

General Variational Principle, General Noether Theorem, Their Classical and Quantum New Physics and Solution to Crisis Deducing All Basic Physics Laws

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Abstract

This paper discovers that current variational principle and Noether theorem for different physics systems with (in) finite freedom systems have missed the double extremum processes of the general extremum functional that both is deduced by variational principle and is necessarily taken in deducing all the physics laws, but these have not been corrected for over a century since Noether's proposing her famous theorem, which result in the crisis deducing relevant mathematical laws and all physics laws. This paper discovers there is the hidden logic cycle that one assumes Euler-Lagrange equations, and then he finally deduces Euler-Lagrange equations via the equivalent relation in the whole processes in all relevant current references. This paper corrects the current key mistakes that when physics systems choose the variational extreme values, the appearing processes of the physics systems are real physics processes, otherwise, are virtual processes in all current articles, reviews and (text) books. The real physics should be after choosing the variational extreme values of physics systems, the general extremum functional of the physics systems needs to further choose the minimum absolute extremum zero of the general extremum functional, otherwise, the appearing processes of physics systems are still virtual processes. Using the double extremum processes of the general extremum functionals, the crisis and the hidden logic cycle in current variational principle and current Noether theorem are solved. Furthermore, the new mathematical and physical double extremum processes and their new mathematical pictures and physics for (in) finite freedom systems are discovered. This paper gives both general variational principle and general Noether theorem as well as their classical and quantum new physics, which would rewrite all relevant current different branches of science, as key tools of studying and processing them.

Keywords: Variational Principle, Noether Theorem, Mathematical Physics, Fundamental Interaction, Physics Law, Unification Theory, Classical and Quantum New Physics

PACS numbers

1. Introduction

A variational principle in science is to enables a problem being solved by using the calculus of variations, which optimizes the values of these quantities in the variational systems [1].

Fundamental physics laws can be expressed by a variational principle, which can give Euler-Lagrange equations and the corresponding convervation quantity [2, 3]. Noether generalized the variational principle to a now called Noether theorem by finding the transformation symmetry properties of variational systems and giving both Euler-Lagrange equations and the many conservation quantities depending on the corresponding many symmetries [4, 5].

Current variational principle and Noether theorem have been extensively used in different branches of science and have become key tools of studying and processing the different branches, for examples:

In mathematics [1, 6-9]: (i) The extremum method for solving boundary-value problems; (ii) Variational principle in mathematical optimization; (iii) Variational principle in mathematical extremum problems; (iv) Variational principle in mathematical motion equations and invariant quantities; (v) The finite element method; In physics [2, 10-14]: (i) Fermat's principle in geometrical optics; (ii) Maupertuis' principle in classical mechanics; (iii) The principle of least action in mechanics, electromagnetic theory and so on; (iv) The variational method in quantum mechanics; (v) Gauss's principle of least constraint and variational principle of least curvature; (vi) Hilbert's action principle in general relativity leading to Einstein field equations; (vii) Palatini variational principle; (viii) Variational principle in different field theories;...... . In astronmy and astrophysics, in chemistry, even in engineering and so on [9, 15-17].

Various variational principles and their applications are very well investigated, e.g., see [18-23]. Role of Noether's Theorem at the Deconfined Quantum Critical Point is studied, Noether's It is no losing the generality, because the results of the systems theorems and conserved currents in gauge theories in the with higher derivatives of q are the similar but more terms presence of fixed fields are explored, furthermore, Noether's relevant to higher derivatives of q. theorem and conserved quantities for the crystal- and ligand field Hamiltonians invariant under continuous rotational symmetry are investigated [24-26]. \mathbf{t} times, and then deducing and then deducing \mathbf{t} \arccos equivalent to Euler-Lagrange equality equals to \arccos therm and conserved quantities for the crystal- and figand field mitomans invariant under continuous rotational symmetry the time conservation $[24-20]$. *Preprints* **(www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 October 2020 doi:10.20944/preprints202008.0334.v3**

In current variational principle and Noether theorem, there are which are related to a hidden logic cycle problem and conservations which the needs in advance to assume existing some conditions which the needs in advance to assume existing some conditions which
are equivalent to Euler-Lagrange equations and conservation are equivalent to Euler-Lagrange equations and conservation
quantities, and then deducing Euler-Lagrange equations and quantities, and then deducing Euler-Lagrange equations and
conservation quantities, which are related to a hidden logic cycle problem and are not both exact and natural. blem and are not both exact and natural. urrent variational principle and Noether theorem, there are tions and conservation quantities, and then deducing urrent variational principle and Noether theorem, there are $\frac{1}{2}$ and conservation and conservation quantities, $\frac{1}{2}$ and $\frac{1}{2}$ a urrent variational principle and Noether theorem, there are needs in advance to assume existing some conditions which

Furthermore, we find that all the investigations on variational principle and Noether theorem for different physics systems have missed the key studies on the double extremum processes related to the general extremum functional that is deduced via
the least action principle and should be key largely taken in the least action principle and should be key largely taken in deducing all the physics laws, but the current variational principle
and current Noether theorem have missed the general extreme and current Noether theorem have missed the general extreme functionals and their minimum extremums for over a century $\frac{1}{2}$ and the minimum external to over a century since Noether's proposing her theorem $[4, 5]$, which result in the $\frac{1}{2}$ is proposing net incordin $\left[\frac{1}{2}, \frac{1}{2}\right]$, which result in the crisis of no objectively deducing all the physics laws. Using the studies on the double extremum processes related to the general extremum functionals in this paper, the crisis and the hidden extremant ranctionals in this paper, the erists and the medern
logic cycle problem are solved, and the new physical pictures are discovered. The general by the general thermore, we find that all the investigations on variational c cycle problem are solved, and the new physical pictures μ be vered. p_{min} respectively as some equations and the model be viewed as some Euler-Lagrange equations, the Euler- α also verted. e Noether's proposing her theorem [4, 5], which result in the is of no objectively deducing all the physics laws. Using the are solved, and the new physics laws. Can be exexis on the double extremally processes related to the general emum functionals in this paper, the crisis and the hidden thermore, we find that all the investigations on variational be viewe procedure and some equations, the Euler-Lagrange equations, the Euler- Δ ϵ viewed as some Euler-Lagrange equations, the Euler-Lagrange equations, the Euler-Legrie problem are solved, and the new physical pictures α and α and α not α not α not α .

No losing generality, all physics laws always can be expressed as some equations, these equations always can be expressed as some equations, these equations always can be viewed as
some Euler-Lagrange equations, the Euler-Lagrange equations some Euler-Lagrange equations, the Euler-Lagrange equations
always can be deduced by the general variational principle and/ or Noether theorem [4, 5]. Especially, the four fundamental or Noether theorem [4, 5]. Especially, the four fundamental interaction theories in the universe, i.e., the strong, weak, meraction theories in the universe, i.e., the strong, weak, electromagnetic and granvitational interaction theories, are directly deduced by variational principle and Noether theorem.
Therefore, there always is the crisis deducing all the physics Therefore, there always is the crisis deducing all the physics laws. This paper wants to solve the crisis. tromagnetic and granvitational interaction theories, are losing generality, all physics laws always can be expressed eral variational principle and/or Noether theorem [4, 5]. losing generality, all physics laws always can be expressed $\frac{1}{1}$ bonic equations, these equations always can be viewed as the universe, i.e., the universe, i.e., the strong generality, all physics laws always can be viewed as
interactions, always can be viewed as
the Euler-Lagrange equations, the Euler-Lagrange equations

The arrangements of this paper are: Sect. 2 shows unification studies on variational principle and Noether theorem for finite freedom systems; Sect. 3 investigates crisis of deducing physics hection systems, see: 5 investigates erists of dedicing physics
laws and its solution to the crisis for finite freedom systems; Sect. Find the section is one of the state of the recovering systems; Sect.
4 gives unification studies on variational principle and Noether theorem for in finite freedom systems; Sect. 5 studies crisis of deducing physics laws and its solution to the crisis for in finite deducing physics laws and its solution to the crisis for in finite tional principle factor and the solution to the virist for in time
freedom systems; Sect. 6 shows discussions and applications; Sect. 7 gives summary and conclusions. studies on variational principle and Noether theorem for arrangements of this paper are: Sect. 2 shows unification In systems, Sect. o shows diseassions and approations, mg physics laws and his solution to the crisis for in time dom systems; Sect. 6 shows discussions and applications; $\frac{1}{2}$ s and its solution to the crisis for finite freedom systems; Sect. ves unification studies on variational principle and Noether The Mathematical Principle and Principle and Principle and Principle on variation of the Michelle School of the School of th The exact mathematical descriptions of the least ac- $\ddot{}$, the action) of the action $\ddot{}$ $\frac{1}{4}$ $\frac{1}{2}$ about $\frac{1}{2}$ and conclusions.

sis for finite freedom systems; Sect. 4 gives unification

2. Unification Studies on Variational Principle and Noether Theorem for Finite Freedom Systems

The exact mathematical descriptions of the least action principle for a general case are: the variation of the integral of a general case are: the variation of the integral

(i.e., the action) of the Lagrangian *L* during $[t_1, t_2]$ about *N* generalized coordinates $q = (q_1, q_2, ..., q_n)$ is [13, 27]. μ _{1.} μ _{1.} 2015.

$$
\Delta A = A' - A = \int_{t_1'}^{t_2'} L'(q', \dot{q}', \ddot{q}', t')dt' - \int_{t_1}^{t_2} L(q, \dot{q}, \ddot{q}, t)dt = 0.
$$
\n(1)

It is no losing the generality, because the results of the systems It is no losing the generality, because the results of the with higher derivatives of q are the similar but more terms relevant to higher derivatives of q.

