

Forecasting Budgetary Cycles in the Member States of the West African Economic and Monetary Union (Waemu): A Markov Chainbased Approach

Damien J. Agboton*

Statistician Economist Engineer (ISE), National School of Applied Economics and Management (ENEAM), University of Abomey Calavi (UAC), Benin

***Corresponding Author**

Damien J. Agboton, Statistician Economist Engineer (ISE), National School of Applied Economics and Management (ENEAM), University of Abomey Calavi (UAC), Benin.

Submitted: 2024, Oct 01; Accepted: 2024, Oct 23; Published: 2024, Nov 20

Citation: Agboton, D. J. (2024). Forecasting Budgetary Cycles in the Member States of the West African Economic and Monetary Union (Waemu): A Markov Chainbased Approach. *J Electrical Electron Eng*, 3(6), 01-20.

Abstract

This article models the various budgetary states of the eight WAEMU member countries using Markov Chains. A panel of 512 observations was constructed from the BCEAO database covering the period from 1960 to 2023. Our results highlight a dispersed convergence of states toward the budget deficit threshold imposed by the macroeconomic convergence criteria, identifying Mali, Senegal, and Benin as the top performers in the region in terms of effective deficit management. Furthermore, fiscal consolidation is necessary for Côte d'Ivoire, Guinea-Bissau, Burkina Faso, Togo, and Niger to stabilize their budget deficits in the long term and further stimulate economic growth.

Keywords: Budget Deficit, Economic Growth, Markov Chains, Panel, WAEMU

1. Introduction

The COVID-19 health crisis of 2020, compounded by the Russo-Ukrainian war that began on February 24, 2022, has led the West African governments to implement significant demand stimulating programs, substantially increasing public debt levels. In the WAEMU, the overall budget deficit exceeded 8.4% in some Union countries in 2022, a shocking level in peacetime [1]. Various macro econometric models have been developed in this context to analyze budgetary scenarios in the region.

The availability and relative loosening of the constraint of statistical data, theoretical innovations, and new concerns in development policy have fostered the development of a multitude of models applied to developing countries [2].

The new range of models applied to developing countries includes accounting or quasi-accounting models, macro econometric models, computable general equilibrium models, vector autoregressive (VAR) models, and microsimulation models [3-9]. These different tools allow for projection exercises (accounting or quasi-accounting models), forecasting (macro econometric models, VAR models), and simulation (macro econometric models, computable general equilibrium models, microsimulation models).

Most developing countries have experimented with constructing decision-making tools in the field of macroeconomic modeling. Indeed, various models have been developed in this dynamic: [10-13]. However, most macroeconomic models applied in these economies are accounting and quasi-accounting models [14]. Thus, their scope is limited as they do not provide insights into economic policy choices or account for the impact of exogenous shocks.

The availability of a forecasting model is of great assistance to public decision-makers, as such a tool fulfills critical functions such as supporting the definition of economic and financial programs, serving as an instrument for dialogue with development partners, and monitoring economic and social policies. This paper emphasizes establishing a multi-country model that takes into account the specificities of the state, the WAEMU region, and those of the central bank. In recent years, researchers have rekindled interest in examining the extent to which fiscal policy could and should be used as a tool for stabilization and macroeconomic smoothing. One of the main challenges in modeling fiscal policy is that it is intrinsically dynamic due to the accumulation of public debt. An increase in debt manifests as a constraint on the use of fiscal policy within the framework of stabilizing an economy, and thus warrants explicit modeling.

However, empirical attempts using cross-sectional or panel data techniques to predict budget deficits have failed to reach a consensus or clear direction. Indeed, several theoretical arguments highlight that macroeconomic forecasts cannot be made solely through the analysis of cross-sectional and panel data. Empirical results and policy recommendations vary and are sometimes contradictory. Previous studies have not been conclusive in terms of policy recommendations that can be consistently applied across all countries. These differences seem to arise from varying datasets, country-specific characteristics, and different methodologies used. Several variants of Markovian models have been developed by including Markov chains, Markov fields, Markov trees, coupled and tripled Markov models, and Markov models coupled with Dempster-Shafer theory [15-17].

This lack of consensus is also true for studies using data from WAEMU countries. In the specific case of the West African Economic and Monetary Union (WAEMU), a grouping of eight (8) states that notably share the use of the CFA franc and monetary policy under the direction of the Central Bank of West African States (BCEAO), the issue of convergence criteria holds particular interest in several respects. In this study, we contribute to the debate on forecasting budgetary cycles in two ways.

First, we employ recently developed forecasting methods to predict the budget deficits and surpluses of WAEMU member countries, using a large and homogeneous sample of 64 years for each WAEMU country. These methods avoid the low power issues associated with classic forecasting models. We simulate potential trajectories of the state's budget through public expenditures and tax revenues. This analysis aims to shed light on future economic policies. The basic model, which is very popular, emphasizes the deficit-surplus dichotomy of the state budget. We expand it to account for intermediate budget situations. Indeed, most countries in the WAEMU zone have accumulated significant deficits over time, although they remain modest concerning the 3% threshold set by the Union. This observation leads us to consider intermediate situations between budget deficit and surplus. These include a low deficit (between -5% and -3%), a low surplus (between 1% and 2%), and a balanced budget (deficit=0). Notably, a deficit is considered high when it is below -5%, and a surplus is considered high when it is above 2%. Our approach considers transitions between the main states as exogenous ingredients.

Formally, we use Markov chains composed of transition matrices. Each scenario is determined by a set of specific matrices. We model the transitions between budget deficit and surplus. To our knowledge, there are no recent studies in the WAEMU region that have utilized Markov chains for budgetary forecasting.

The second contribution of this study is the recommendation of a reference framework to optimize the use of fiscal policy. However, the earlier measures taken to prevent excessive deficits are implemented, the less painful they are likely to be. It is thus essential to identify potentially dangerous budgetary developments early enough to take preventive action. It is also important to

understand the causes of these developments so that prevention is targeted to have a maximal impact with minimal inconvenience. It should be noted that our analysis is statistical and econometric in nature. It does not carry the value of a complete macroeconomic modeling or a thorough analysis of fiscal policy. However, it is valuable because, in our view, our method provides a realistic order of magnitude of possible budgetary evolutions.

The rest of the article is organized as follows: Section 1 presents a brief literature review and the empirical approach used to study the relationship between inflation and economic growth. Section 2 discusses the main findings and robustness. Finally, Section 3 summarizes and concludes.

1.1 Theoretical Literature on Budgetary Cycles

1.1.1 Deficit and Budget Surplus

A budget deficit is a situation in which the state's revenues (excluding loan repayments) are lower than its expenditures (excluding borrowing) in a given year. It is, therefore, a negative balance.

It differs from the public deficit, as it does not encompass the balance of revenues and expenditures of other public administrations (local authorities and social security bodies, in particular).

The budget deficit translates into new borrowing that the state must contract during the year. If the state must borrow to obtain the necessary liquidity to cover maturing previous borrowings, this is referred to as the "snowball effect."

The budget balance is the difference between the revenues and expenditures of public administrations. A budget deficit occurs when an administration spends more than it receives in a given year. In contrast, it will show a surplus if its revenues exceed its expenditures. The budget balance includes a structural component (related to occasional variations in revenues and expenditures) and a cyclical component. There is a structural deficit when the economy operates at full capacity and the administration continues to spend more than it receives. The cyclical component of the deficit is sensitive to the economic cycle and results from the difference between actual and potential production. For example, during an economic contraction, the cyclical deficit results from a decrease in revenues and an increase in expenditures aimed at social programs such as unemployment benefits.

On average, the deficit of WAEMU member countries has been increasing since 2020, partly due to global economic disruptions related to the COVID-19 crisis in 2020 and the Russo Ukrainian war in 2022, and partly because some countries follow a procyclical policy (increasing public expenditures as a percentage of GDP when growth is strong or "failure to save for rainy days"). Between 2020 and 2024, the GDP growth rate in WAEMU member countries averaged 4.6% per year, but most of these countries recorded

1.2 Recent Economic Situation and Medium-Term Prospects of the WAEMU Zone

Despite a challenging international environment, growth remains robust in the economies of the WAEMU zone, according to the information note from the WAEMU region—July 2023. The growth rate of the WAEMU region is estimated at nearly 6.0% in 2022, compared to 6.1% and 1.7% in 2021 and 2020, respectively, while the average growth of Sub-Saharan Africa was 4.0% in 2022, down from 4.7% the previous year. WAEMU remains one of the most dynamic economic zones in Sub-Saharan Africa, driven by the performance of its member states' economies.

Despite the economic slowdown observed in 2020 due to the adverse effects of COVID-19, the economies of the WAEMU zone have been dynamic over the last five years, achieving an average growth of over 5.0%, exceeding the Sub-Saharan African average. This momentum is expected to continue in the medium term, linked to the ongoing implementation of major projects included in the National Development Programs (PND) and positive outlooks in the hydrocarbon sector. According to projections from the Central Bank of West African States (BCEAO), growth in the WAEMU zone is expected to accelerate, surpassing 6.0% in 2024.

