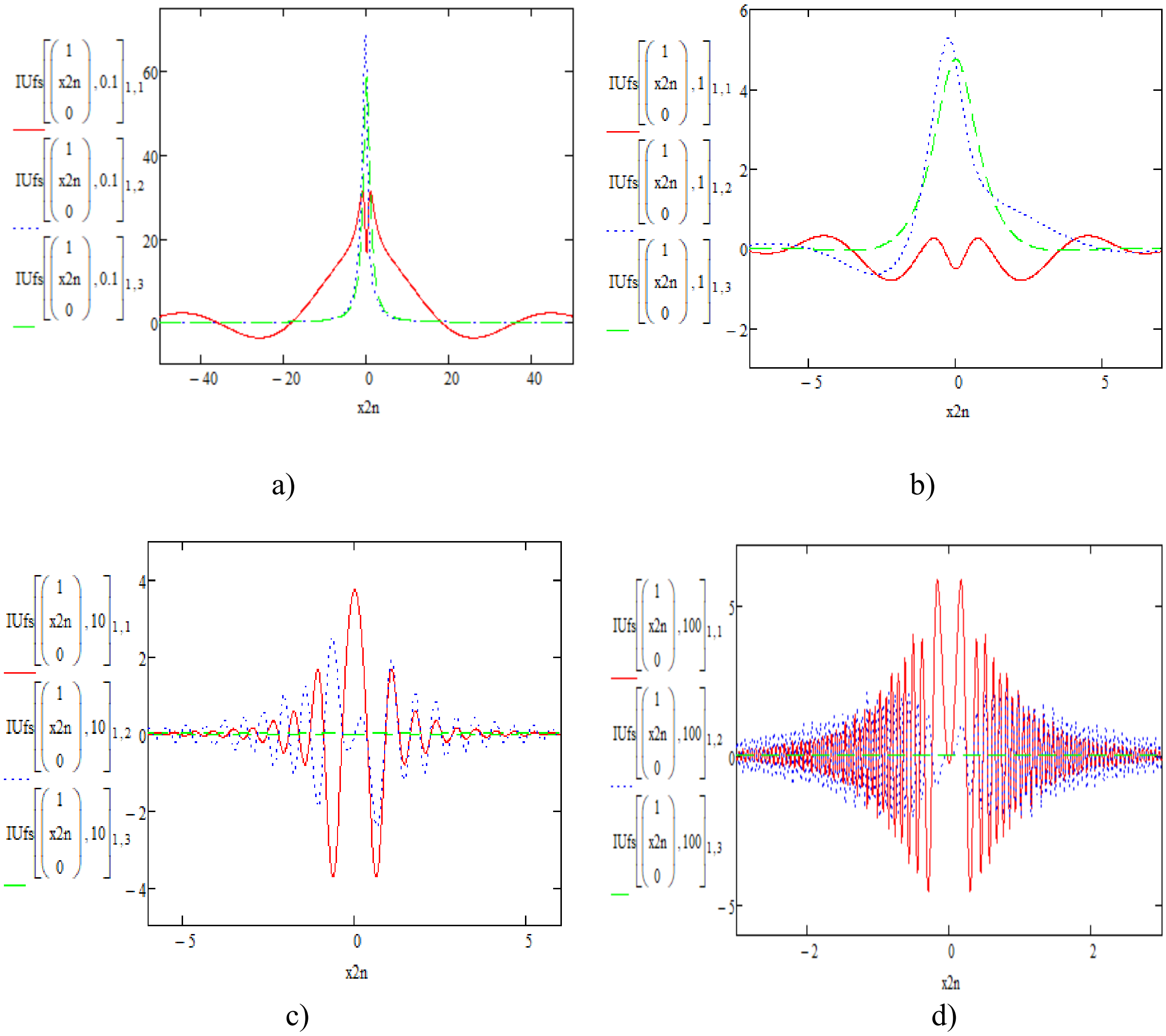
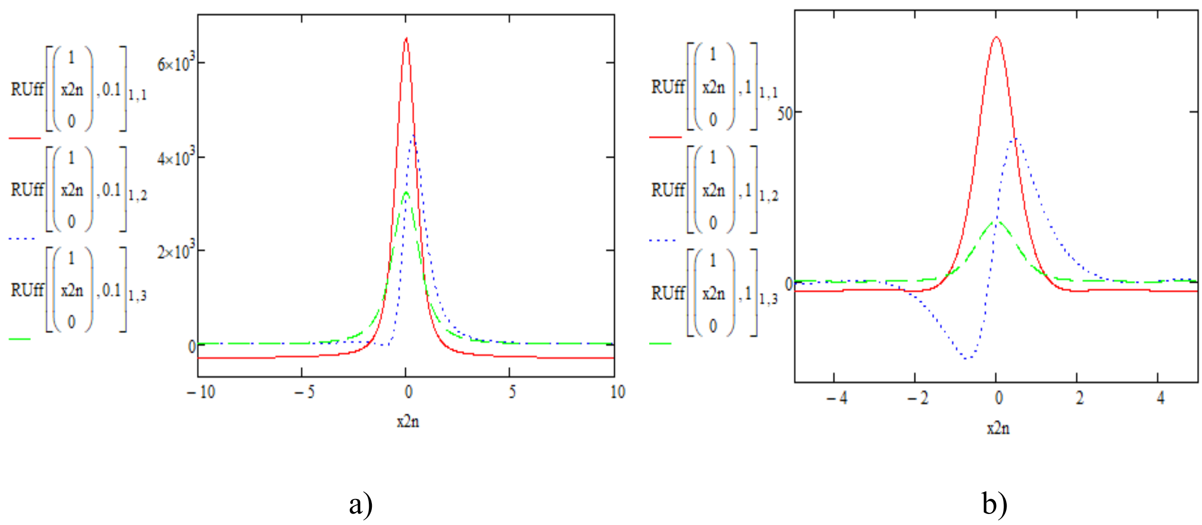