Among them, the general infinitesimal transformations are [27-20] 29]. are \mathbf{r} $\frac{27}{27}$

$$
t' = t'(q, \dot{q}, \ddot{q}, t, \alpha) = t + \Delta t = t + \varepsilon_{\sigma} \tau^{\sigma}, \qquad (2)
$$

$$
q_i^{(r)} = q_i^{(r)}(q, \dot{q}, \ddot{q}, t, \alpha) = q_i^{(r)} + \Delta q_i^{(r)} = q_i^{(r)} + \varepsilon_\sigma (\xi_i^{\sigma})^{(r)},
$$
\n(3)

in which $r = 0, 1, 2, \alpha = (\alpha_1, \alpha_2, ..., \alpha_m)$ are independent continuous variable parameters of Lie group \overline{G} and $\frac{1}{\sqrt{2}}$ which $r = 0, 1, 2, \alpha = (\alpha_1, \alpha_2, ..., \alpha_m)$ are independent conti: α and α

$$
(\xi_i^{\sigma})^{(r)} = \frac{\partial q_i^{(r)}(q, \dot{q}, \ddot{q}, t, \alpha)}{\partial \alpha_{\sigma}}|_{\alpha=0}, \sigma = 1, 2, ..., m; r = 0, 1,
$$
\n(4)

$$
\tau^{\sigma} = \frac{\partial t'(q, \dot{q}, \ddot{q}, t, \alpha)}{\partial \alpha_{\sigma}} \Big|_{\alpha=0} , \sigma = 1, 2, ..., m.
$$
 (5)

Eqs. (4) and (5) are the infinitesimal generating functions under Lets. (+) and (b) are the immediating generating randoms and
the operation of group G, ε_{σ} ($\sigma = 1,2,...,m$) are independent infinitesimal parameters corresponding to α , one dot and two dots infinitesimal parameters corresponding to α , one dot and two dots
denote the first and second order time derivatives respectively, the curve $q(t)$ is parameterized by time, and the path takes extremum corresponding $\Delta A = 0$. remum corresponding $\Delta A = 0$. $\mathbf{F}_{\mathbf{a}}$ and $\mathbf{F}_{\mathbf{a}}$ are the infinitesimal generating function functions function functions functi $s.$ (4) and (5) are the infinitesimal generating functions under being interesting to a ϵ_{σ} ($\sigma = 1, 2, ..., m$) are independent $s.$ (4) and (5) are the infinitesimal generating functions under

Doing as the well-known Refs. [2, 9, 13, 27, 29], we define _{ing as}

$$
L'(q', \dot{q}', \ddot{q}', t') = L(q', \dot{q}', \ddot{q}', t') + \varepsilon_{\sigma} \frac{d\Omega^{\sigma}}{dt} \quad , \sigma = 1, 2, ..., m.
$$
\n
$$
(6)
$$
\n
$$
B_{\sigma}(6)
$$
 into Eq. (1) angle

Putting Eq. (6) into Eq. (1), one has Putting Eq.(6) into Eq.(1), one has

dt i

 \overline{a}

i

i

 $\overline{}$

$$
\Delta A = \int_{t_1'}^{t_2'} [L(q', \dot{q}', \ddot{q}', t') + \varepsilon_{\sigma} \frac{d\Omega^{\sigma}}{dt}] dt' - \int_{t_1}^{t_2} L(q, \dot{q}, \ddot{q}, t) dt
$$
\n(7)

Using the technique of deducing Euler-Lagrange equations to simplify Eq. (7) and neglecting second-order infinitesimal quantities, we get \overline{a} U_s are technique of deducing Euler-Lagrange equations Using the technique of deducing Euler-Lagrange equa t is simplify σ in the second-order in-dependent in-dependen Using the technique of deducing Euler-Lagrange equaing the technique of deducing Euler-Lagrange equations $f(x)$ and $f(x)$ and $f(x)$ $\frac{1}{2}$ fing the technique of deduction

tities, we get
\n
$$
\Delta A = \int_{t_1}^{t_2} \left[\frac{d\Omega}{dt} + \sum_i \left[\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}_i} \right] \delta q_i + \frac{d}{dt} \left[\sum_i \left(\frac{\partial L}{\partial \dot{q}_i} \delta q_i + \frac{\partial L}{\partial \ddot{q}_i} \delta \dot{q}_i - \frac{d}{dt} \frac{\partial L}{\partial \ddot{q}_i} \delta q_i \right) + L \Delta(t) \right] dt. \quad (8)
$$
\n
$$
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$$

where $\Omega = \varepsilon_{\sigma} \Omega^{\sigma}$. Eq. (8) is simplified as

$$
\Delta A = 0 = \int_{t_1}^{t_2} \left\{ \sum_i \left[\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}_i} \right] \delta q_i + \right\}
$$

$$
\frac{d}{dt}\left[\sum_{i}\left(\frac{\partial L}{\partial \dot{q}_i}\delta q_i + \frac{\partial L}{\partial \ddot{q}_i}\delta \dot{q}_i - \frac{d}{dt}\frac{\partial L}{\partial \ddot{q}_i}\delta q_i\right)\right] + L\Delta(t) + \Omega\right]dt.
$$
 (9) 3. Crisis of Deducing Physics Laws and Its Solution
Ciss for Finite Freedom Systems
Case (iii): Using Eq. (9) and the rule of merging like

For Eq. (9), about the degree of freedom, there are still three different cases: $\sum_{i=1}^{\infty}$ there:
experiences: For Eq. (9), about the degree of freedom, there are still three exactly have a general functional expression different cases: \mathbf{F} . About the degree of freedom, the degree of freedom, the still stil

Case (i): When assuming Case (i): When assuming $\mathcal{C}(\mathbf{r})$: When assuming σ of Ω \mathbf{H} and cases se (i): When assuming

$$
\Delta A = \int_{t_1}^{t_2} \frac{d}{dt} \left[\sum_i \left(\frac{\partial L}{\partial \dot{q}_i} \delta q_i + \frac{\partial L}{\partial \ddot{q}_i} \delta \dot{q}_i - \frac{d}{dt} \frac{\partial L}{\partial \ddot{q}_i} \delta q_i \right) + L \Delta(t) + \Omega \right] dt,
$$
\n(10)

using Eq. (10), one has

$$
\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}_i} = 0, \tag{11}
$$

 (12)

where $i = 1, 2,...,N$, because δ_{qi} $(i = 1,2,...,N)$ are linear v independent each other because q_i are independent coordinates. U sing U sing U sing U where i $\mathcal{L} = \mathcal{L}$, and $\mathcal{L} = \mathcal{L}$ U sing U sing U deduce conservation \mathbb{R}^n where $i = 1, 2,...,N$, because o_{qi} $(i = 1,2,...,N)$ are independent of i where $i = 1, 2,...,N$, because δ_{qi} $(i = 1,2,...,N)$ are lineexercity Eq. (10), we deduce σ_{qi} (10), we deduce conservation quantity are independent coordinates. $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{2}$

Using Eq. (10), we deduce conservation quantity \cos ing Ly. (1)

$$
\sum_{i} \left(\frac{\partial L}{\partial \dot{q}_i} \delta q_i + \frac{\partial L}{\partial \ddot{q}_i} \delta \dot{q}_i - \frac{d}{dt} \frac{\partial L}{\partial \ddot{q}_i} \delta q_i \right) + L\Delta(t) + \Omega = const., \tag{12}
$$

because because because because because i wse≀

because (12)
\n
$$
\Delta q_i^{(r)} = \delta q_i^{(r)}(t) + q_i^{(r+1)}(t)\Delta t, r = 0, 1, 2,
$$
 (13)

Eq. (12) can be rewritten as Eq.(12) can be rewritten as E_{α} (12) can be rewritten as \mathcal{L}

$$
\sum_{i} \left(\frac{\partial L}{\partial \dot{q}_i} (\Delta q_i - \dot{q}_i \Delta t) + \frac{\partial L}{\partial \ddot{q}_i} (\Delta \dot{q}_i - \ddot{q}_i \Delta t) - \frac{d}{dt} \frac{\partial L}{\partial \ddot{q}_i} (\Delta q_i - \dot{q}_i \Delta t) \right) + L\Delta(t) + \Omega = const.
$$
 (14)

Eq. (14) is the result of variational principle. \cdot (14) is the result of variational principle. Eq. (14) is the result of variational principle. \mathbf{P} into Eqs.(14) into Eqs.(14) into Eqs.(14), we deduce must be d

Putting Eqs. (2) and (3) into Eq. (14) , we deduce m conservation quantities of the systems $\frac{d}{dt}$ and $\frac{d}{dt}$ $\frac{d}{dt$ i − qïito)
|- qïito Putting Eqs. (2) and (3) into Eq. (14) , we deduce m conservation Putting Eqs. (2) and (3) into Eq. (14) , we deduce μ Eqs. (2) and (3) into Eq. (14), we deduce in conservative

$$
\sum_{i} \left(\frac{\partial L}{\partial \dot{q}_i} (\xi_i^{\sigma} - \dot{q}_i \tau^{\sigma}) + \frac{\partial L}{\partial \ddot{q}_i} (\dot{\xi}_i^{\sigma} - \ddot{q}_i \tau^{\sigma}) \right) \qquad \text{e}
$$

$$
- \frac{d}{dt} \frac{\partial L}{\partial \ddot{q}_i} (\xi_i^{\sigma} - \dot{q}_i \tau^{\sigma}) + L \tau^{\sigma} + \Omega^{\sigma} = const^{\sigma}. \qquad (15)
$$

where we have used that ε_{σ} ($\sigma = 1; 2,...,m$) are independent infinitesimal parameters. Namely, eq. (15) is the Noetther is theorem's result. Noetther theorem's result. ere we have used that ε_{σ} ($\sigma = 1; 2,...,m$) are indeperhave used that ε_{σ} ($\sigma = 1; 2,...,m$) are independent N but the converter the convervation f $\mathbf{u} = \mathbf{u} \cdot \mathbf{u}$ where we have used that ε_{σ} ($\sigma = 1, 2,...,m$) are independent ϵ esult. t_{min} given-lagrange equations. Theorem \mathbf{q}_{min} , \mathbf{q}_{min} , \mathbf{q}_{min} , \mathbf{q}_{min} , \mathbf{q}_{min} , \mathbf{q}_{min} μ the converging term μ We can see that both variational principle and Noether We can see that both variational principle and Noether We can see that both variational principle and Noether pendent infinitesimal parameters. Namely, eq.(15) is the where we have used that ε_{σ} ($\sigma = 1, 2,...,m$) are independent

e
We can see that both variational principle and Noether theorem v all give the same Euler-Lagrange equations (11), but they give g the convervation quantities are very different, i.e., Eq. (14) and In Conversation of the for the same and taganize equations (11) , one and \mathbf{g} . I validit quantities al vation quantities are very different $i \in$ Eq. (14) and the convervation quantities are very different, i.e., Eq. (14) and We can see that both variational principle and Noether the We can see that both variational principle and Noether the
all give the same Euler-Lagrange equations (11), but they
the convervation quantities are very different, i.e., Eq. (14) the convervation quantities are very different i.e. $E_q(14)$ following, there are the almost same discussions below $\frac{1}{2}$ the convenient question are $\frac{1}{2}$ into $\frac{1}{2}$. Eq. (14) $\frac{1}{2}$ following, the almost same very different, i.e., Eq. $(1 +$ ent, give the same Eurer-Lagrange equations \mathcal{C} convervation quantities are very different, i.e., Eq. (14),

Eq. (15) respectively.

i

<u>t</u> l
C ∂L − d ∂L \overline{a}

∂qⁱ

∂L − d ∂L

∂qⁱ

−
− dt ∂q˙ⁱ

dt ∂q˙ⁱ

i

t1

^t²

t1 i

Case (ii): When assuming that there exists Eq. (11) , then putting Eq. (11) into Eq. (9) , one has Eq. (10) . In the following, there are the almost same discussions below Eq. (11) in Case (i).

