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Demonstrating Goldbach's Strong Conjecture by Deduction using 4x ± 1 Equations in Loops and Gaps of 4

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Abstract

After defining the writing of even numbers with equations 4x ± 1; the article shows that odd numbers obey loops of 4 numbers to infinity that obey the equations 4x ± 1. Each odd number in the loop can be prime (P) or composite (C), but for an even number E to be E = P + P′ such that P′ > P and such that P′ > E/2 and P < E/2, P and P′ must belong to two loops symmetrical with respect to E/2 and occupy the same positions in them and specific unit digits. The article shows that the three possible sums of an even number E are E = C + C′; or E = P + C; or E = P + P′ (C is composite and P is prime). The article demonstrates that E = C + C′ ↔ E = C + P ↔ E = P + P′ ; and that one sum can be converted into another by substracting and adding gaps of 4n from or to the two terms of addition. This is a deductive demonstration of Goldbach's strong conjecture shown here for the first time.

Keywords: Goldbach's Strong Conjecture, Primes, Composites, Primes 4x ± 1, Gaps of 4, Addition, Substraction, Evens, Odds, Loops

1. Introduction

Using 6x ± 1 equations there are three types of evens including 6x ; 6x + 2 and 6x + 4. Using the same equations, we can convert an even 6x in sum of two primes one of which is 6x – 1 and the other is 6x + 1 while 6x + 2 is a sum of two primes 6x + 1. Finally, 6x + 4 = 6x – 2 is a sum of two primes 6x – 1 [1-2]. However there are only two evens with 4x ± 1 equations including 4x and 4x + 2 which reduces the number of cases and allows to investigate in a new way the Goldbach's strong conjecture stating that every even denoted E ≥ 4 is a sum of two primes. The aim of this article is to demonstrate Goldbach's conjecture using the 4x ± 1 equations; the gaps between the 4x ± 1 primes; and predict the positions of the equidistant 4x ± 1 primes, which are the only ones that count for verifying this conjecture by calculation.

2. Results

2.1 The Subsets of Even and Primes Numbers (P)

By the equation 6x ± 1 there are three subsets of even numbers denoted Se such that Se1= {6x} ; Se2 = {6x + 2} and Se3 = {6x + 4}. On the other hand, there are two subsets of even numbers denoted Sf such that Sf1 = {4x} and Sf2 = {4x + 2} (see also Figure 1). P are divided into two subsets Spn including Spn1 = {4x – 1} and Spn2 = {4x + 1} (Table 1).

Spn1	Spn2
{4x – 1}	{4x + 1}
3	1
7	5
11	9
15	13
19	17
23	21
27	25
31	29
35	33

39	37
43	41
47	45
51	49
55	53
59	57
63	61
67	65
71	69
75	73
79	77
83	81
87	85
91	89
95	93
99	97
→+∞	→ +∞

Table 1: The two subsets Spn1 and Spn 2 of Prime Numbers (P) by the equations 4x ± 1