Economic activity in the WAEMU zone in 2023 occurred amid persistent security risks due to terrorist threats and a slowdown in global growth. However, growth remained strong in the region in 2023. BCEAO's December 2023 estimates indicate a real GDP growth rate for the Union of 5.7% in 2023, consistent with the previous year's performance, despite a challenging international climate. This growth was driven by the tertiary sector, contributing 3.0 percentage points to the Union's growth, followed by the secondary (+1.6 points) and primary sectors (+1.1 points). All WAEMU member states recorded economic growth exceeding 4.0% in 2023, while the global economy and Sub-Saharan Africa grew by only 3.1% and 3.3%, respectively.

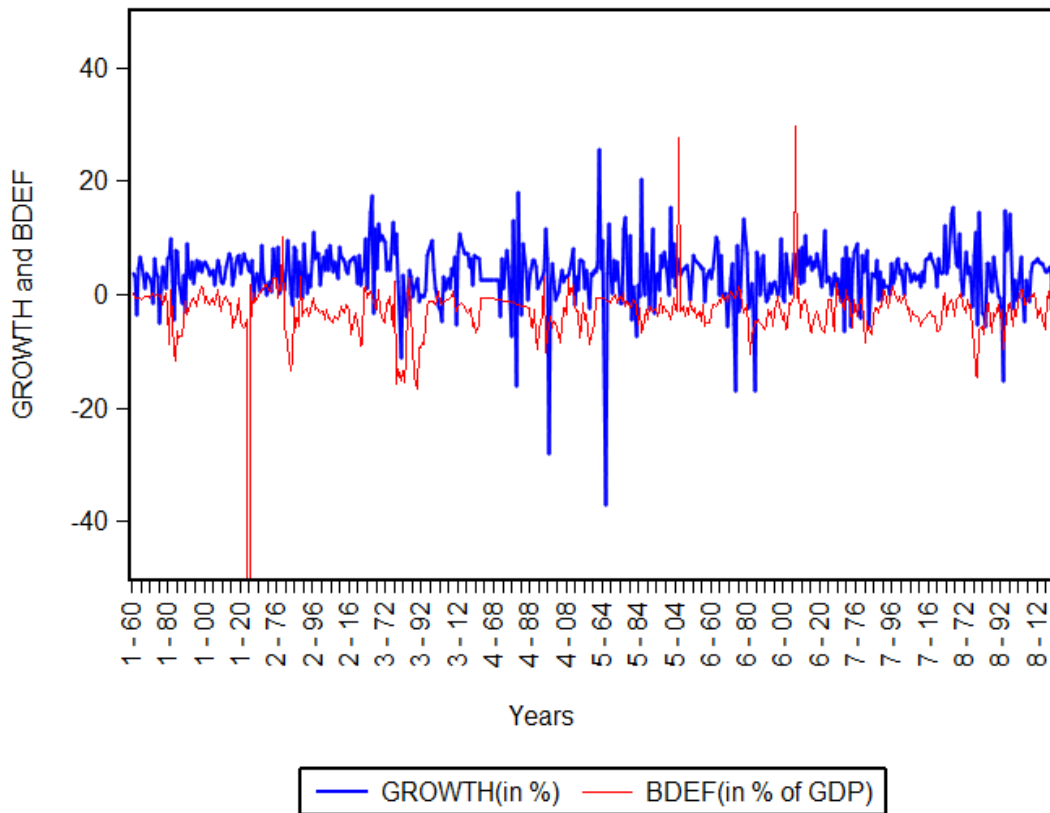
The economic performance of WAEMU is attributed to the adoption of measures by member states to accelerate the implementation of necessary reforms for macroeconomic stability and improvement of the business climate, aimed at attracting more private investors, particularly in agro-food and manufacturing industries. In 2023, inflationary pressures eased in the WAEMU zone, with an average inflation rate of 3.7% for the entire year, down from 7.4% in 2022. This decrease in inflation levels is attributed to monetary policy measures and a slowdown in food prices following an increase

in cereal production during the 2022/2023 agricultural campaign. This trend has been reinforced by a relaxation of food and energy prices in international markets.

Regarding the financial operations of the Union's states, the budget deficit was reduced in 2023, representing 5.3% of GDP compared to 6.7% of GDP in 2022. In line with these developments, the Union's debt ratio is estimated at 55.7% in 2023, down from 57.6% the previous year, according to the June 2023 report from the WAEMU Commission's multilateral surveillance.

Concerning the foreign trade of WAEMU member states, a global deficit of 3,008.9 billion FCFA in the balance of payments was recorded in 2023, following a deficit of 3,343.1 billion FCFA the previous year. This improvement in the overall balance of payments is attributed to the reduction of the goods and services balance due to an increase in exports and a decrease in imports. According to BCEAO projections, growth is expected to accelerate in the WAEMU zone to reach 6.5% in 2024, driven by strong activity in the tertiary sector and an increase in the secondary sector. The secondary sector is expected to benefit from increased extractive and manufacturing production, as well as a boom in the construction and public works (BTP) sector. The tertiary and secondary sectors are expected to contribute 3.5 and 2.0 points, respectively, to the Union's growth in 2024, while the contribution of the primary sector is projected to be 1.0 percentage point in 2024, down from 1.1 points the previous year.

Inflation is expected to continue to decline in the Union in 2024, falling below the community norm of 2.5%. This decrease is attributed to the anticipated easing of global prices for imported food and petroleum products in the countries of the Union. Regarding public finances, the budget balance is expected to continue improving in the Union's member states in 2024, aided by the ongoing implementation of reforms aimed at improving tax revenue mobilization and rationalizing public spending. Thus, the overall budget deficit of the Union as a percentage of GDP is projected to be 3.8% in 2024, after 5.3% and 6.7% in 2023 and 2022, respectively. With the commencement of gas and oil production in some WAEMU member states, the Union's foreign trade is projected to yield a surplus in the overall balance of payments in 2024, following two consecutive years of deficit. The overall balance is expected to show a surplus of 874.5 billion FCFA in 2024, compared to deficits of 3,008.9 and 3,343.1 billion FCFA in 2023 and 2022, respectively.



Legend: 1=Benin; 2=Burkina Faso; 3=Côte d'Ivoire; 4=Guinea-Bissau; 5=Mali; 6=Niger; 7=Senegal; 8=Togo.

Source: Our Own Investigations

Graph 1: Evolution of Economic Growth and Budget Deficit in WAEMU, 1960-2023

The above graph illustrates the fluctuations of economic growth and budget deficits in the member countries of WAEMU from 1960 to 2023. It is evident that both deficit and economic growth are highly volatile in the WAEMU area and appear to be correlated. The highest deficit was recorded in Burkina Faso in 1960, while the highest surplus was observed in Niger in 2006, attributed to the economic recovery in the mining and oil sector and the resumption of financial cooperation with most Technical and Financial Partners.

2. Empirical Methodology

2.1 Discrete-Time Markov Chain

A discrete-time Markov chain is a sequence of random variables taking values in a state space (finite or infinite) that is usually represented by the integers such that:

$P(X_{n+1}=i_{n+1} / X_n = i_n, \dots, X_0 = i_0) = P(X_{n+1}=i_{n+1} / X_n = i_n)$, for all states i_{n+1}, i_n, \dots, i_0 and any integer $n \geq 0$. This is the Markov

property. The index typically represents time. When the transition probabilities above are stationary (i.e., the same for all integers), the chain is said to be homogeneous. This is the assumption made in this study.

2.2 Modeling Budget States using Markov Chains

We assume that the budget deficit at the end of the current year can either be Low Deficit (LD), High Deficit (HD), Low Surplus (LS), High Surplus (HS), or Balanced Budget (BB), depending only on the most recent year's deficit, rather than deficits from previous years. Let be LD_n the event that the state budget has a low deficit at the end of year n , HD_n the event that the budget has a high deficit at the end of year $n \geq 0$, LS_n the event that the budget has a low surplus at the end of year $n \geq 0$, HS_n the event that the budget has a high surplus at the end of year $n \geq 0$, and BB_n the event that the budget is balanced at the end of year $n \geq 0$. The following conditional probabilities will be calculated in the subsequent sections of this paper:

$$\mathbb{P}(LD_{n+1}/LD_n) = \beta_{1s} ; \mathbb{P}(HD_{n+1}/HD_n) = \beta_{2s} ; \mathbb{P}(LS_{n+1}/LD_n) = \beta_{3s} ; \mathbb{P}(HS_{n+1}/LD_n) = \beta_{4s} ;$$

$$\mathbb{P}(BB_{n+1}/LD_n) = 1 - \sum_{i=1}^4 \beta_{is} .$$

$$\mathbb{P}(LD_{n+1}/HD_n) = \gamma_{1s} ; \mathbb{P}(HD_{n+1}/HD_n) = \gamma_{2s} ; \mathbb{P}(LS_{n+1}/HD_n) = \gamma_{3s} ; \mathbb{P}(HS_{n+1}/HD_n) = \gamma_{4s} ;$$