$\frac{d}{d\Gamma}\sum_{i}(\frac{\partial L}{\partial a_{i}} + \frac{\partial L}{\partial a_{i}} - \frac{d}{d\Gamma}\frac{\partial L}{\partial a_{i}}) + L\Lambda(t) + O[\frac{1}{d\Gamma}\frac{d}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma}\frac{1}{d\Gamma$

 $\frac{a_i}{i}$ $\frac{b_i}{i}$ $\frac{d_i}{i}$ $\frac{d_i}{i}$ exactly have a general functional expression

$$
\int_{t_1}^{t_2} \sum_i \left[\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}_i} \right] \delta q_i dt = -\int_{t_1}^{t_2} \frac{d}{dt} \left[\sum_i \left(\frac{\partial L}{\partial \dot{q}_i} \delta q_i + \frac{\partial L}{\partial \ddot{q}_i} \delta \dot{q}_i - \frac{d}{dt} \frac{\partial L}{\partial \ddot{q}_i} \delta q_i \right) + L\Delta(t) + \Omega \right] dt. (16)
$$

Eq. (16) comes from the general systems' taking extremum of the Lagrangian, but when the system has no Eq. (10) or Eq. (11), or no Eqs. (10) and (11) , then the systems cannot give Euler-Lagrange equations and the corresponding conservation quantities. Namely, this case cannot give real physics laws, which is just the reason that current variational principle and current Noether theorem have missed the case (iii) [27-29]. $\frac{1}{2}$ (11), or no Eqs. (10) and (11), then the systems can Eq. (10) or Γ or (10) and (11) , then the theory of Γ or no eqs. (10) and (11), then the systems cannot give $= 0.05$ (11) \sim 0, (11) \sim 0, (11) \sim the Eagrangian, but when the system has no Eq. (10) or Eq. (11), or no Eqs. (10) and (11), then the systems cannot giv t_{tot} to Eqs. (10) and (11), and the systems cannot give Euler-Lagrange equations and the corresponding conservation Eq. (16) comes from the general systems' taking extr t_{u} can enveloped the case of α in α is α and α the Lagrangian, but when the system has no Eq. (10)
(11), or no Eqs. (10) and (11), then the systems can
Euler-Lagrange equations and the corresponding con
quantities. Namely, this case cannot give real phys that current variation was current variational principle and current Noether theorem have missed the age (iii) [27] current Noether theorem have missed the case (iii) [27]. Euler-Lagrange equations and the corresponding con (11), or no Eqs. (10) and (11), then the systems can $\frac{1}{\sqrt{2}}$. (16) comes $\frac{d}{dt}$ = defined the definition e general systems' taking extr cannot give real physics laws, which is just the reason in the reason is just the reason in the reason is just the reason in the reason in the reason in the reason in the reason is just the reason in the reason in the reas

Cases (i) and (ii) are necessary and sufficient conditions that just give real physics laws, and accordint to current variational principle and current Noether theorem, case (iii) at all cannot give real physics laws $[28,29]$. give real physics laws $[20,29]$. U_{other} Carrolling from the variation the variation of $\frac{1}{2}$ derived from the variation principle and current Noether theorem, case (iii) at all cannot give real physics laws [28,29]. 29.9 , case (iii) at all cannot give real physics laws. Cases (1) and (11) are necessary and sumclent condin C_1 and C_2 are necessary and sufficient conditions are necessary and sufficient conditions and sufficient conditions are necessary and sufficient conditions are necessary and sufficient conditions are necessary and s Cases (i) and (ii) are necessary and sufficient conditions in the real physics laws and accordint to current v E_{U} is the variation that E_{U} derived from the variation E_{U} give real physics laws $[28,29]$. principle and current indented theorem, case (iii) at a

Using Eq. (16) derived from the variational extremum, we can Come Eq. (10) derived nont the variational extremum, we exactly define a general extremum functional \mathcal{F} \mathbf{u} t² exactly define a general extremum functional
 $\int_0^t \frac{t_2}{a} dA \frac{dA}{dt} dA^2 \frac{dA}{dt}$ Using Eq. (16) derived from the variational extremun

$$
F = \int_{t_1}^{t_2} \sum_i \left[\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}_i} \right] \delta q_i dt = -\int_{t_1}^{t_2} \frac{d}{dt} \left[\sum_i \left(\frac{\partial L}{\partial \dot{q}_i} \delta q_i + \frac{\partial L}{\partial \ddot{q}_i} \delta \dot{q}_i - \frac{d}{dt} \frac{\partial L}{\partial \ddot{q}_i} \delta q_i \right) + L\Delta(t) + \Omega \right] dt. \tag{17}
$$

The new general equal equation functional F between the

The new general equal equation functional \overline{F} between the functional F_1 of deducing Euler-Lagrange equations having
merged like terms relevant to Euler-Lagrange equations and the $\frac{1}{1}$ of actions Euclidean Eugenige equations having functional F_2 of deducing the general conservation quantities having merged like terms relevant to the general conservation
synthics is deduced by extinting variational principle i.e. F quantities is deduced by satisfying variational principle, i.e., F $=$ F_1 = - F_2 , namely, $F_1 + F_2 = 0$, which just shows the variational $\frac{1}{1}$ $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$ $\frac{1}{2}$ extreme variet, our mese sun cannot give rear physics, see the studies below, these are very important classical and quantum studies below, these are very important classical and quantum
new physics processes of general physics systems, because this Lagrangian is a general classical or quantum Lagrangian. The new general equal equation functional F bet new physics processes of go t of deducing Euler-Lagrange equation-lagrange equations below, these are very important classical and quantum studies below, these are very important classical and quantum studies below, these are very important classical and quantum new physics processes of general physics systems, because this physics processes of general physics systems, oceanse un new physics processes or general physics systems, because this Lagrangian is a general classical or quantum Lagrangian. merged like terms relevant to Euler-Lagrange equation σ defined absolute value of the absolute value of the general extremum function σ $\frac{1}{2}$ conservation $\frac{1}{2}$ conservatives having the second conservative functional F_2 of deducing the general conservation of \mathcal{C} absolute value of the absolute value of the general extremum function \mathcal{C} The flew general equal equation-lunctional F below Lagrangian is a general classical or quantum Lagrangia \overline{u} is new general equation function function function \overline{u} between \overline{v} The new general equal equation functional F bet quantities is deduced by satisfying variational princip
= $F_1 = -F_2$, namely, $F_1 + F_2 = 0$, which just shows the v
attenue value, but these still cannot give real physics extreme value, but these still cannot give real physic studies below, these are very important classical and new physics processes of general physics systems, been $\ddot{}$ quantities is deduced by satisfying variational princip $= F_1 = -F_2$, hallely, $F_1 + F_2 = 0$, which just shows the v
extreme value, but these still cannot give real physic

t When the absolute value of the general extremum functional F r is taken as zero, because the minimum absolute value of any I is taken as zero, because the minimum absolute value of any
functional is zero, i.e., a general extremum (because the general runctional is zero, i.e., a general extremum (because the general
extremum functional F may generally take a lot of different values, e.g., arbitrary positive and/or negative values), then we ϵ generally have theorem all give the same Euler-Lagrange equations (11), $\mathcal{L}_{\mathcal{A}}$ quantum Lagrangian. t When the absolute value of the general extremum functional \vec{l}
r is taken as zero, because the minimum absolute value of an functional is zero, i.e., a general extremum (because the minimum sero, i.e., a general extremum (because the e generally have
¹ i
i $W_{\rm eff}$ the absolute value of the general extremum function μ α values, e.g., arbitrary positive and/or negative values)

dt² ∂q¨ⁱ

d
2 ∂L

dt
20 ∂q¨ⁱ

Eq.(11) in Case (i).

$$
\int_{t_1}^{t_2} \sum_i \left[\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}_i} \right] \delta q_i dt = 0
$$
\n
$$
= -\int_{t_1}^{t_2} \sum_i \frac{d}{dt} \left[\sum_i \left(\frac{\partial L}{\partial \dot{q}_i} \delta q_i + \frac{\partial L}{\partial \ddot{q}_i} \delta \dot{q}_i \right) \right]
$$
\n
$$
= -\frac{d}{dt} \frac{\partial L}{\partial \ddot{q}_i} \delta q_i \right) + L\Delta(t) + \Omega] dt = 0
$$
\n(18)

Thus, the first line of Eq. (18) is equivalent to case (ii), and the 1 sum of the second and third lines of Eq. (18) are equivalent to λ case (i), all these can give physics laws. Namely, the general case (i), all these can give physics laws. Namely, the general sextremum functional F takes the minimum absolute value, i.e., zero, all the physics laws can be deduced, otherwise, all the physics laws cannot be deduced. That is, Eq. (17) is deduced 1 from the variational extremum, Eq. (18) is further taking I the absolute extreme value zero, i.e., the minimum absolute. extremum, of the general extremum functional F. Therefore, we, for the first time, discover that it is the double extreme values λ (i.e., the extreme functional F's extremum) that result in that all the physics laws can be deduced, otherwise, all the physics \angle laws cannot be deduced. These are very important classical and raws cannot be deduced. These are very important classical quantum new physics processes of general physics systems. quantum new physics processes or general physics system $\frac{1}{2}$ the physics laws cannot be deduced. These are very important chassical and antum new physics processes of general physics systems. $e(t)$, an these can give physics faws. Namely, the general $\frac{1}{2}$ Callifornian, and the Lagrangian method is not naturally dependent with $\frac{1}{2}$ and e^{i} ; (i), all these can give physics laws. Namely, the general s $\frac{1}{2}$ is cannot be deduced. These are very important classical and s cannot be deduced. These are very important classical and s_{max} ntum new physics processes of general physics systems. extrement interpretequent to case (ii), and the $\frac{1}{2}$ ne extreme functional F's extremum) that result in that can be deduced, otherwise, all the physics laws cannot be the first line of Eq. (18) is equivalent to case (ii) , and the $\frac{1}{2}$ of the general extremus $\frac{1}{10}$ and $\frac{1}{10}$ is the double extreme values