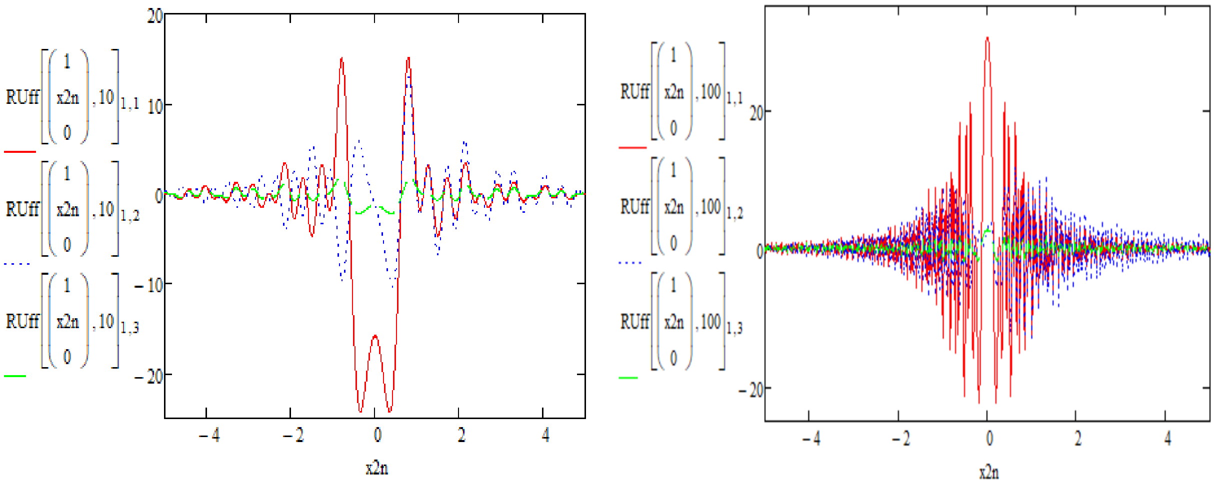