$$\mathbb{P}(BB_{n+1}/LD_n) = 1 - \sum_{i=1}^4 \gamma_{is} .$$

$$\mathbb{P}(LD_{n+1}/LS_n) = \alpha_{1s} ; \mathbb{P}(HD_{n+1}/LS_n) = \alpha_{2s} ; \mathbb{P}(LS_{n+1}/LS_n) = \alpha_{3s} ; \mathbb{P}(HS_{n+1}/LS_n) = \alpha_{4s} ;$$

$$P_r(BB_{n+1}/LS_n) = 1 - \sum_{i=1}^4 \alpha_{is} .$$

$$\mathbb{P}(LD_{n+1}/HS_n) = \delta_{1s} ; \mathbb{P}(HD_{n+1}/HS_n) = \delta_{2s} ; \mathbb{P}(LS_{n+1}/HS_n) = \delta_{3s} ; \mathbb{P}(HS_{n+1}/HS_n) = \delta_{4s} ;$$

$$\mathbb{P}(BB_{n+1}/HS_n) = 1 - \sum_{i=1}^4 \delta_{is} .$$

$$\mathbb{P}(LD_{n+1}/BB_n) = \theta_{1s} ; \mathbb{P}(HD_{n+1}/BB_n) = \theta_{2s} ; \mathbb{P}(LS_{n+1}/BB_n) = \theta_{3s} ; \mathbb{P}(HS_{n+1}/BB_n) = \theta_{4s} ;$$

$$\mathbb{P}(BB_{n+1}/BB_n) = 1 - \sum_{i=1}^4 \theta_{is} .$$

Where $(\beta_{is}, \gamma_{is}, \alpha_{is}, \delta_{is}, \theta_{is})_{i=1,2,3,4}$ et $s=1,2,\dots,8 \in [0, 1]^{20}$; $s \in (1,2, \dots, 8)$ is the country's index.

The random variable is defined as X_n by $X_n = 0$ if DF_n occurs, $X_n = 1$ si DE_n occurs, $X_n = 2$ if EF_n occurs, $X_n = 3$ if EE_n occurs et $X_n = 4$ if EB_n occurs.

We can notice that :

$$\mathbb{P}(X_{n+1}=j / X_n = i, X_{n-1} = i - 1, \dots, X_0 = i_0) = \mathbb{P}(X_{n+1}=j / X_n = i) \forall (i, j) \in (0, 1, 2, 3, 4)^2 .$$

The sequence $\{X_n\}_{n \geq 0}$ is thus a homogeneous Markov chain in discrete time on the states 0, 1, 2, 3 and 4 with a transition matrix P_s given by:

$$P_s = \begin{pmatrix} \beta_{1s} & \beta_{2s} & \beta_{3s} & \beta_{4s} & 1 - \sum_{i=1}^4 \beta_{is} \\ \gamma_{1s} & \gamma_{2s} & \gamma_{3s} & \gamma_{4s} & 1 - \sum_{i=1}^4 \gamma_{is} \\ \alpha_{1s} & \alpha_{2s} & \alpha_{3s} & \alpha_{4s} & 1 - \sum_{i=1}^4 \alpha_{is} \\ \delta_{1s} & \delta_{2s} & \delta_{3s} & \delta_{4s} & 1 - \sum_{i=1}^4 \delta_{is} \\ \theta_{1s} & \theta_{2s} & \theta_{3s} & \theta_{4s} & 1 - \sum_{i=1}^4 \theta_{is} \end{pmatrix} .$$

This matrix will be determined for each member country of WAEMU.

2.3 n-step Transition Matrix

The n-step Transition matrix for all $n \geq 1$ is the matrix $P^{(n)}$ in which the entry in row i and column j is given by :

$$P_{ij}^{(n)} = \mathbb{P}(X_n = j / X_0 = i), \text{ for all states } i \text{ and } j . \text{ By stationarity, we have :}$$

$$P_{ij}^{(n)} = \mathbb{P}(X_{n+k} = j / X_k = i), \text{ for all } k \geq 0$$

The n-step Transition matrix for country s is as follows :

$$P_s^{(n)} = \begin{pmatrix} \beta_{1s} & \beta_{2s} & \beta_{3s} & \beta_{4s} & 1 - \sum_{i=1}^4 \beta_{is} \\ \gamma_{1s} & \gamma_{2s} & \gamma_{3s} & \gamma_{4s} & 1 - \sum_{i=1}^4 \gamma_{is} \\ \alpha_{1s} & \alpha_{2s} & \alpha_{3s} & \alpha_{4s} & 1 - \sum_{i=1}^4 \alpha_{is} \\ \delta_{1s} & \delta_{2s} & \delta_{3s} & \delta_{4s} & 1 - \sum_{i=1}^4 \delta_{is} \\ \theta_{1s} & \theta_{2s} & \theta_{3s} & \theta_{4s} & 1 - \sum_{i=1}^4 \theta_{is} \end{pmatrix}^n , \text{ with } s \in (1, 2, \dots, 8) \text{ representing the}$$

country's index.

The coefficients of this matrix will likewise be determined for each member country of the WAEMU. Calculating the n -th power of matrix P_s involves its diagonalisation.

The matrix P_s is diagonalizable if and only if there exists a diagonalizable matrix D in $\mathcal{M}_{5,5}(\mathbb{R})$ and an invertible matrix A in $\mathcal{M}_{5,5}(\mathbb{R})$ such that :

$P_s = ADA^{-1}$. Starting from this expression, and assuming the diagonalisation of P_s , can determine $P_s^{(n)}$ as follows :

$P_s^{(n)} = AD^n A^{-1}$ for any non-zero natural integer n

In this case, the diagonal coefficients of D are eigenvalues of P_s , and the columns of A form eigenvectors of P_s associated with the eigenvalues of P_s in the same order.

It is interesting to understand what happens in the long term, given the n -step transition matrix. In the context of the convergence of transition probabilities between the different states, we can determine the limit of this convergence. It is given by :

$$\lim_{n \rightarrow +\infty} P_s^{(n)} = \begin{pmatrix} a_{1s} & a_{2s} & a_{3s} & a_{4s} & a_{5s} \\ b_{1s} & b_{2s} & b_{3s} & b_{4s} & b_{5s} \\ c_{1s} & c_{2s} & c_{3s} & c_{4s} & c_{5s} \\ d_{1s} & d_{2s} & d_{3s} & d_{4s} & d_{5s} \\ e_{1s} & e_{2s} & e_{3s} & e_{4s} & e_{5s} \end{pmatrix}.$$

With $(a_{is}, b_{is}, c_{is}, d_{is}, e_{is})_{i=1,2,3,4,5 \text{ et } s=1,2,\dots,8} \in [0, 1]^{25}$; $s \in (1, 2, \dots, 8)$ represents the country's index.

2.4 Budget Deficit Projections for the Years 2024, 2025, 2026, 2027, and 2028 In Waemu Member Countries (Methodological Approach)

Understanding the anticipated budget deficit at the end of the current year and the subsequent four years in each WAEMU member country is crucial for empowering public decision-makers in the effectiveness of budgetary policies.

2.4.1 Benin

2.4.1.1 Calculation of Transition Probabilities and Definition of the n -step transition matrix

The transition matrix for Benin ($s=1$) is structured as

$$\text{follows: } P_1 = \begin{pmatrix} \beta_{11} & \beta_{21} & \beta_{31} & \beta_{41} & 1 - \sum_{i=1}^4 \beta_{i1} \\ \gamma_{11} & \gamma_{21} & \gamma_{31} & \gamma_{41} & 1 - \sum_{i=1}^4 \gamma_{i1} \\ \alpha_{11} & \alpha_{21} & \alpha_{31} & \alpha_{41} & 1 - \sum_{i=1}^4 \alpha_{i1} \\ \delta_{11} & \delta_{21} & \delta_{31} & \delta_{41} & 1 - \sum_{i=1}^4 \delta_{i1} \\ \theta_{11} & \theta_{21} & \theta_{31} & \theta_{41} & 1 - \sum_{i=1}^4 \theta_{i1} \end{pmatrix}$$

Similarly, the n -step transition matrix for Benin($s=1$) is presented as follow :

2.4.1.2 Budget Deficit Projections Given that Benin Experienced a High Deficit in 2023 (Reference Situation)

To project deficits for the years 2024 to 2028, we will vary the natural number sequentially between the values (2.3.4.5 and 6). The projections will be summarized in a table.

2.4.1.3 State of Government Budget in the Long Term

We aim to determine the most probable state of the Beninese budget in the long term based on the convergence limit of the n -step transition matrix

$$\lim_{n \rightarrow +\infty} P_1^{(n)} = \begin{pmatrix} a_{11} & a_{21} & a_{31} & a_{41} & a_{51} \\ b_{11} & b_{21} & b_{31} & b_{41} & b_{51} \\ c_{11} & c_{21} & c_{31} & c_{41} & c_{51} \\ d_{11} & d_{21} & d_{31} & d_{41} & d_{51} \\ e_{11} & e_{21} & e_{31} & e_{41} & e_{51} \end{pmatrix}.$$

With $(a_{i1}, b_{i1}, c_{i1}, d_{i1}, e_{i1})_{i=1,2,3,4,5} \in [0, 1]^{25}$.