Therefore, the systems first choose extreme value (i.e., via Eq. (1)) of the Lagrangian, and then we naturally deduce Eq. (9) , there are needs as usual in advance to assume existing case (i) or i (ii), because which are equivalent to Euler-Lagrange equations and conservation quantities, and then deducing Euler-Lagrange equations and conservation quantities, which are related to a hidden logic cycle and are not both exact and natural. Therefore, the systems first choose extreme value (i.e., $\sqrt{(1-x^2)^2 + (1-x^2)^2}$ \overline{c} general physics systems. T erefore, the systems first choose extreme value (i.e., via Eq. $s_{\rm eff}$ or (iii), because which are equiva-dimensional are equiva-dimensional are equiva-dimensional are equivacase (i) or (iii) can be naturally deduced (\sim tities, and then deducing Euler-Lagrange equations and refore, the systems first choose extreme value (i.e., via Eq. reflects the systems' intrinsical properties, namely, the in- $\frac{1}{\sqrt{1-\frac{1$ case (i) or (ii) can be naturally deduced (e.g., see the s t udies and conservation quantities, which are related to a

Actually, there naturally exists the general extremum functional *F* so that we can choose the absolute extreme value zero of the general extremum functional F , then case (i) or (ii) can be naturally deduced (e.g., see the studies below Eq. (18). Making these natural deductions reflects the systems' intrinsical properties, namely, the intrinsical mathematical and physical double extreme value processes. Otherwise, the systems cannot get real physical laws. These results are not only supplementary
developments of the current variational principle and current get ical physical laws. These results are not only supplementally
developments of the current variational principle and current Noether theorem but also classsical and quantum new physics corresponding to classical and quantum physics systems, Noether theorem but also classsical and quantum new because this Lagrangian is a general Lagrangian. $\frac{p_{\text{ref}}}{p_{\text{ref}}}}$ to classical and quantum physics sy $\frac{1}{2}$ $\frac{d}{dx}$ and $\frac{d}{dx}$ Notifical theorem but also classsical and quantum new physics μ physics systems, μ is the classical and quantum physics systems, to that we can choose the absolute extreme value zero extremum choice and the choice and further the during $\frac{1}{2}$ takes the minimum absorptional extremum functional F. extreme value corresponding ∆A = 0, which means that ause this Lagrangian is a general Lagrangian. party, there have any exists the general extrement renewond o that we can choose the absolute extreme value zero of \overline{a} experience to exaste and quantum physics system ause this Lagrangian is a general Lagrangian. procceses. Otherwise, the systems cannot get real physier theorem but also classsical and quantum new physics $\frac{1}{2}$ = 0. which means that means the corresponding means that means the $\frac{1}{2}$

For all times, both the Lagrangian and the action contain the systems' dynamics, and the real appearance case is that the path taken by the systems during $[t_1, t_2]$ takes extreme value u corresponding $\Delta A = 0$, which means that the systems can not i only choose but also make the least extremum choice and further choose the minimum absolute extremum of the general extremum functional F . For an umes, boun the Lagrangian and the action come For all times, both the Lagrangian and the action cont an times, both the Lagrangian and the action contain the extremum choice and further choose the minimum absoall times, both the Lagrangian and the action contain We discover that, up to now, all the investigations on all times, both the Lagrangian and the action contain the $t_{\rm{re}}$ and $\frac{1}{K}$ Γ results in the critical Γ . physics out also limits are four extremum encreve and ϵ choose the minimum absolute extremum of the general ϵ

We discover that, up to now, all the investigations on variational principle and Noether theorem for different physics systems have missed the key studies on the double extremum processes related to the general extremum functional F that is deduced ϵ insseq the key studies on the double extremum process the double extreme processes system σ ver that, up to now, all the investigations on variational exresults in the crisis and the hidden logic cycle of no obscover that, up to now, all the investigations on variational

via the least action principle and should be key largely taken in deducing all the physics laws, but the current variational 4 $\int_{\Gamma} \sum_{n=1}^{\infty} \left[\frac{\partial L}{\partial q_1} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_1} + \frac{d}{dt^2} \frac{\partial L}{\partial \ddot{q}_1} \right] \delta q_i dt = 0$ in weaking and current Noether theorem have neglected the general extreme function F and F' s minimum extremum, which $\frac{1}{2}$ results in the crisis and the hidden logic cycle of no objectively $I = \int_{0}^{t_2} \sum \frac{d}{dt} \sum (\frac{\partial L}{\partial t} + \frac{\partial L}{\partial t})$ deducing all physics laws. Using the studies on the double extremum processes related to the general extremum processes related to the general extremum functional F in this paper, the crisis and the hidden logic cycle are not only solved, but also the 8) new mathematical and physical double extremum processes and their new mathematical and physical dealer external processes and the Therefore, the general variantional principle and the general the Therefore, the general variantional principle and the general to
to Noether theorem for finite freedom systems are given, which al solve the crisis and the hidden logic cycle. discovered. The general varianties of the general principle pr nerefore, the general variantional principle and the general Noether theorem for finite freedom systems are given, which had not interest the $\frac{1}{2}$ exuelmum processes related to the general exuelmum proc related to the general extremum functional \vec{r} in this paper discussing an physics tawa. Sang the statics on the double
extremum processes related to the general extremum processes α and the processes related to the general extremating proc related to the general extremum functional F in this paper, the general variantional principles of F the did calculated the crisis and the cris heral extreme function \vec{r} and \vec{r} s minimum extremum, which

e.,
he **4. Unification Studies on Variational Principle and Noether** Theorem for Infinite Freedom Systems . unii
.. For general field variables X(x) = {Ψ(x), ϕ(x), 4. Unification Studies on Variational Principle and Noether

 $F_{\rm eff}$ general field variables $\mathcal{F}_{\rm eff}$ $\mathcal{F}_{\rm eff}$

and the process of the column systems
 $X(x) = \{ \Psi(x), \varphi(x), \omega_{\mu}(x), g_{\mu\nu}(x), \}$ \ldots), the exact mathematical descriptions of the least action principle for a general case are: the variation of the action about \overline{X} *N* field components $X = (X_1, X_2, \ldots, X_N)$ is For general case are: the variation of the action above eld components $X = (X, X, ..., X)$ is N field components $Y =$ α neighbors α field components $X = (X_1, X_2, \dots, X_N)$ is $\mathcal{L}_{\mathcal{A}}$ سس
. - - - \mathcal{X} and \mathcal{X} are \mathcal{X} .

$$
\text{as } \Delta A = A' - A = \int_{x_1'}^{x_2'} \mathcal{L}'(X'(x'), \partial_{\alpha}' X'(x'), \partial_{\alpha}' \partial_{\beta}' X'(x'), x') dx'^4
$$
\n
$$
- \int_{x_1}^{x_2} \mathcal{L}((X(x), \partial_{\alpha} X(x), \partial_{\alpha} \partial_{\beta} X(x), x) dx^4 = 0.
$$
\n
$$
\text{a.g.} \tag{19}
$$

in which the general infinitesimal transformations are $[28, 29]$. which the general infinitesimal transformations $% \mathcal{L}_{\mathcal{A}}\left(\mathcal{A}\right)$ in which the general infinitesimal transformations are [28, 29].

$$
x^{\prime \mu} = x^{\mu} + \Delta x^{\mu} = x^{\mu} + \varepsilon_{\sigma}(x)\tau^{\mu\sigma}(x, X(x), \partial_{\alpha}X(x), \partial_{\alpha}\partial_{\beta}X(x)),
$$
\n(20)

$$
X^{\prime\alpha}(x') = X^a(x) + \varepsilon_\sigma(x)\xi^{a\sigma}(x, X(x), \partial_\mu X(x), \partial_\mu \partial_\nu X(x))
$$
\n(21)

of are independent continuous variable parameters of Lie group G
an and where α where $X'^{\alpha}(x') = X^{\alpha}(x) + \Delta X^{\alpha}(x)$, $\omega = (\omega_1, \omega_2, ..., \omega_m)$ $\frac{1}{\text{d}}$

$$
\tau^{\mu\sigma} = \frac{\partial x^{\mu}(x, X(x), \partial_{\mu}X(x), \partial_{\mu}\partial_{\nu}X(x), \omega)}{\partial \omega_{\sigma}}|_{\omega_{\sigma}=0}, \tag{22}
$$

$$
\xi^{a\sigma} = \frac{\partial X^{\alpha}(x, X(x), \partial_{\alpha} X(x), \partial_{\alpha} \partial_{\beta} X(x), \omega)}{\partial \omega_{\sigma}} |_{\omega_{\sigma}=0},
$$
\n18.

where $\tau^{\mu\sigma}$ and ζ^{ab} ($\sigma = 1, 2, ..., m$) are infinitesimal transformation functions. $f(x) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} x_j$ are infinitesimal transferred e $\tau^{\mu\sigma}$ and ζ^{ab} ($\sigma = 1, 2, ..., m$) are infinitesimal transformation $\mathbf t$ refer t^2 and ζ $(0 - 1, 2, ..., m)$ are immitesimal transformations. formation functions.