Figure 1.6: Displacement of fluid under the action of force F^f along the X2 axis at $t = nT + \dfrac{T}{4}$ and a) $\omega = 0.1$, b) $\omega = 1$, c) $\omega = 10$, d) $\omega = 100$

Calculations are given for water-saturated soil in dimensionless parameters. . The unit of stress is the shear modulus of the solid component μ , the density of the mass – is the density of water ρ_{22}, time in seconds. As is customary in seismology, we assume $\lambda = \mu$

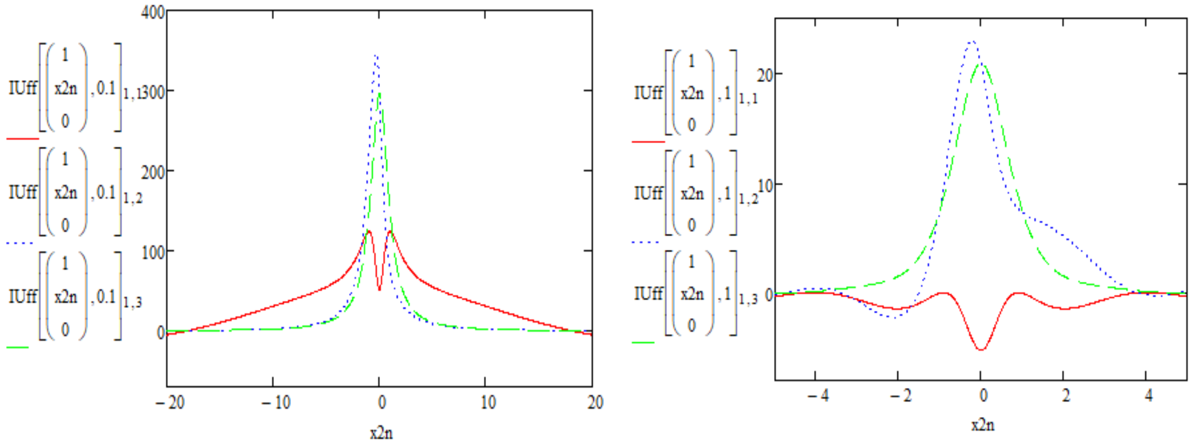




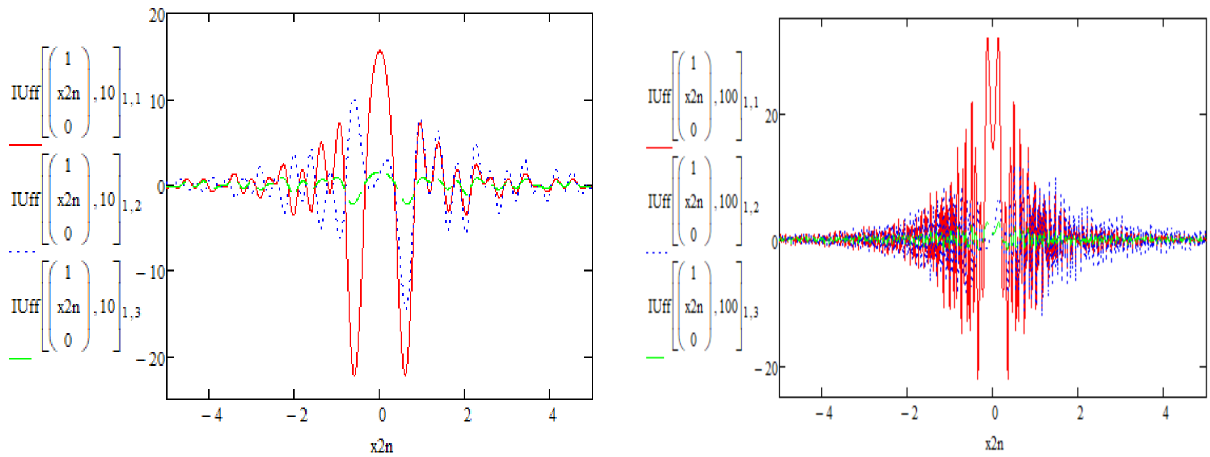
c) d)

Figure 1.7: Displacement of fluid under the action of force F^f along the X2 axis at $t = nT$ and a) $\omega = 0.1$, b) $\omega = 1$, c) $\omega = 10$, d) $\omega = 100$

The graphs show the amplitudes of oscillations in the Biot medium under the action of concentrated sources of oscillations at the origin of coordinates on the segments whose coordinates are indicated in columns of variable functions on the ordinate axis at fixed times, separated by a quarter of a period from each other. These are the maximum amplitudes, the magnitude of which depends on the frequency.



a) b)



c) d)

Figure 1.8: Displacement of fluid under the action of force F^f along the X2 axis at $t = nT + \frac{T}{4}$ and a) $\omega = 0.1$, b) $\omega = 1$, c) $\omega = 10$, d) $\omega = 100$

A program has also been developed for calculating the displacements of the solid and liquid components under the action of distributed forces in the solid and liquid components on the segments. Based on the materials of these studies, an article is being prepared for the foreign press.

8. Conclusion

Note that mass forces may be different from the space of generalized vector-function, singular and regular. Since Green tensor is singular, contains delta-functions, this convolution are calculated on the rule of convolution in generalized function space. If a support of acting forces are bounded (contained in a ball of finite radius), then all convolutions exist. If supports are not bounded, then the existence condition (17) require some limitations on behavior of forces at infinity which depend on the type of mass forces.

The obtained solutions allow us to study the dynamics of porous water and gas-saturated media at the action of periodic sources of disturbances of a sufficiently arbitrary form. In particular, under the action of certain forces on surfaces, for example cracks, in porous media that can be simulated by simple and double layers on the crack surface.

There is another feature of the Green tensor of the Biot equations, which can be used for solving boundary value problems based on the boundary untegral equations method.

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