2.4.2 Burkina-Faso

2.4.2.1 Calculation of Transition Probabilities and Definition of the Transition Matrix in n Steps

The transition matrix for Burkina Faso (s=2) is expressed as:

$$P_2 = \begin{pmatrix} \beta_{12} & \beta_{22} & \beta_{32} & \beta_{42} & 1 - \sum_{i=s}^4 \beta_{i2} \\ \gamma_{12} & \gamma_{22} & \gamma_{32} & \gamma_{42} & 1 - \sum_{i=s}^4 \gamma_{i2} \\ \alpha_{12} & \alpha_{22} & \alpha_{32} & \alpha_{42} & 1 - \sum_{i=s}^4 \alpha_{i2} \\ \delta_{12} & \delta_{22} & \delta_{32} & \delta_{42} & 1 - \sum_{i=s}^4 \delta_{i2} \\ \theta_{12} & \theta_{22} & \theta_{32} & \theta_{42} & 1 - \sum_{i=s}^4 \theta_{i2} \end{pmatrix}$$

Similarly, the n -step transition matrix for Benin(s=1) is presented as follow:

$$P_2^{(n)} = \begin{pmatrix} \beta_{12} & \beta_{22} & \beta_{32} & \beta_{42} & 1 - \sum_{i=s}^4 \beta_{i2} \\ \gamma_{12} & \gamma_{22} & \gamma_{32} & \gamma_{42} & 1 - \sum_{i=s}^4 \gamma_{i2} \\ \alpha_{12} & \alpha_{22} & \alpha_{32} & \alpha_{42} & 1 - \sum_{i=s}^4 \alpha_{i2} \\ \delta_{12} & \delta_{22} & \delta_{32} & \delta_{42} & 1 - \sum_{i=s}^4 \delta_{i2} \\ \theta_{12} & \theta_{22} & \theta_{32} & \theta_{42} & 1 - \sum_{i=s}^4 \theta_{i2} \end{pmatrix}^n$$

2.4.2.2 Budget Deficit Projections Given that Burkina Faso Experienced a High Deficit in 2023 (Reference Situation)

To project deficits for the years 2024 to 2028, we will vary the natural number sequentially between the values (2.3.4.5 and 6). The projections will be summarized in a table.

2.4.2.3 State of Government Budget in the Long Term

We aim to determine the most probable state of the Burkinabe budget in the long term based on the convergence limit of the n -step transition matrix:

$$\lim_{n \rightarrow +\infty} P_2^{(n)} = \begin{pmatrix} a_{12} & a_{22} & a_{32} & a_{42} & a_{52} \\ b_{12} & b_{22} & b_{32} & b_{42} & b_{52} \\ c_{12} & c_{22} & c_{32} & c_{42} & c_{52} \\ d_{12} & d_{22} & d_{32} & d_{42} & d_{52} \\ e_{12} & e_{22} & e_{32} & e_{42} & e_{52} \end{pmatrix}.$$

Where $(a_{i2}, b_{i2}, c_{i2}, d_{i2}, e_{i2})_{i=1,2,3,4,5} \in [0, 1]^{25}$;

For simplicity, this section limits itself to the two previously discussed countries. The same calculations will be performed for the remaining six countries in the WAEMU zone.

3. Data and Empirical Results

3.1 Data

The dataset consists of a panel of observations for the 8 WAEMU countries over the period 1960-2023. Our variable of interest is the overall budget deficit (BDEF) as a percentage of GDP). Economic growth is also used for long-term trend comparison purposes. The

two variables are extracted from the CBWAS data table.

Table 1 presents the descriptive statistics of the variables used.

Countries	Variables	Mean	Standard Deviation	Minimum	Maximum
Benin	Budget Deficit (% of GDP)	-2.1	2.8	-11.6	1.4
	Economic Growth (in %)	3.9	3.0	-4.9	10.0
Burkina-Faso	Budget Deficit (% of GDP)	-5.8	25.9	-207.5	10.3
	Economic Growth (in %)	4.4	3.0	-1.8	11.0
Côte d'Ivoire	Budget Deficit (% of GDP)	-3.9	4.9	-16.7	2.9
	Economic Growth (in %)	4.3	5.3	-11.0	17.6
Guinea-Bissau	Budget Deficit (% of GDP)	-3.0	2.9	-10.1	3.1
	Economic Growth (in %)	2.7	5.9	-28.1	18.2
Mali	Budget Deficit (% of GDP)	-2.0	4.1	-6.7	27.8
	Economic Growth (in %)	4.0	7.2	-37.1	25.6
Niger	Budget Deficit (% of GDP)	-2.5	4.8	-10.6	29.6
	Economic Growth (in %)	2.9	5.5	-17.0	13.5
Senegal	Budget Deficit (% of GDP)	-2.6	2.3	-8.5	2.0
	Economic Growth (in %)	3.2	3.4	-6.6	8.9
Togo	Budget Deficit (% of GDP)	-3.3	3.3	-14.5	1.7
	Economic Growth (in %)	4.0	5.5	-15.1	15.5

Source: Author's Investigations

Table 1: Descriptive Statistics, 1960-2023

It appears from this table that most high budget deficits are associated with high growth rates over the study period. This implies that the strategic debt of the State improves the latter's economic growth. This confirms the Keynesian theory of State support for the Economy by increasing the budget deficit.

Burkina-Faso comes first in terms of the highest budget deficit (-5.8%) but is very volatile and curiously has the best average growth rate over the period and of the entire Union (5.8%) then even though Mali (-2.0%) has the lowest and less volatile deficit with an average growth rate nevertheless very close to that of Burkina-Faso (4.0%).

3.2 Empirical Results

3.2.1 Projection of Budget Deficits for the years 2024, 2025, 2026, 2027 and 2028 In Waemu Member Countries (Results)

It is very important to know the budget deficit that will occur at the end of the current year (2024) and the next four years in each WAEMU member country, taking 2023 as a reference year. in

order to provide a powerful weapon to public decision-makers for the effectiveness of budgetary policies.

• Projection Assumptions

The main assumptions relate to external and internal factors underlying the dynamics of the WAEMU economies. On the external side, they concern forecasts of economic growth at world level and in the Union's main partner countries, as well as developments in international commodity prices and international financial conditions. At the sub-regional level, the assumptions used relate to the profile of the budget deficits of the Member States of the Union and their mode of operation, their resilience to exogenous macroeconomic shocks, the orientation of monetary policy, the evolution of the security and socio-political situation, as well as agro-climatic conditions.

• Benin

3.2.1.1 Calculation of Transition Probabilities and Definition of the n-step Transition Matrix

	Low Deficit	High Deficit	Low surplus	High surplus	Budget balance	Total
Number of favorable cases	43	10	11	0	0	64
Total number of cases	64	64	64	64	64	64
P(Have a budget status type in a given year)	0.67	0.16	0.17	0.00	0.00	1

Source: Our own investigations

Table 2: Assessment of the Probabilities of Each Fiscal Transition State

In view of this table, the probabilities of the states' subsidy (High surplus and Budget balance) are all zero. These are therefore impossible events. The transition matrix for Benin is therefore reduced to a square matrix of order 3.

	LD∩LD	HD∩LD	LS∩LD
Number of favorable cases	35	3	5
Number of total acses	64	64	64
Probability (X∩LD)	0.53	0.06	0.08
Probability (have a low Deficit in a given year)	0.67		
Probabilité(X/LD)	0.81	0.07	0.12

Source: Our Own Investigations

Table 3: Evaluation of Conditional Probabilities (Probability (X/Low Deficit)) WITH X= {LD, HD, LS, HS}

	LD∩HD	HD∩HD	LS∩HD
Number of favorable cases	3	6	1
Number of total cases	64	64	64
Probability (X∩HD)	0.05	0.10	0.02
Probability (have a High Deficit in a given year)	0.16		
Probability(X/HD)	0.31	0.56	0.13

Source: Our own investigations

Table 4: Evaluation of Conditional Probabilities (Probability (X/High Deficit)) With X= {LD, HD, LS, HS}

	LD∩LS	HD∩LS	HS∩LS
Number of favorable cases	7	0	4
Number of total acses	64	64	64
Probability (X∩LS)	0.11	0.00	0.06
Probability (have a Low Surplus in a given year)	0.17		
Probabilité(X/Low Surplus)	0.65	0.00	0.35

Source: Our own investigations

Table 5: Evaluation of Conditional Probabilities (Probability (X/Low Surplus)) WITH X= {LD, HD, LS, HS}