> Eqs. (22) and (23) are the infinitesimal generating function under the operation of group $G, \varepsilon_{o}(\sigma=1,2,...,m)$ are independent and the operation of group $G, \mathcal{C}_g(\mathcal{O}^{\vee}, 1, 2, ..., m)$ are mapper infinitesimal parameters corresponding to ω . ∂ω^σ t th t ¹ t ² t are independent in \sum_{σ} in \sum_{σ} is \sum_{σ} in \sum_{σ} i infinitesimal parameters corresponding to ω . (22) and (23) are the immediate generating function (22) and (23) are the infinitesimal generation function $f(x)$ the state is (22) and (23) are the immediation of generating function

ral Without loss of generality, we define \overline{M} ^{re} in dependent infinitesimal parameters corresponding to \overline{M} parameters corresponding to \overline{M} parameters corresponding to \overline{M} parameters corresponding to \overline{M} parameters corresponding to $\overline{$ w. Without loss of generality, we define

$$
\mathcal{L}'(X'(x'), \partial_{\alpha}' X'(x'), \partial_{\alpha} \partial_{\beta} X'(x'), x') = \mathcal{L}(X'(x'), \partial_{\alpha}' X'(x'),
$$

all $\partial_{\alpha}' \partial_{\beta}' X'(x'), x') + \varepsilon_{\sigma} \partial_{\mu} \Omega^{\sigma}(X(x), \partial_{\alpha} X(x), \partial_{\alpha} \partial_{\beta} X(x), x),$
(24)

where $\sigma = 1, 2, ..., m$. Putting Eq.(24) into Eq.(19), one has wiit
1. $p = 1, 2, ..., m$. Futting Eq.(24) into Eq.(19), one $\tau = 1.2$ m. Butting $F_a(24)$ into $F_a(10)$, one re $\sigma = 1, 2, ..., m$. Putting Eq.(24) into Eq.(19), one $\mathcal{L}(\mathcal{X})$ $\sum_{i=1}^{n}$ $\sigma = 1, 2, ..., m$. Putting Eq.(24) into Eq.(19), one $\frac{1}{2}$ e where $\overline{}$ \mathbb{R}^n), x)+εσ∂µΩ^σ(X(

where α is the contribution of α into α into α into α

where α is the contribution of α into α into α into α

where α is the contract of α into α into α into α into Eq.(19), one Eq.(19),

 $\overline{}$ \mathcal{L}), ∂ $\overline{}$ \mathcal{L}), ∂ α∂ βX \mathcal{L} $\overline{}$

x), ∂α∂βX(x), *αα*βX(x), x), *ααβX(x), x*), x) − l(x), x) − l(x), x) − l(x), x), x)

∆A =

Case
\n
$$
\Delta A = \int_{x'_1}^{x'_2} [\mathcal{L}(X'(x'), \partial'_{\alpha} X'(x'), \partial'_{\alpha} \partial'_{\beta} X'(x'), x') + \varepsilon_{\sigma} \partial_{\mu} \Omega^{\sigma}(X(\alpha)))
$$
\n
$$
x), \partial_{\alpha} X(x), \partial_{\alpha} \partial_{\beta} X(x), x) - \mathcal{L}(X(x'), \partial'_{\alpha} X(x'), \partial'_{\alpha} \partial'_{\beta} X(x'), x')
$$
\n
$$
+ \mathcal{L}(X(x'), \partial'_{\alpha} X(x'), \partial'_{\alpha} \partial'_{\beta} X(x'), x')]dx'^4 - \int_{x_1}^{x_2} \mathcal{L}(X(x),
$$
\n
$$
\partial_{\alpha} X(x), \partial_{\alpha} \partial_{\beta} X(x), x)dx^4 = \int_{x'_1}^{x'_2} [\delta(\mathcal{L}(X'(x'), \partial'_{\alpha} X'(x'),
$$
\n
$$
\partial'_{\alpha} \partial'_{\beta} X'(x'), x') + \varepsilon_{\sigma} \partial_{\mu} \Omega^{\sigma}(X(x), \partial_{\alpha} X(x), \partial_{\alpha} \partial_{\beta} X(x), x) +
$$
\n
$$
\mathcal{L} + \frac{D\mathcal{L}}{D x^{\mu}} \Delta x^{\mu}]dx'^4 - \int_{x_1}^{x_2} \mathcal{L}(X(x), \partial_{\alpha} X(x), \partial_{\alpha} \partial_{\beta} X(x),
$$
\n
$$
\text{are gives}
$$
\n
$$
\text{Using}
$$

$$
x)dx^{4} = \int_{x_{1}}^{x_{2}} \left[(\frac{\partial \mathcal{L}}{\partial X^{a}} \delta X^{a} + \frac{\partial \mathcal{L}}{\partial X_{,\nu}^{a}} \delta \partial_{\nu} X^{a} + \right.
$$

$$
\frac{\partial \mathcal{L}}{\partial X_{,\nu\rho}^a} \delta \partial_\nu \partial_\rho X^a dx^4 + \int_{x_1}^{x_2} \left[\varepsilon_\sigma \partial_\mu \Omega^\sigma + \mathcal{L} + \frac{D\mathcal{L}}{D x^\mu} \Delta x^\mu \right] (
$$

$$
1 + \frac{\partial \Delta x^{\beta}}{\partial x^{\beta}})dx^{4} - \int_{x_{1}}^{x_{2}} \mathcal{L}(X(x), \partial_{\alpha} X(x), \partial_{\alpha} \partial_{\beta} X(x), x)dx^{4}
$$
\n(25)
\nwhere $D\mathscr{L}/Dx^{\mu}$ is whole derivative for the whole Lagrangian.

where $D\mathscr{L}/Dx^{\mu}$ is whole derivative for the whole Lagrangian. where DZ/DX^2 is whole derivative for the whole Lagrangian.
Using the technique of deducing Euler-Lagrange equations to Using the teeninque or deducing Edici-Eagrange equations to make Eq. (25) into order and neglecting two order in nitesimal quantities, we get $\frac{1}{2}$ into order and nemake Eq. (25) into order and neglecting two order in_nites: the extended to deducing Euler-Lagrange equations to (25) into order and neglecting two order in_nitesimal we get $D\mathscr{L}Dx^{\mu}$ is whole derivative for the whole Lagrangian.
he technique of deducing Euler Lagrange equations to ∂L (25)
where $D\mathscr{L}/Dx^{\mu}$ is whole derivative for the whole Lagrangian. \overline{a} ∂L $\overline{}$ $\frac{d}{dt}$ quantities, we get into order and negled
post neglecting two order in_nite
neglecting two order in_nite make Eq. (25) into order and neglecting two order in nitesima ing the technique of deducing Euler-Lagrange equations to ng the technique of deducing $\frac{1}{2}$ agrange equations to $\frac{1}{2}$ and coming to the authority of material expanses operations. $\frac{1}{2}$ glecting two order in $\frac{1}{2}$

$$
\Delta A = \int_{M^4} \{ \varepsilon_{\sigma} \partial_{\mu} \Omega^{\sigma} (X(x), \partial_{\alpha} X(x), \partial_{\alpha} \partial_{\beta} X(x), x) + [\frac{\partial \mathcal{L}}{\partial X^a} \n- \partial_{\mu} \frac{\partial \mathcal{L}}{\partial X^a,_{\mu}} + \partial_{\mu} \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^a,_{\mu\nu}}] \delta X^a + \partial_{\mu} [(\frac{\partial \mathcal{L}}{\partial X^a,_{\mu}} \n- \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^a,_{\mu\nu}}) \delta X^a + \frac{\partial \mathcal{L}}{\partial X^a,_{\mu\nu}} \delta X^a,_{\nu} + \mathcal{L} \Delta x^{\mu}] \} d^4 x \tag{26}
$$

 $\partial X^a, \mu \nu}^{\partial X^a, \mu \nu}$ ²² $\partial X^a, \mu \nu}^{\partial X^a, \mu \nu}$
Omitting the higher order infinitesimal quantities, Eq. (26) is simplified as $simplified as$ $\sum_{i=1}^{n}$ Omitting the higher order infinitesimal quantities, Eq. (26) is simplified as \mathcal{L} $\frac{d}{dt}$ of the higher order infinitesimal quantities. Eq. (26) is $\sum_{n=1}^{\infty}$ is simplified as

$$
\Delta A = 0 = \int_{M^4} \{ [\frac{\partial \mathcal{L}}{\partial X^a} - \partial_{\mu} \frac{\partial \mathcal{L}}{\partial X^a,_{\mu}} + \partial_{\mu} \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^a,_{\mu\nu}} \]
$$

$$
] \delta X^a + \partial_{\mu} [(\frac{\partial \mathcal{L}}{\partial X^a,_{\mu}} - \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^a,_{\mu\nu}}) \delta X^a + \frac{\partial \mathcal{L}}{\partial X^a,_{\mu\nu}} \delta X^a,_{\nu} + \mathcal{L} \Delta x^{\mu} + \Omega^{\mu}] \} d^4 x \tag{27}
$$

in which $\Omega = \varepsilon_o \Omega^{\sigma}(\sigma = 1, 2, ..., m)$ is one order infinitesimal quantity: Eq.(27) cannot directly give Euler-Lagrange equations, and there are some additional degrees of freedom. $\Omega = \varepsilon \Omega^{\sigma}(\sigma = 1, 2, ..., m)$ is one order infinites.

For Eq. (27), about the degree of freedom, there are still three

cases: cases: $\frac{1}{2}$ = $\frac{1}{2}$, $\frac{1}{2}$ = $\frac{1}{2}$, $\frac{1}{2}$ = s . \mathbb{R}^2 Case (I): When assuming $\overline{}$ finitesimal quantity. Eq.(27) cannot directly give Eulerin which <u>is one order in which is one order in a statement in the statement of the statement in-</u> $\frac{1}{2}$ cannot directly give Euler-Cannot directly give Euler-Cannot directly give Euler-Cannot directly give Euler-Cannot directly give $\frac{1}{2}$ finitesimal quantity. Eq. (27) cannot directly give Euler-Second give Euler-Second give Euler-Second give Euler-

)

)
+εσ∂Ωσ(X(X(X)σ(X)σ(X)

), ∂ α∂ $\frac{1}{2}$

), ∂ $\frac{1}{\sqrt{2}}$

> Case (I): When assuming Case (I): When assuming [∂]µ[([∂]^L Case (1) . When as $\overline{\text{Case}}$ (I) \cdot When as

still three cases:

Case (I): When assuming

in which $\mathcal{L} = \{x_1, x_2, \ldots, x_n\}$ is one order in-

in which $\mathcal{L} = \{x_1, x_2, \ldots, x_n\}$ is one order in $\mathcal{L} = \{x_1, x_2, \ldots, x_n\}$

Case (I): When assuming

Case (1): when assuming
\n
$$
\int_{M^4} \partial_{\mu} [(\frac{\partial \mathcal{L}}{\partial X^a,_{\mu}} - \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^a,_{\mu\nu}}) \delta X^a]
$$

For Eq.(27)), about the degree of freedom, there are

$$
+\frac{\partial \mathcal{L}}{\partial X^{a},_{\mu\nu}} \delta X^{a},_{\nu} + \mathcal{L}\Delta x^{\mu} + \Omega^{\mu}]\}d^{4}x = 0,
$$
 (28)
using Eq. (28), one has