In view of the preceding tables, the transition matrix for Benin (s=1) is as follows:

$$P_1 = \begin{pmatrix} 0,81 & 0,07 & 0,12 \\ 0,31 & 0,56 & 0,13 \\ 0,65 & 0,00 & 0,35 \end{pmatrix}$$

Since the P_1 matrix is diagonalizable, the n-step transition matrix for Benin (s=1) is as follows:

$$P_1^{(n)} = \begin{pmatrix} 0,73 + 0,08(0,49)^n + 0,20(0,23)^n & 0,12 - 0,07(0,49)^n - 0,04(0,23)^n & 0,16 - 0,15(0,23)^n \\ 0,73 - 0,97(0,49)^n + 0,25(0,23)^n & 0,12 + 0,94(0,49)^n - 0,05(0,23)^n & 0,16 + 0,04(0,49)^n - 0,19(0,23)^n \\ 0,73 + 0,36(0,49)^n - 1,09(0,23)^n & 0,12 - 0,35(0,49)^n + 0,23(0,23)^n & 0,16 - 0,01(0,49)^n + 0,86(0,23)^n \end{pmatrix}$$

3.2.1.2 Projection of Budget Deficits Knowing That the Beninese Budget Experienced a High Deficit in 2023 (Reference Situation)

The following projections are based on variations of the natural integer between the values (2.3.4.5.6). derived from the transition matrix. The results are summarized in the table below:

Years	Different Projections	Budget Deficit (% of GDP)
2024	Our projections	Low Deficit(-5%<DF<-3%)
2025		Low Deficit (-4%<DF<-3%)
2026		Low Deficit (-3.5%<DF<-3%)
2027		Low Deficit (-3%<DF<-2.5%)
2028		Low Deficit (-2.5%<DF<-1%)

Source : Our own investigations

Table 6: Projected Budget Deficits

Using 2023 as the reference year, the budget deficit projections for Benin from 2024 to 2028 indicate a consistently decreasing low budget deficit (including grants) over the five years. This outcome directly results from effective public spending control and enhanced state revenue optimization through anti-corruption and tax evasion measures. These projections are contingent on a stable international economic environment.

3.2.1.3 Long-Term Government Budget Outlook

This section examines the most probable state of the Benin budget in the long term based on the convergence limit of the transition matrix over steps:

$$\lim_{n \rightarrow +\infty} P_1^{(n)} = \begin{pmatrix} 0.73 & 0.12 & 0.16 \\ 0.73 & 0.12 & 0.16 \\ 0.73 & 0.12 & 0.16 \end{pmatrix}.$$

Regardless of whether the Benin budget experiences a low, high deficit, or even a low surplus in a given year, the most probable long-term budget state is a low deficit (probability = 0.73). This suggests the continued pursuit of macroeconomic reforms for the improvement of public finances in Benin.

• Burkina-Faso

3.2.1.4 Transition Probability Calculations and Transition Matrix Definition

	Low Deficit	High Deficit	Low surplus	High surplus	Budget balance	Total
Number of favorable cases	34	14	12	4	0	64
Total number of cases	64	64	64	64	64	64
P(Have a budget status type in a given year)	0.53	0.22	0.19	0.06	0.00	1

Source: Our Own Investigations

Table 7: Evaluation of Transition State Probabilities

From this table, the probability of reaching a budget equilibrium is zero, indicating an impossible event. Consequently, the transition matrix for Burkina Faso simplifies to a square matrix of order 4.

	LD∩LD	HD∩LD	LS∩LD	HS∩LD
Number of favorables cases	26	5	2	0
Number of total acses	64	64	64	64
Probability (X∩LD)	0.42	0.08	0.03	0.00
Probability(Have a low deficit a given year)	0.53			
Probability(X/Low Deficit)	0.79	0.15	0.06	0.00

Source: Source : Our own investigations

Table 8: Evaluation of Conditional Probabilities (P (X | Low Deficit)) Where X= {LD, HD, LS, BB}

	LD∩HD	HD∩HD	LS∩HD	HS∩HD
Number of favorables cases	4	6	2	2
Number of total acses	64	64	64	64
Probability (X∩DE)	0.06	0.10	0.03	0.03
Probability(Have a High Deficit a given year)	0.22			
Probability(X/High Deficit)	0.29	0.43	0.14	0.14

Source: Our own investigations

Table 9: Evaluation of Conditional Probabilities (P (X | High Deficit)) Where X= {LD, HD, LS, BB}

	LD∩LS	HD∩LS	LS∩LS	HS∩LS
Number of favorables cases	3	0	6	3
Number of total acses	64	64	64	64
Probability (X∩LS)	0.05	0.00	0.10	0.05
Probability(Have an Low Surplus a given year)	0.19			
Probability(X/Low Surplus)	0.25	0.00	0.50	0.25

Source: Our Own Investigations

Table 10: Evaluation of Conditional Probabilities (P (X | Low Surplus)) Where X= {LD, HD, LS, HS}

	DF∩EE	DE∩EE	EF∩EE	EE∩EE
Number of favorables cases	1	2	0	1
Number of total acses	64	64	64	64
Probability (X∩EE)	0.02	0.03	0.00	0.02
Probability(Have an High Surplus a given)	0.06			

Source: Our Own Investigations

Table 11: Evaluation of Conditional Probabilities (P (X | Low Surplus)) Where X= {LD, HD, LS, HS}

Based on the previous tables, the transition matrix for Burkina Faso (s=2) is as follows:

$$P_2 = \begin{pmatrix} 0.79 & 0.15 & 0.06 & 0.00 \\ 0.29 & 0.43 & 0.14 & 0.14 \\ 0.25 & 0.00 & 0.50 & 0.25 \\ 0.24 & 0.52 & 0.00 & 0.24 \end{pmatrix}$$

Since matrix P_2 is diagonalizable, the transition matrix for steps for Burkina Faso (s=2) is represented as:

$$P_2^{(n)} = \begin{pmatrix} 0.56 + 0.45(0.53)^n - 0.05(0.28)^n + 0.03(0.15)^n & 0.23 - 0.21(0.53)^n + 0.28(0.28)^n - 0.30(0.15)^n & 0.13 - 0.08(0.53)^n - 0.17(0.28)^n + 0.11(0.15)^n & 0.08 - 0.17(0.53)^n - 0.07(0.28)^n + 0.15(0.15)^n \\ 0.56 - 0.50(0.53)^n - 0.06(0.15)^n & 0.23 + 0.23(0.53)^n + 0.02(0.28)^n + 0.53(0.15)^n & 0.13 + 0.08(0.53)^n - 0.01(0.28)^n - 0.20(0.15)^n & 0.08 + 0.19(0.53)^n - 0.26(0.15)^n \\ 0.56 - 0.75 + 0.40(0.28)^n - 0.21(0.15)^n & 0.23 + 0.34(0.53)^n - 2.41(0.28)^n + 1.84(0.15)^n & 0.13 + 0.13(0.53)^n + 1.45(0.28)^n - 0.70(0.15)^n & 0.08 + 0.28(0.53)^n + 0.56(0.28)^n - 0.93(0.15)^n \\ 0.56 - 0.53(0.53)^n - 0.30(0.28)^n + 0.26(0.15)^n & 0.23 + 0.24(0.53)^n + 1.81(0.28)^n - 2.27(0.15)^n & 0.13 + 0.09(0.53)^n - 1.09(0.28)^n + 0.87(0.15)^n & 0.08 + 0.20(0.53)^n - 0.42(0.28)^n + 1.14(0.15)^n \end{pmatrix}$$

3.2.1.5 Projection of Budget Deficits Knowing Burkina Faso Experienced a High Deficit in 2023

For the projections from 2024 to 2028, we again vary the natural integer between the values (2.3.4.5 and 6). The results are summarized in the table below:

Years	Different Projections	Budget Deficit (% of GDP)
2024	Our projections	Low Deficit(-5%<DF<-3%)
2025		Low Deficit(-4%<DF<-3%)
2026		Low Deficit(-3.5%<DF<-3%)
2027		Low Deficit(-3%<DF<-2.5%)
2028		Low Deficit(-2.5%<DF<-1%)

Source: Our Own Investigations

Table 12: Projected Budget Deficits

Using 2023 as the reference year, the budget deficit projections for Burkina Faso for the years 2024, 2025, 2026, 2027, and 2028 indicate a low budget deficit (including grants) that is decreasing but at a slow pace over the next five years. This result is a direct consequence of the current security situation in Burkina Faso, leading to enormous military expenditures by the government in the face of low tax revenues, which are gradually decreasing due to a loss of confidence from investors in the country.

3.2.1.6 Long-Term Government Budget Status

This section aims to examine the most probable state of the Burkinabé budget in the long term based on the convergence limit of the transition matrix as.

$$\lim_{n \rightarrow +\infty} P_2^{(n)} = \begin{pmatrix} 0.56 & 0.23 & 0.13 & 0.08 \\ 0.56 & 0.23 & 0.13 & 0.08 \\ 0.56 & 0.23 & 0.13 & 0.08 \\ 0.56 & 0.23 & 0.13 & 0.08 \end{pmatrix}$$

We observe that whether the Burkinabé budget is in a low deficit, high deficit, low surplus, or high surplus in a given year, in the long term, the most probable budget state is a low deficit (probability = 0.56). This probability, although greater than 0.50, indicates that the current socio-political and security instability in Burkina Faso weighs heavily on the public finances of the state. The effects of this instability could disrupt the state budget in the long term if appropriate measures are not taken to address the current crisis.