 \mathbb{E} Eq.(28), one has \mathbb{E}

$$
\frac{\partial \mathcal{L}}{\partial X^a} - \partial_\mu \frac{\partial \mathcal{L}}{\partial X^a,_\mu} + \partial_\mu \partial_\nu \frac{\partial \mathcal{L}}{\partial X^a,_\mu\nu} = 0 \tag{29}
$$

 $=$ 0 (29)

because δX^a ($a = 1, 2, \ldots, N$) are independent each other. Eq. (29) α are just the usual Euler-Lagrange equations. because δX^a ($a = 1, 2, ..., N$) are independent each other. Eq. (29) the usual Euler-Lagrange equations. ∂ol and Luler-Lagrange equation
∩C20 and late

dre just the usual Eurer-Lagrange equations.
Using Eq. (28), we deduce a general continuous equation ϵ _{tio}n Jsing Eq. (28), we deduce a general continuous equation

$$
\partial_{\mu}J^{\mu} = 0 = \partial_{\mu} [(\frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu}} - \partial_{\nu}\frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}})\delta X^{a} + \frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}}\delta X^{a}{}_{,\nu} + \mathcal{L}\Delta x^{\mu} + \Omega^{\mu}]
$$
(30)

and its general conservation current and its general conservation current (25)

$$
J^{\mu} = \left(\frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu}} - \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}}\right) \delta X^{a} + \frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}} \delta X^{a}{}_{,\nu} + \mathcal{L}\Delta x^{\mu} + \Omega^{\mu}
$$
\nBecause\n
$$
(31)
$$

 $\overline{\mathbf{a}}$ $\frac{1}{2}$ is generated current curren

$$
\delta X^{a} = \Delta X^{a} - X^{a}, \quad \Delta x^{\nu}; \quad \delta X^{a}_{,\beta} = \Delta X^{a}_{,\beta} - X^{a}_{,\beta \nu} \Delta x^{\nu}, \tag{32}
$$

Eqs. (30) and (31) can be rewritten as ∂Xa,^µ Xa, 1) can be rewritten as (30) and (31) can be rewritten as $\frac{1}{2}$ (30) $\frac{1}{31}$ can be rewritten as rewritten as rewritten as rewritten as $\frac{1}{2}$ (30) and (31) can be rewritten a Eqs. (30) and (31) can be rewritten as

$$
\partial_{\mu}J^{\mu} = \partial_{\mu} [(\frac{\partial \mathcal{L}}{\partial X^{a},_{\mu}} - \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^{a},_{\mu\nu}})(\Delta X^{a} - X^{a},_{\nu'} \Delta x^{\nu'}) +
$$

$$
\frac{\partial \mathcal{L}}{\partial X^{a},_{\mu\nu}} (\Delta X^{a}_{,\nu} - X^{a},_{\nu\nu'} \Delta x^{\nu'}) + \mathcal{L}\Delta x^{\mu} + \Omega^{\mu}] = 0
$$
and a general conservation current (33)

and a general conservation current and a general conservation current ent

(26)
\n
$$
J^{\mu} = \left(\frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu}} - \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}}\right) (\Delta X^{a} - X^{a}{}_{,\nu'} \Delta x^{\nu'}) +
$$
\n
$$
\frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}} (\Delta X^{a}{}_{,\nu} - X^{a}{}_{,\nu\nu'} \Delta x^{\nu'}) + \mathcal{L}\Delta x^{\mu} + \Omega^{\mu} \quad (34)
$$

Namely, eq. (34) is the variational principle's result. √
∂Xa,µy, na ang \overline{a} $\frac{1}{\sqrt{2}}$ \mathbf{L} Namely, eq. (34) is the variational principle's result.

Using Eqs. (20) and (21), we achieve m continuos equations and $\frac{\text{Sing Eqs. (20) and (21),}}{\text{O.2}}$ \overline{a} δ Leps. (20) and (21), we arrive in communes equation
conservative currents $\frac{dy}{dx}$ Eqs. (20) and (21), we achieve m continuos equaterially currents \log Eqs. (20) and (21), we $\lim_{x \to 0}$ Eqs. (20) and (21), we achieve m continuos equation

$$
\partial_{\mu}J^{\mu\sigma} = \partial_{\mu} [(\frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu}} - \partial_{\nu}\frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}})(\xi^{a\sigma} - X^{a}{}_{,\nu'} \tau^{\nu'\sigma}) +
$$

$$
\frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}}(\xi^{a\sigma}_{,\nu} - X^{a}{}_{,\nu\nu'} \tau^{\nu'\sigma}) + \mathcal{L}\tau^{\mu\sigma} + \Omega^{\mu\sigma}] = 0
$$
(35)

− ∂^µ

− ∂^ν

 (33)

$$
J^{\mu\sigma} = \left(\frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu}} - \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}}\right) \left(\xi^{a\sigma} - X^{a}{}_{,\nu'} \tau^{\nu'\sigma}\right) + \mathbf{g}
$$

$$
\frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}} \left(\xi^{a\sigma}_{,\nu} - X^{a}{}_{,\nu\nu'} \tau^{\nu'\sigma}\right) + \mathcal{L}\tau^{\mu\sigma} + \Omega^{\mu\sigma} \qquad (36) \qquad \frac{\mathbf{U}}{\mathbf{e}}
$$

σ) + L_τ
2006 - Lisa + Lisa + 2007 + 2007 + 2007 + 2007 + 2007 + 2007 + 2007 + 2007 + 2007 + 2007 + 2007 + 2007 + 2007

where we have used that $\varepsilon_{\sigma}(\sigma = 1, 2, ..., m)$ are independent
infinite-implementary Neural parameters (20) is the Neuthern infinitesimal parameters. Namely, eq. (36) is the Noetther $\frac{1}{2}$ permanent infinitesimal parameters. Namely, eq.(36) is the extension pendent infinitesimal parameters. Namely, eq.(36) is the parameters. Namely, eq.(36) is the parameters. Namely, α p ent informal parameters. Namely, eq. (36) is the Notting Non s result. where we have used that $\varepsilon_{\sigma}(0^{\circ}-1, 2, ..., m)$ are independently, eq. (36) is the No infinitesimal parameters. Namely, eq. (36) is the Noettle theorem's result.

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Στην αναγκατοικό του αναγκατοικό του αναγκατοικό του αναγκατοικό του αναγκατοικό του αναγκατοικό του αναγκατ

(ξ,aσ

Using Eqs.(34) and (36) and
$$
\int_{M^3} \partial_0 J^0 dV = -\int_{M^2} J^i dS_i \rightarrow 0
$$
, $(S_i \rightarrow \infty, J^i \rightarrow 0)$,
conservation charges of variational principle and Noether

theorem, respectively conservation charges of variational principle and rocenter Conservation charges of variation
theorem, respectively Note that the property of the respectively conservation charges of variational principle and Noether theorem, respectively coloni, respectively

$$
Q_{vp} = \int_{M^3} \left[\left(\frac{\partial \mathcal{L}}{\partial X^a,_{0}} - \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^a,_{0\nu}} \right) (\Delta X^a - X^a,_{\nu'} \Delta x^{\nu'}) + \right]
$$

$$
\frac{\partial \mathcal{L}}{\partial X^{a},_{0\nu}}(\Delta X^a_{,\nu} - X^a_{,\nu\nu'} \Delta x^{\nu'}) + \mathcal{L}\Delta x^0 + \Omega^0]dV, \quad (37)
$$

$$
Q_{Nt}^{\sigma} = \int_{M^3} [(\frac{\partial \mathcal{L}}{\partial X^a,0} - \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^a,0\nu})(\xi^{a\sigma} - X^a,{}_{\nu'} \tau^{\nu'\sigma}) + \\ \frac{\partial \mathcal{L}}{\partial X^a,0\nu}(\xi^{a\sigma}_{,\nu} - X^a,{}_{\nu\nu'} \tau^{\nu'\sigma}) + \mathcal{L}\tau^{0\sigma} + \Omega^{0\sigma}dV_0^38)
$$

 σX^{α} , 0ν
where $\sigma = 1, 2, \ldots, m$. We can see that both variational principle and Noether theorem all give the same Euler-Lagrange equations (29) , but they give the convervation currents (charges) are very different, i.e., Eq. (34) and Eq. (36) (Eq. (37) and Eq. (38)) respectively. For E_q . E_q . 37 and E_q . 30 $(E_q$. $37)$ α (IVCI): α give the conversation catteries (enarges) are very
example. Eq. (24), and Eq. (26), (Eq. (27), and Eq. (29). erent, i.e., Eq. (34) and Eq. (36) (Eq. (37) and Eq. (38) E_{S} and Noether theorem all give the same Euler-Lagrange equations (29), but they give the convervation currents (charges) are very c_{c} are very different, i.e., c_{c} where $\sigma = 1, 2, \dots, m$. We can see that both variational principle $\frac{22}{3}$, out they give the convervation currents (charges) are ver different, i.e., Eq. (34) and Eq. (36) (Eq. (37) and Eq. (38)) currents, i.e., Eq. (34) and Eq. (36) (Eq. (37) and Eq. (3 $\frac{1}{2}$ different i.e., Eq. (34) and Eq. (36) (Eq. (37) and Eq. $\text{Hence, } \text{Hence, } \text{$ \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} that the are Eq.(29), then are Eq.(29), t

Case (II): When assuming that there are Eq. (29), then putting Eq. (29) into Eq. (27) , one has Eq. (28) . In the following, there the almost same discussions below Eq. (29) in Case (I) . se (ii). When assuming that there are Eq. (29), then putting $\overline{\text{CD}}$: When assuming that the area $\overline{\text{CD}}$ (29), then are then be (ii). When assuming that there are Eq. (29), then putting (20) . In the following (20) . e (II): When assuming that there are Eq. (29), then putting currents (charges) are very different, i.e., Eq.(34) and Case (II): When assuming that there are Eq. (29), then putting $\frac{1}{2}$ Case (ii). When assuming that there are Eq. (29), then put p_{200} (II), When assuming that there are Eq. (20), then \hat{p} Case (ii). When assuming that there are Eq. (29), then $p = \frac{\Gamma a}{29}$ and Γa (29). In the following $Eq. (27)1$

Eq.(36) (Eq.(37) and Eq.(38)) respectively.