• Ivory Coast

3.2.1.7 Calculation of Transition Probabilities and Definition of the Transition Matrix in Steps

	Low Deficit	High Deficit	Low surplus	High surplus	Budget balance	Total
Number of favorable cases	38	17	7	2	0	64
Total number of cases	64	64	64	64	64	64
P(Have a budget status type in a given year)	0.59	0.27	0.11	0.03	0.00	1

Source: Our own investigations

Table 13: Evaluation of Probabilities for Each State of Budgetary Transition

Given this table, the probability of achieving a budget balance (Budget Balance) is zero. It is thus an impossible event. The transition matrix for Côte d'Ivoire therefore reduces to a square matrix of order 4.

According to preliminary calculations, the transition matrix for Côte d'Ivoire (s=3) is as follows:

$$P_3 = \begin{pmatrix} 0.74 & 0.11 & 0.11 & 0.04 \\ 0.23 & 0.71 & 0.00 & 0.06 \\ 0.58 & 0.14 & 0.28 & 0.00 \\ 0.50 & 0.50 & 0.00 & 0.00 \end{pmatrix}$$

La matrice P_3 étant diagonalisable, la matrice de transition en n pas pour la Côte d'Ivoire(s=3) est déterministe.

The matrix P_3 being diagonalizable, the n -step transition matrix for Côte d'Ivoire (s=3) is deterministic.

3.2.1.8 Projection of Budget Deficits Knowing That the Ivorian Budget Experienced a High Deficit in 2023 (Reference Situation)

For projections from 2024 to 2028, it suffices to successively vary the natural number between the values (2.3.4.5.6).

The projections are summarized in the following table:

Years	Different Projections	Budget Deficit (% of GDP)
2024	Nos projections	Déficit Faible(-5%<DF<-3%)
2025		Déficit Faible(-4%<DF<-3%)
2026		Déficit Faible(-3.5%<DF<-3%)
2027		Déficit Faible(-3%<DF<-2.5%)
2028		Déficit Faible(-2.5%<DF<-1%)

Source: Our own Investigations

Table 14: Projection of Budget Deficits

Using 2023 as the reference year, the budget deficit projections for Côte d'Ivoire for the years 2024, 2025, 2026, 2027, and 2028 indicate a low budget deficit (including grants) that is decreasing but at a slow pace over the next five years. This result is a direct consequence of the current security situation in Côte d'Ivoire, leading to enormous military expenditures by the government in the face of low tax revenues, which are gradually decreasing due to a loss of confidence from investors in the country.

3.2.1.9 Long-term Government Budget Status

This section aims to examine the most probable state of the Ivorian budget in the long term based on the convergence limit of the n -step transition matrix.

$$\lim_{n \rightarrow +\infty} P_3^{(n)} = \begin{pmatrix} 0.55 & 0.32 & 0.08 & 0.05 \\ 0.55 & 0.32 & 0.08 & 0.05 \\ 0.55 & 0.32 & 0.08 & 0.05 \\ 0.55 & 0.32 & 0.08 & 0.05 \end{pmatrix}$$

We observe that whether the Ivorian budget is in a low deficit, high deficit, low surplus, or high surplus in a given year, in the long term, the most probable budget state is a low deficit (probability = 0.55). This probability, although greater than 0.50, indicates that the current socio-political and security instability in Côte d'Ivoire weighs heavily on the public finances of the state. The effects of this instability could disrupt the state budget in the long term and excessively increase the budget deficit (32% chance) if appropriate measures are not taken to restore public finances.

For simplicity, the remaining projections for the other five countries in WAEMU are presented in the appendix.

4. Conclusion and Implications for Economic Policies

In this article, we examine the budgetary states of the WAEMU (West African Economic and Monetary Union) countries. We conducted an exploratory analysis before focusing on Markov chains to conclude that the budget deficit is dynamic and confirms Ricardo's snowball effect. Our results indicate that Mali is the best performer in terms of effective budget deficit management within WAEMU, followed by Benin and Senegal in both the short and long term.

Guinea-Bissau, Côte d'Ivoire, Togo, Burkina Faso, and Niger are on a trajectory of high budget deficits, and their public finances are likely to deteriorate further in the long term if effective and urgent policies are not implemented to change the dynamics of budgetary expansion.

Our data also highlight a positive correlation between budget

• Annex

Guinea-Bissau

Calculation of Transition Probabilities and Definition of the Transition Matrix in n Steps

	Low Deficit	High Deficit	Low surplus	High surplus	Budget balance	Total
Number of favorable cases	42	18	2	2	0	64
Total number of cases	64	64	64	64	64	64
P(Have a budget status type in a given year)	0.66	0.28	0.03	0.03	0.00	1

Source: Our own investigations

Table 1: Evaluation of the Probabilities of Each Budgetary Transition State

Based on this table, the probability of the state (Budget Equilibrium) is zero. This is therefore an impossible event. The transition matrix for Guinea-Bissau thus reduces to a square matrix of order 4.

Considering preliminary calculations, the transition matrix for Guinea-Bissau (s=4) is presented as follows:

$$P_4 = \begin{pmatrix} 0.80 & 0.12 & 0.03 & 0.05 \\ 0.28 & 0.56 & 0.06 & 0.10 \\ 0.50 & 0.50 & 0.00 & 0.00 \\ 0.50 & 0.50 & 0.00 & 0.00 \end{pmatrix}$$

deficits and economic growth in WAEMU countries, confirming the view that strategic state borrowing can foster growth if funds are allocated effectively to profitable public investments.

The persistence of debt contributions in the medium and long term suggests that a medium- to long-term objective for the Union should be the development of local capital markets. This would allow WAEMU countries to borrow in their own currency, encouraging domestic savings while reducing the budget deficit. Indeed, CFA zone countries are severely affected by the risks associated with exchange rate and interest rate fluctuations. External borrowing necessarily involves a very high risk. Additionally, the asynchronous nature of deadlines and the lack of alignment of currencies in most countries pose a major source of macroeconomic instability. Therefore, leveraging the financial potential of the Union should be a preferred choice in the fight to control the risks associated with indebtedness.

Moreover, monetary and budgetary authorities must pay particular attention to the implementation of economic policies in the face of uncertainties. Consequently, the BCEAO (Central Bank of West African States) must adopt leadership strategies and ensure its credibility, as well as the strategic transmission of monetary policy. Otherwise, the social losses relative to the coordinated equilibrium are significant, and budgetary policies become largely expansionary, further widening the budget deficit.

Finally, efforts should be directed toward mobilizing domestic resources to rely less on foreign borrowing and to better manage the budget deficit.

The matrix P_4 being diagonalizable, the n-steps transition matrix for Guinea-Bissau (s=4) is calculated as follows:

$$P_4^{(n)} = \begin{pmatrix} 0,63 + 0,36(0,49)^n - 0,29(-0,13)^n & 0,28 - 0,3(0,49)^n + 0,03(-0,13)^n & 0,03 - 0,016(0,49)^n - 0,019(-0,13)^n & 0,06 - 0,027(0,49)^n - 0,03(-0,13)^n \\ 0,63 - 0,67(0,49)^n - 0,06(-0,13)^n & 0,28 + 0,60(0,49)^n + 0,12(-0,13)^n & 0,03 + 0,03(0,49)^n - 0,07(-0,13)^n & 0,06 + 0,05(0,49)^n - 0,11(-0,13)^n \\ 0,63 - 0,34(0,49)^n - 0,29(-0,13)^n & 0,28 + 0,29(0,49)^n - 0,57(-0,13)^n & 0,03 + 0,015(0,49)^n + 0,32(-0,13)^n & 0,06 + 0,03(0,49)^n + 0,54(-0,13)^n \\ 0,63 - 0,34(0,49)^n - 0,29(-0,13)^n & 0,28 + 0,29(0,49)^n - 0,57(-0,13)^n & 0,03 + 0,015(0,49)^n + 0,32(-0,13)^n & 0,06 + 0,03(0,49)^n - 0,54(-0,13)^n \end{pmatrix}$$

Projection of Budget Deficits Considering that the Guinean Budget Experienced a High Deficit in 2023 (Reference Situation)

For projections from 2024 to 2028, we need to successively vary the natural integer n between the values (2.3.4.5 and 6). The projections are summarized in the following table:

Years	Different Projections	Budget Deficit (% of GDP)
2024	Our projections	Low Deficit(-5%<DF<-3%)
2025		Low Deficit(-4%<DF<-2.5%)
2026		Low Deficit(-2.5%<DF<-1%)
2027		Low Deficit(-3%<DF<-2.5%)
2028		Low Deficit(-2.5%<DF<-1%)

Source: Our own investigations

Table 2: Projection of Budget Deficits

Taking 2023 as the reference year, the budget deficit projections for Guinea-Bissau for the years 2024, 2025, 2026, 2027 and 2028 show a low budget deficit (including grants) decreasing but with a low speed over the next five years. This result is the direct consequence of the clean-up of public finances by the new Guinean government since the recent coup d'état.

State of the Government Budget in the Long Term

This is to see the most likely state of the Guinean budget in the long term from the convergence limit of the transition matrix in n steps.

$$\lim_{n \rightarrow +\infty} P_4^{(n)} = \begin{pmatrix} 0,63 & 0,28 & 0,03 & 0,06 \\ 0,63 & 0,28 & 0,03 & 0,06 \\ 0,63 & 0,28 & 0,03 & 0,06 \\ 0,63 & 0,28 & 0,03 & 0,06 \end{pmatrix}$$

We note that, whether the Guinean budget is in low deficit, high deficit, low surplus or high surplus during a given year, in the long term, the most likely state of the budget is a low deficit (probability = 0.63). This probability, although greater than 0.50, shows that the current sociopolitical and security instability in Guinea-Bissau weighs heavily on the State's public finances. The effects of these instabilities risk disrupting the State budget in the long term if appropriate measures are not taken to curb the current security crisis.

• Mali

Calculation of transition probabilities and definition of the n-steps transition matrix

	Low Deficit	High Deficit	Low surplus	High surplus	Budget balance	Total
Number of favorable cases	58	4	1	1	0	64
Total number of cases	64	64	64	64	64	64
P(Have a budget status type in a given year)	0.90	0.06	0.02	0.02	0.00	1

Source: Our own investigations

Table 4: Evaluation of the Probabilities of Each Budgetary Transition State

In view of this table, the probability of the state being provided (Budget Balance) is zero. This is therefore an impossible event. The transition matrix for Mali is thus reduced to a square matrix of order 4.

In view of the preliminary calculations, the transition matrix for Mali (s=5) is as follows:

$$P_5 = \begin{pmatrix} 0.92 & 0.06 & 0.00 & 0.02 \\ 0.78 & 0.22 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 & 0.00 \end{pmatrix}$$

The matrix P_5 being diagonalizable, the n-step transition matrix for Mali(s=5) is calculated and is presented as follows:

$$P_5^{(n)} = \begin{pmatrix} 0.91 + 0.06(0.17)^n + 0.03(-0.03)^n & 0.07 - 0.06(0.17)^n & 0 & 0.02 + 0.04(0.17)^n + 0.94(-0.03)^n \\ \text{-----} & \text{-----} & 0 & \text{-----} \\ \text{-----} & \text{-----} & 0 & \text{-----} \\ 0.91 + 0.33(0.17)^n - 1.24(-0.03)^n & \text{-----} & 0 & 0.02 - 0.10(0.17)^n + 0.08(-0.03)^n \end{pmatrix}$$

Projection of Budget Deficits Knowing That the Malian Budget Experienced A High Deficit in 2023 (Reference Situation)

For the projections for the years 2024 to 2028, it is sufficient to successively vary the natural integer n between the values (2.3.4.5 and 6). The projections are summarized in the following table 5:

Years	Different Projections	Budget Deficit (% of GDP)
2024	Our projections	Low Deficit(-5%<DF<-3%)
2025		Low Deficit(-3%<DF<-2.75%)
2026		Low Deficit(-2.75%<DF<-2%)
2027		Low Deficit(-2%<DF<-1%)
2028		Low Deficit(-2%<DF<-1%)

Source: Our own investigations

Table 5: Projection of Budget Deficits

Taking 2023 as the reference year, the budget deficit projections for Mali for the years 2024, 2025, 2026, 2027 and 2028 show a low budget deficit (including grants) decreasing but with a low speed over the next five years. This result is the direct consequence of the clean-up of public finances by the new Guinean government since the recent coup d'état.

State of the Government Budget in the Long Term

This is to see the most likely state of the Guinean budget in the long term from the convergence limit of the n-steps transition matrix.

$$\lim_{n \rightarrow +\infty} P_5^{(n)} = \begin{pmatrix} 0.91 & 0.07 & 0.00 & 0.02 \\ 0.91 & 0.07 & 0.00 & 0.02 \\ 0.91 & 0.07 & 0.00 & 0.02 \\ 0.91 & 0.07 & 0.00 & 0.02 \end{pmatrix}$$

We note that, whether the Burkinabe budget is in low deficit, high deficit, low surplus or high surplus during a given year, in the long term, the most probable state of the budget is a low deficit (probability = 0.91). This probability is much higher than 0.50 and shows that Mali will succeed in stabilizing its budget deficit in the long term.

• Niger

Calculation of Transition Probabilities and Definition of the Transition Matrix in N Steps

	Low Deficit	High Deficit	Low surplus	High surplus	Budget balance	Total
Number of favorable cases	42	15	5	1	1	64
Total number of cases	64	64	64	64	64	64
P(Have a budget status type in a given year)	0.65	0.23	0.08	0.02	0.02	1

Source: Our Own Investigations

Table 6: Evaluation of the Probabilities of Each Budgetary Transition State

Based on preliminary calculations, the transition matrix for Niger (s=6) is as follows:

$$P_6 = \begin{pmatrix} 0.75 & 0.14 & 0.07 & 0.02 & 0.02 \\ 0.40 & 0.60 & 0.00 & 0.00 & 0.00 \\ 0.61 & 0.00 & 0.39 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{pmatrix}$$

Since the P_6 matrix is diagonalizable, the n-step transition matrix for Niger (s=6) is calculated but not presented here given the high dimension of the latter for the sake of ease of reading.

Projection of Budget Deficits Knowing that the Niger Budget Experienced A Small Deficit in 2023 (Reference Situation)

For projections for the years 2024 to 2028, it is sufficient to successively vary the natural integer n between the values (2.3.4.5.6).

The projections are summarized in the following table 7:

Years	Different Projections	Budget Deficit (% of GDP)
2024	Our projections	Déficit Faible(-5%<DF<-3%)
2025		Déficit Faible(-3%<DF<-2.5%)
2026		Déficit Faible(-2.5%<DF<-2%)
2027		Déficit Faible(-2%<DF<-1%)
2028		Déficit Faible(-2%<DF<-1%)

Source : Our own investigations

Table 7: Projection of Budgetary Deficits

Taking 2023 as the reference year, the budget deficit projections for Niger for the years 2024, 2025, 2026, 2027 and 2028 show a fairly low budget deficit (including grants) decreasing but with a low speed over the next five years. This result is the direct consequence of the budgetary rigor by the new transitional government since the recent coup d'état.

State of the Government Budget in the Long Term

$$\lim_{n \rightarrow +\infty} P_6^{(n)} = \begin{pmatrix} 0.66 & 0.23 & 0.08 & 0.01 & 0.01 \\ 0.66 & 0.23 & 0.08 & 0.01 & 0.01 \\ 0.66 & 0.23 & 0.08 & 0.01 & 0.01 \\ 0.66 & 0.23 & 0.08 & 0.01 & 0.01 \\ 0.66 & 0.23 & 0.08 & 0.01 & 0.01 \end{pmatrix}.$$

We note that, whether the Niger budget is in low deficit, high deficit, low surplus or high surplus during a given year, in the long term, the most probable state of the budget is a low deficit (probability = 0.66). This probability, although higher than 0.50, shows that the current sociopolitical and security instability in Niger could disrupt the State's public finances in the long term.

• Senegal

Calculation of Transition Probabilities and Definition of the Transition Matrix In n Steps

	Low Deficit	High Deficit	Low surplus	High surplus	Budget balance	Total
Number of favorable cases	48	9	7	0	0	64
Total number of cases	64	64	64	64	64	64
P(Have a budget status type in a given year)	0.75	0.14	0.11	0.00	0.00	1

Source: Our Own Investigations

Table 8: Evaluation of the Probabilities of Each Budgetary Transition State

In view of this table, the probabilities of the states “High Surplus” and “Budget Balance” occurring are all zero. These are therefore impossible events. The transition matrix for Senegal is thus reduced to a square matrix of order 3.