5. Crisis of Deducing Physics Laws and Its Solution to the Crisis for Infinite Freedom Systems TSIS Of Deducing Physics Laws and Its Solut 5. Crisis of Deducing Physics Laws and Its Solution to the 5. Crisis of Deducing Physics Laws and Its Solution \overline{r} c to \overline{r} for \overline{r} in $\$ S. Crisis of Deuteing Fhysics Laws and its

Case (III): Using Eq. (27), we generally have $\overline{\mathbf{v}}$ }
{ ∂L ∂L ∂X^a [−]∂^µ

$$
\int_{M^4}\{[\frac{\partial \mathcal{L}}{\partial X^a}-\partial_\mu \frac{\partial \mathcal{L}}{\partial X^a,_{\mu}}+\partial_\mu \partial_\nu \frac{\partial \mathcal{L}}{\partial X^a,_{\mu\nu}}]\delta X^a d^4x=-\int_{M^4}\partial_\mu[(
$$

$$
\frac{\partial \mathcal{L}}{\partial X^a,_\mu} - \partial_\nu \frac{\partial \mathcal{L}}{\partial X^a,_\mu\nu} \Big) \delta X^a + \frac{\partial \mathcal{L}}{\partial X^a,_\mu\nu} \delta X^a,_\nu + \mathcal{L} \Delta x^\mu + \Omega^\mu \Big] \} d^4 x
$$
\n(39)

Eq. (39) comes from the general systems' taking extremum of the Lagrangian, but when the systems have no Eq. (28) or Eq. (29), or no Eqs. (28) and (29), then the systems cannot give Euler-Lagrange equations and the corresponding conservation quantities. Namely, this case cannot give real physics laws, which is just the reason that current variational principle and current Noether theorem have missed the case (III) [28, 29].

Cases (I) and (II) are necessary and sufficient conditions that just give real physics laws, and accordint to current variational

principle and current Noether theorem, case (III) at all cannot give real physics laws [28, 29]. remotive and carrent recenter theorem, case (11) at an cannot $\frac{1}{2}$ inciple and current Noether theorem, case (III) at all cannot merpre une current riccurer theorem, case (11) at an cumo $\overline{1}$ and $\overline{1}$ and $\overline{1}$ and sufficient conditions conditions conditions conditions conditions conditions of $\overline{1}$ the time that is the contract example $\frac{1}{2}$ and t_{t} is that just give real physics laws, and according to τ nciple and current indefine theorem, case (III) at all cannot t_{max} (II) t_{max} (II) t_{max} (III) C_2 and current incluent incording case (III) at an early tions that just give real physics laws, and accordint to principle and current Noether theorem, case (III) at all can principle and current Noether theorem, case (111) at an ear principle and current Noether theorem case (III) at all rem $\frac{1}{\pi}$ case ($\frac{1}{\pi}$) at all cannot give real physics laws [28, 29]

Using Eq. (39) derived from the variational extremum, we can Using Eq.(39) derived from the variational extremum, α exactly define a general extremum functional $\frac{1}{2}$ can be can extremum functional extremum, we laws. \mathbb{E} Eq. (39) derived from the variational extremum, we can rem [28, 29], case (III) at all cannot give real physics $\log E$ laws. g Eq. (39) derived from the variational extremum, we can current variational principle and current Noether theo-Using Eq. (39) derived from the variational extremum, we only U sing Eq. (39) derived from the variational extremum, we Using E_{α} (20) derived from the variational extremum $\lim_{x \to 0} E(x, y)$ defined non the variational

$$
G = \int_{M^{4}} \left[\frac{\partial \mathcal{L}}{\partial X^{a}} - \partial_{\mu} \frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu}} + \partial_{\mu} \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}} \right] \delta X^{a} d^{4}x
$$

$$
= - \int_{M^{4}} \partial_{\mu} \left[\left(\frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu}} - \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}} \right) \delta X^{a} \right]
$$

$$
+ \frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}} \delta X^{a}{}_{,\nu} + \mathcal{L} \Delta x^{\mu} + \Omega^{\mu} \right] d^{4}x \tag{40}
$$

The new general equal equation functional G between the functional G_1 of deducing Euler-Lagrange equations having runctional O_1 of deducing Euler-Lagrange equations having merged like terms relevant to Euler-Lagrange equations and the functional G_2 of deducing the general conservation quantities 7) having merged like terms relevant to the general conservation
was desired by estisfying verictional principle i.e. G quantities is deduced by satisfying variational principle, i.e., $G = C$, namely $G + G = 0$, which just above the variational G_1 = - G_2 , namely, G_1 + G_2 = 0, which just shows the variational $G_1 = -G_2$, namely, $G_1 + G_2 = 0$, which just shows the variational extreme value, but these still cannot give real physics, see the + studies below, these are very important classical and quantum
new physics processes of general physics systems, because this new physics processes of general physics systems, because this (8) Lagrangian is a general classical or quantum Lagrangian. merged like terms relevant to Euler-Lagrange equations and the ϵ_{max} $+$ functional G_1 of deducing Euler-Lagrange equations having new general equal equation functional G between the systems, because this Lagrangian is a general classical or The new general equal equation functional G between e new general equal equation functional G between the The new general equal equation-functional G between 18) Lagrangian is a general classical or quantum Lagrangian. $\frac{1}{2}$ functional σ_1 or deducing Euler-Lagrange equations na $\frac{1}{\pi}$ functional G of deducing Fuler-Lagrange equations $\frac{1}{2}$ of deducing Eurer-Eugrange equations merged me terms relevant to Eurer Eagluitge equations referred to $\frac{1}{2}$ or deducing the general conservation quantities is deduced. $b_{(f)}^{\text{max}}$ having merged the terms refevant to the general conservational principle, i.e., $c_{(f)}$ quantities is deduced by satisfying variational principle, i.

The new general equal equation functional G between

e When the absolute value of the general extremum function G is taken as zero, because the minimum absolute value of any function is zero, i.e., a general extremum (because the general extremum functional G may generally take a lot of different values, e.g., arbitrary positive and/or negative values), then we generally have $\frac{1}{\sqrt{2}}$ and we generally have race, e.g., arbitrary positive and of hegative values), then we herally have ally take a construction and the positive values, then we and, c.g., and then y positive alleger hegative values), values e^{α} arbitrary positive and/or negative values), then values, e.g., arbitrary positive and/or negative values), then we generally have β and α are α $\frac{1}{10}$ is taken as zero, because the minimum absolute value of $\frac{1}{10}$ $\mathbf y$ function is zero, i.e., a general extremum (because the general $\mathbf y$) ally take a lot of different values, e.g., and α may generally take a lot of different values, e.g., and α generally

g
\n
$$
\int_{M^{4}} \left[\frac{\partial \mathcal{L}}{\partial X^{a}} - \partial_{\mu} \frac{\partial \mathcal{L}}{\partial X^{a}, \mu} + \partial_{\mu} \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^{a}, \mu \nu} \right] \delta X^{a} d^{4}x = 0
$$
\n
$$
= - \int_{M^{4}} \partial_{\mu} \left[\left(\frac{\partial \mathcal{L}}{\partial X^{a}, \mu} - \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^{a}, \mu \nu} \right) \delta X^{a} \right.
$$
\n
$$
+ \frac{\partial \mathcal{L}}{\partial X^{a}, \mu \nu} \delta X^{a}, \nu + \mathcal{L} \Delta x^{\mu} + \Omega^{\mu} \right] d^{4}x = 0 \qquad (41)
$$

 \mathcal{L} [(thus the first line of Eq. (41) is equivalent to case (II), and the sum of the second and third lines of Eq. (41) are equivalent to case (I), these all can give physics laws. Namely, the general x extremum function G takes the minimum absolute value, i.e., zero, all the physics laws can be deduced. Otherwise, all the physics laws cannot be deduced. That is, Eq.(39) is deduced from the variational extremum, Eq.(41) is further taking the absolute extreme value zero, i.e., the minimum absolute extremum, of the general extremum functional G, therefore, we, for the first time, discover that it is the double extreme values (i.e., the extreme functional G's extremum) that result in that all the physics laws can be deduced, otherwise, all the physics laws cannot be deduced. These are very important classical and quantum new physics processes of general physics systems. δX^a,^ν ⁺L∆x^µ+Ω^µ]}d⁴^x a_{μ} [(thus the first line of Eq. (41) is equivalent to case (11) , a

Therefore, the systems first choose extreme value (i.e., via Eq.

 $\frac{1}{19}$ of the Lagrangian, and then we naturally deduce Eq. (27), there are needs as usual in advance to assume existing case (I) or (II) , because which are equivalent to Euler-Lagrange equations and conservation quantities, and then deducing Euler-Lagrange and conservation quantities, and then deducing Euler-Lagrange and conservation quantities, and then deducing Eurer Eagrange equations and conservation quantities, which are related to a equations and conservation quantities, which are related to hidden logic cycle and are not both exact and natural. hidden logic cycle and are not both exact and natural. $f(19)$ of the Lagrangian, and then we naturally deduce Eq. (2)

Actually, there naturally exists the general extremum functional G so that we can choose the absolute extreme value zero of the general extremum functional G , then case (I) or (II) can to general and continuum physics α , see the studies below Eq. (41). Lagrangian is a general deductions reflects the systems' intrinsical Making these natural deductions reflects the systems' intrinsical making these natural deductions vences the systems manifold properties, namely, the intrinsical mathematical and physical properties, namer, the intrinsical mathematical and physical double extreme value processes. Otherwise, the systems cannot processes. Such wise, the systems cannot get real physical laws. These results are not only supplementary developments of the current variational principle and current Noether theorem for infinite freedom systems, but also classsical and quantum new physics corresponding to classical and quantum physics systems, because this Lagrangian is a general Lagrangian. We discover that, up to now, all the investigations on variational principle and Noether theorem for different physics systems and infinite freedom systems have missed the key studies on the double extremum processes related to the general extremum functional G that both is deduced via the least shown the double and should be key largely taken in deducing all action principle and should be key largely taken in deducing all the physics laws, but the current variational principle and current the physics laws, but the current variational principle and current hidden logic calculation of the current variational principle and current Noether theorem for infinite freedom systems have missed the procedured incordin for immediate extreme functional *G* and G's minimum extremum, which results in the crisis and the hidden logic cycle of no
discovered. objectively deducing all physics laws. Using the studies on the double extremum processes related to the general extremum functional G in this paper, the crisis and the hidden logic cycle. are not only solved, but also the new mathematical and physical double extremum processes and their new mathematical and physical pictures are discovered. Therefore, general variantional extremum physical pictures are discovered. Therefore, general variantional priysical pictures are discovered. Therefore, general variantional principle and general Noether theorem for infinite freedom systems are given, which solve the crisis and the hidden logic cycle. Actually, there naturally exists the general extremum function general extreme functional G and G 's minimum extremum principle and general Noether theorem for infinite freedom for $\frac{1}{2}$ pplysical states. These results are not only supprement