In view of the preliminary calculations, the transition matrix for Senegal (s=7) is as follows:

$$P_7 = \begin{pmatrix} 0,81 & 0,13 & 0,06 \\ 0,56 & 0,33 & 0,11 \\ 0,57 & 0,00 & 0,43 \end{pmatrix}$$

La matrice P_7 étant diagonalisable, la matrice de transition en n pas pour le Sénégal(s=7) est calculée et se présente comme suit:

$$P_7^{(n)} = \begin{pmatrix} 0.74 - 0.06(0.31)^n + 0.31(0.26)^n & 0.15 + 0.41(0.31)^n - 0.56(0.26)^n & 0.11 - 0.36(0.31)^n + 0.25(0.26)^n \\ 0.74 + 0.09(0.31)^n - 0.84(0.26)^n & 0.15 - 0.66(0.31)^n + 1.52(0.26)^n & 0.11 + 0.57(0.31)^n - 0.68(0.26)^n \\ 0.74 + 0.28(0.31)^n - 1.03(0.26)^n & 0.15 - 2.00(0.31)^n + 1.85(0.26)^n & 0.11 + 1.72(0.31)^n - 0.83(0.26)^n \end{pmatrix}$$

Projection of budget deficits knowing that the Senegalese budget experienced a high deficit in 2023 (reference situation)

For projections for the years 2024 to 2028, it is sufficient to successively vary the natural integer n between the values (2.3.4.5.6). The projections are summarized in the following table 9:

Years	Different Projections	Budget Deficit (% of GDP)
2024	Our projections	Low Deficit(-5%<DF<-3%)
2025		Low Deficit(-3%<DF<-2.75%)
2026		Low Deficit(-2.75%<DF<-2%)
2027		Low Deficit(-2%<DF<-1%)
2028		Low Deficit(-2%<DF<-1%)

Source: Nos Proper’s Investigations

Table 9: Projection of Budgetary Deficits

Taking 2023 as the reference year, the budget deficit projections for Senegal for the years 2024, 2025, 2026, 2027 and 2028 show a low budget deficit (including grants) decreasing but with a low speed over the next five years. This result is the direct consequence of the strengthening of the Senegalese economy through industrialization.

State of the Government Budget in the Long Term

This is to see the most likely state of the Senegalese budget in the long term from the convergence limit of the transition matrix in n steps.

$$\lim_{n \rightarrow +\infty} P_7^{(n)} = \begin{pmatrix} 0.74 & 0.15 & 0.11 \\ 0.74 & 0.15 & 0.11 \\ 0.74 & 0.15 & 0.11 \end{pmatrix}.$$

We note that, whether the Senegalese budget is in low deficit, high deficit, low surplus or high surplus during a given year, in the long term, the most probable state of the budget is a low deficit (probability = 0.74). This probability is much higher than 0.50 and shows that Senegal will succeed in stabilizing its budget deficit in the long term, thus respecting the macroeconomic convergence criteria in the WAEMU.

• **Togo**

Calculation of Transition Probabilities and Definition of the Transition Matrix in n Steps

	Low Deficit	High Deficit	Low surplus	High surplus	Budget balance	Total
Number of favorable cases	43	15	6	0	0	64
Total number of cases	64	64	64	64	64	64
P(Have a budget status type in a given year)	0.67	0.24	0.09	0.00	0.00	1

Source: Nos Proper's Investigations

Table 10: Evaluation of the Probabilities of Each Budgetary Transition State

In view of this table, the probabilities of the states "High surplus" and "Budget balance" occurring are zero. These are therefore impossible events. The transition matrix for Togo is thus reduced to a square matrix of order 3.

In view of the preliminary calculations, the transition matrix for Togo (s=8) is as follows: $P_8 =$

$$\begin{pmatrix} 0.75 & 0.13 & 0.12 \\ 0.40 & 0.60 & 0.00 \\ 0.66 & 0.17 & 0.17 \end{pmatrix}$$

Since the matrix P_8 is diagonalizable, the n-step transition matrix for Togo(s=8) is calculated and is presented as follows:

$$P_8^{(n)} = \begin{pmatrix} 0.65 + 0.19(0.46)^n + 0.16(0.06)^n & 0.25 - 0.27(0.46)^n + 0.02(0.06)^n & 0.10 + 0.08(0.46)^n - 0.17(0.06)^n \\ 0.65 - 0.54(0.46)^n - 0.12(0.06)^n & 0.25 + 0.76(0.46)^n - 0.01(0.06)^n & 0.10 - 0.22(0.46)^n + 0.13(0.06)^n \\ 0.65 + 0.12(0.46)^n - 0.77(0.06)^n & 0.25 - 0.17(0.46)^n - 0.08(0.06)^n & 0.10 + 0.05(0.46)^n + 0.86(0.06)^n \end{pmatrix}$$

Projection of Budget Deficits Knowing That the Togolese Budget Experienced A High Deficit in 2023 (Reference Situation)

For projections for the years 2024 to 2028, it is sufficient to successively vary the natural integer n between the values (2.3.4.5 and 6).

The projections are summarized in the following table 11:

Years	Different Projections	Budget Deficit (% of GDP)
2024	Our projections	Déficit Faible(-5%<DF<-3%)
2025		Déficit Faible(-3%<DF<-1%)
2026		Déficit Faible(-3%<DF<-0.5%)
2027		Déficit Faible(-2%<DF<-1%)
2028		Déficit Faible(-2%<DF<-1%)

Source: Nos Proper's Investigations

Table 11: Projection of Budgetary Deficits

Taking 2023 as the reference year, the budget deficit projections for Togo for the years 2024, 2025, 2026, 2027 and 2028 show a low budget deficit (including grants) decreasing but with a much lower speed over the next five years compared to other countries of the Union. This result is the direct consequence of the recent political instability in Togo.

State of the Government Budget in the Long Term

This is to see the most likely state of the Togolese budget in the long term from the convergence limit of the transition matrix in n steps.

$$\lim_{n \rightarrow +\infty} P_8^{(n)} = \begin{pmatrix} 0.65 & 0.25 & 0.10 \\ 0.65 & 0.25 & 0.10 \\ 0.65 & 0.25 & 0.10 \end{pmatrix}.$$

We note that, whether the Togolese budget is in low deficit, high deficit, low surplus or high surplus during a given year, in the long term, the most probable state of the budget is a low deficit (probability = 0.65). This probability is slightly higher than 0.50 and shows that Togo risks going through periods of high deficit (25% chance) in the long term if the current budgetary consolidation reforms are not maintained and optimized.

References

1. International Monetary Fund. (2022). Annual Report 2022 on the Operation and Development of the Union, December 2022.
2. Agenor, P. R., Izquierdo, A., & Fofack, H. (2002). A Quantitative Macroeconomic Framework for Analyzing Poverty Reduction Strategies. World Bank.
3. Pereira, D. L. A., Essama, B., & Samake, I. (2002). A Poverty Analysis Macro-Economic Simulator (PAMS). Linking Household Surveys with Macro-Models. World Bank Working Paper.
4. Khan, M. S., Montiel, P. J., & Haque, U. N. (1991). Macroeconomic Models for Adjustment in Developing Countries. International Monetary Fund.
5. Haque, U. N., Lahiri, K., & Montiel, P. J. (1991). A Macroeconometric Model for Developing Countries in Khan, M. S., Montiel, P. J., & Haque, U. N. Macroeconomic Models for Adjustment in Developing Countries. *International Monetary Fund*.
6. Wong, C. H. (1977). Demand for Money in Developing Countries: Some Theoretical and Empirical Results. *Journal of Monetary Economics*, 3(January), 59-86.
7. Soludo, C. (1995). Macroeconomic Adjustment, Trade, and Growth: Policy Analysis Using a Macroeconomic Model of Nigeria. AERC Research Paper 32, March.
8. Niang, B. B. (2000). A Macroeconomic Model for a CFA Country: The Case of Senegal. *African Economic Research Consortium, Nairobi, December 2000*.
9. Zeufack, A. G. (1997). Ownership Structure and Investment Behavior in Uncertain Environment: Estimation on Panel Data from the Cameroonian Manufacturing Sector. *Review of Development Economics*, 1, 29-59.
10. Diarisso, S. (2001). Integrated Macroeconomic Projection Model for WAEMU Member States, "PROMES". Estimation and Application to Senegal. BCEAO, January.
11. Evers, I., & Ndoye, O. (1996). MOMAR, Revised Macroeconomic Model of Senegal, Version 2. Ministry of Economy and Finance – Planning Directorate.
12. Diop, A., & Dieng, M. F. (1988a). Presentation of the Structure of the SESAME Model. Ministry of Planning – Planning Directorate.
13. Diop, A., & Dieng, M. F. (1988b). Description of the Macroeconomic Model. Ministry of Planning – Planning Directorate.
14. Niang, B. B. (2003). Review of Macroeconomic Modeling Experiences in Senegal. Ministry of Economy and Finance – Development Strategy Directorate – German Cooperation (GTZ), December.
15. Derrode, S., & Pieczynski, W. (2013). Unsupervised Data Classification Using Pairwise Markov Chains with Automatic Copulas Selection. *Computational Statistics and Data Analysis*, 63, 81-98.
16. Derrode, S., & Pieczynski, W. (2014). Unsupervised Signal and Image Segmentation Using Pairwise Markov Chains. *IEEE Transactions on Signal Processing*, 52(9), 2477-2489.
17. Carincotte, C., Derrode, S., & Bourennane, S. (2006). Unsupervised Change Detection on SAR Images Using Fuzzy Hidden Markov Chains. *IEEE Transactions on Geoscience and Remote Sensing*, 44(2), 432-411.

Copyright: ©2024 Damien J. Agboton. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.