6. Discussions and Applications

Using Eq. (17) derived from the variational extremum, we have a general extremum functional expression for finite freedom systems

$$
f=\sum_i\;[\frac{\partial L}{\partial q_i}-\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i}+\frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{q}_i}]\delta q_i=
$$

$$
-\frac{d}{dt}\left[\sum_{i}\left(\frac{\partial L}{\partial \dot{q}_i}\delta q_i + \frac{\partial L}{\partial \ddot{q}_i}\delta \dot{q}_i - \frac{d}{dt}\frac{\partial L}{\partial \ddot{q}_i}\delta q_i\right)\right] + L\Delta(t) + \Omega]. \tag{42}
$$

where *f* can take any functional value and $F = \int_{t_1}^{t_2} f dt$.
When the absolute value of the general extremum function

When the absolute value of the general extremum functional f is taken as zero, namely, taking the minimum absolute extreme t_0 and the general extremum functional f , i.e., the general value of the general extremum functional f , i.e., the general $\frac{1}{2}$ extremum functional f's extremum, that is, the double extremum functional f's extremum, that is, the double extremum tremum functional f's extremum, that is, the double extremum
process, Eq. (42) can directly deduce Euler-Lagrange equations μ to the linear independent properties of δ qi and the general due to the linear independent properties of δ qi and the general conservation quantity due to having taken the second line of Eq. (42) (42) as zero. due to the linear independent properties of δ qi and the general servation α as zero.

Adv Theo Comp Phy, 2024 Volume 7 | Issue 4 | 7 $\mathbf{E} = \mathbf{P}^{-1}$

Using Eq. (40) derived from the variational extremum, we deduce a general extremum functional expression for infinite freedom systems $U = U(40)$ derived from the variation from the variation $\frac{1}{2}$ Using Eq. (40) derived from the variational extremum,

$$
g = \left[\frac{\partial \mathcal{L}}{\partial X^a} - \partial_\mu \frac{\partial \mathcal{L}}{\partial X^a,_{\mu}} + \partial_\mu \partial_\nu \frac{\partial \mathcal{L}}{\partial X^a,_{\mu\nu}}\right] \delta X^a =
$$

$$
-\partial_{\mu} [(\frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu}} - \partial_{\nu} \frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}}) \delta X^{a} + \frac{\partial \mathcal{L}}{\partial X^{a}{}_{,\mu\nu}} \delta X^{a}{}_{,\nu} + \mathcal{L}\Delta x^{\mu} + \Omega^{\mu}]
$$
\n(43)

where g can take any functional value and $G = \int_{M^4} g d^4x$. When the absolute value of the general extremum functional g When the absolute value of the general extremum functional g when the absolute value of the general extrement ranchonal g is taken as zero, namely, taking the minimum absolute extreme is taken as zero, namely, taking the imminum absolute extreme value of the general extremum functional g, i.e., the general extremum functional g's extremum, that is, the double extremum process, Eq. (43) can directly deduce Euler-Lagrange equations due to the linear independent properties of δX^{α} and the general cale to the mixed maxpendent properties of δ*X* and the general conservation current due to having taken the second line of Eq. (43) as zero. value of the general extremum functional g, i.e., the general extremum, $\frac{1}{2}$ vantistication current due to having the second line of (43) as zero.

Therefore, this paper discovers that the processes no choosing the minimum absolute extremum zero of the general extremum the variathe imminum absolute extremum zero of the general extremum functional F (G) statisfying the variational extremum principle are still the virtual processes, because all current refererenes, e.g., refs, think of cases (iii) and (III) satisfying the variational extreme value cannot derive out Euler-Lagrange equations and their correspoonding conservation quantities $[2, 4, 5, 10-14]$. Thus for choosing the processes of minimam absolute extremum zero of the general extremum functional $F(G)$, the processes of the physics systems are just real physics processes and can give $Euler-Lagrange$ eqautions and their corresponding conservation quantities. Especially, cases (i) and (ii) $($ I) and (II)) are the two special taken value cases and are included in case (iii) $((II))$ as special taken value cases and are included in case (iii) $((III))$ as special cases, and there is the hidden logic cycle between case special cases, and there is the hidden logic cycle between case (i) (assuming to exist Eq.(10) of deducing conservation quantity, then putting Eq.(10) into Eq.(9), one can deduce Euler-Lagrange Eq.(11) and case (ii) (assuming to exist Euler-Lagrange Eq.(11), then putting Eq. (11) into Eq. (9) , one can deduce Eq. (10) of deducing conservation quantity), namely, cases (i) and (ii) are equivalent with each other, which means that one assumes Euler-Lagrange equations in case (ii), and then he finally deduces Euler-Lagrange equations in case (i) via the equivalent relation between cases (i) and (ii) in the whole processes, which is just the hidden logic cycle, so does Eq.(10) of deducing conservation quantity (similar for cases (I) and (II)). Especially, from this paper it can be seen that the current investigations about cases (i-iii) ((I-III)) in all current references, e.g., refs, are no the exact general investigations [2, 4, 5, 10–14]. (i) (assuming to exist Eq.(10) of deducing conservation quant tunctional Γ (O) statistying the variational extremum principle are still the virtual processes.

Therefore, this paper corrects the current key mistake concepts that when physics systems choose the variational extreme values, the appearing processes of the physics systems are real physics processes, otherwise, are virtual processes in all current articles, reviews and (text)books, e.g., [2, 4, 5, 10–14]. The real physics should be what after choosing the variational extreme values of physics systems, the general extremum functional F (G) of the physics systems needs to further choose the minimum absolute extremum zero of the general extremum functional F (G), otherwise, the appearing processes of physics systems are still

virtual processes because the virtual process cases cannot deduce Euler-Lagrange equations and their corresponding conservation quantities. All the investigations on functionals F and G in this paper give the corresponding integral descriptions, using functional f and g we can give the corresponding differantial descriptions, the two descriptions are entirely equivalent, thus we don't repeat more here.

7. Summary and Conclusions

People can deduce Euler-Lagrange equations and corresponding conservation quantities by utilizing the variational principle and Noether theorem. But we discover the fact that the systems generally have extra intrinsical freedoms of choice. And if not assuming to exist Eq. (10) or (11) (Eq. (28) or (29)), then the Lagrange systems cannot give true physical laws. Actually, Eqs. (10) and (11) (Eqs. (28) and (29)) are equivalent to Euler-Lagrange equations and conservation quantities, and then deducing Euler-Lagrange equations and conservation quantities, which are related to a hidden logic cycle and are not both exact and natural. This paper discovers that the processes no choosing the minimum absolute extremum zero of the general extremum functional F (G) statisfying the variational extremum principle are still the virtual processes, because all current refererenes think cases (iii) and (III) satisfying the variational extreme value cannot derive out Euler-Lagrange equations and their correspoonding conservation current. For choosing the processes of minimam absolute extremum zero of the general extremum functional F (G), the processes of the physics systems are just real physics processes and can give Euler-Lagrange eqautions and their corresponding conservation quantities, which are the key new physics.

Especially, cases (i) and (ii) (1) and (1)) are included in case (iii) $((III))$ as special cases of case (iii) $((III))$, and there is the hidden logic cycle between case (i) and case (ii) ((I) and (II)), namely, cases (i) and (ii) (1) and (1)) are equivalent with each other, which means that one assumes Euler-Lagrange equations, and then he finally deduces Euler-Lagrange equations via the equivalent relation between cases (i) and (ii) ((I) and (II)) in the whole processes, which is just the hidden logic cycle, so does Eq.(10) of deducing conservation quantity. This paper corrects the current key mistakes that when physics systems choose the variational extreme values, the appearing processes of the physics systems are real physics processes, otherwise, are virtual processes in all current articles, reviews and (text)books. The real physics should be what after choosing the variational extreme values of physics systems, the general extremum functional F (G) of the physics systems needs to further choose the minimum absolute extremum zero of the general extremum functional F (G), otherwise, the appearing processes of physics systems are still virtual processes because the virtual process case (iii) (III) cannot deduce Euler-Lagrange equations and their corresponding conservation quantities.

The systems first choose extreme value, and then must choose the minimum absolute extremum of the general extremum functional $F(G)$, then cases (i) or (ii) (cases (I) or (II)) can be naturally deduced. Making these deductions shows the systems' intrinsical properties of taking double extreme values, otherwise cannot get real physical laws according exact deduction logic.

These results are not only supplementary developments of the current variational principle and current Noether theorem for finite (infinite) freedom system, but also classsical and quantum new physics corresponding to classical and quantum physics systems, because our Lagrangian is the most general. This paper discovers, up to now, all the studies on variational principle and Noether theorem for different physics systems with finite (infinite) freedom systems have neglected the key studies on the double extremum processes of the general extremum functional F (G) that both is deduced by the least action principle and is key largely taken in deducing all the physics laws, but these have not been done, which result in the crisis of deducing relevant mathematical laws and all physics laws. Using the above studies on the double extremum processes of the general extremum functional F (G) in this paper, i.e., on the double extreme values, the crisis and the hidden logic cycle are not only solved, but also the new mathematical and physical double extremum processes and their new mathematical pictures and physics are discovered. Therefore, general variantional principle and general Noether theorem for (in)finite freedom systems are given in this paper, which solve both the crisis having existed for over a century since Noether's proposing her famous theorem and the hidden logic cycle. Therefore, this paper gives general variational principle, general Noether theorem, their classical and quantum new physics and solution to crisis deducing all fundamental physics laws, opens a new area of research on variational principle and Noether theorem for finite (infinite) freedom systems by choosing optima of the double extreme values to explain origins of physics laws etc., and will significantly influence and rewrite the research of others in relevant different branches of science, because the least action principle or variational principle and Noether theorem are the key firm bases in modern physics, and we just discover new right avenues of research in the established variational principle & Noether theorem for finite (infinite) freedom systems, their applications and so on in modern sciences, and all the relevant current articles and (text)books would be rewritten, supplied and updated.

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