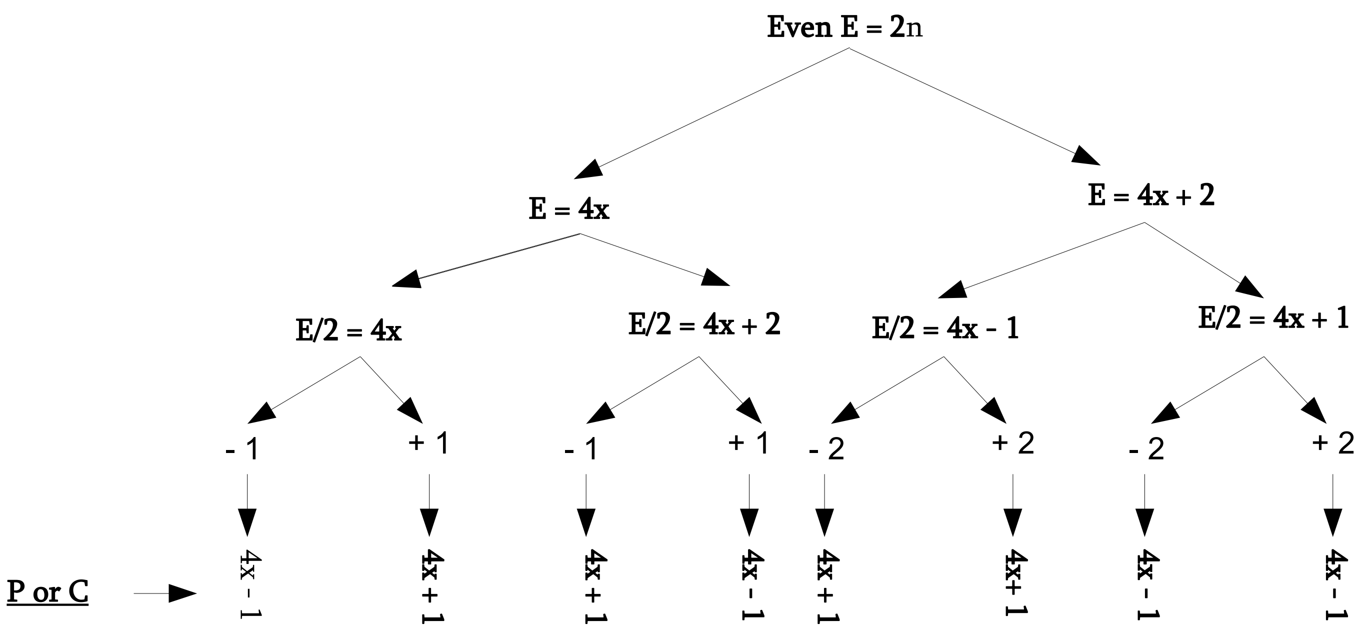


Figure 1: How to obtain prime numbers (P) and composite numbers (C) according to the equations of even numbers (E) such that E = 4x and E = 4x + 2. This is true to infinity. Note P or C is 4x ± 1.

2.2 Other Subsets of Integers

Let SO3n be the subset of odd numbers that are multiples of 3 and therefore SO3n = {3, 9, 15, 21 , 27, 33, 39,...→+∞}.
We can see that the numbers 4x + 1 and 4x – 1 alternate in the subset SO3n. For example 3 + 6 = 9 = 4x + 1; 9 + 6 = 15 = 4x – 1; 15 + 6 = 21 = 4x + 1; 21 + 6 = 27 = 4x –1 to infinity.
Therefore we have SO3n(4x + 1) and SO3n(4x – 1).
Let SOpc be the subset of all P and composite odd numbers (C) that are not 3n → {SO3n} ∩ {Sopc} = ∅.
Let Se2n be the subset of all even integers 2n such that n > 0 so Se2n = {2, 4, 6, 8, 10, 12,...→+∞}. A natural number denoted n ≥ 3 is either even or odd. If even, it is 4x or 4x + 2. If odd, it is 4x + 1 or 4x + 3, knowing that 4x + 3 ↔ 4x – 1.

2.3 Conversion of an Even ≥ 4 in Sum of two Primes According to Goldbach's strong Conjecture (GSC)

GSC means 2n = P + P' such that P' > P and such that n – PN1 = PN2 – n. PN1 and PN2 are said to be equidistant at n. How then can we obtain Ps from n? It all depends on whether the even 2n is 4x or 4x + 2 ; whether n is odd or even ; and whether n is 3n or not. These are the rules by which the GSC operates.

« Even numbers 4x are always the sum of one P 4x – 1 and another P 4x + 1 according to GSC. On the other hand, 4x + 2 evens are the sum of either two P 4x – 1 or two P 4x + 1. This is because 4x – 2 ↔ 4x + 2. And so, only even numbers 4x are the sum of Ps belonging to both Spn1 and Spn2 whereas even numbers 4x + 2 derive from either Spn1 or Spn2 but never from both Spn (Table 1)».

3a. The even 2n is non 3n

3a1. The even 2n is 4x

3a1–1. n is even and is 4x

In this case n – {S03n}(4x + 1) = {Spn1}(4x – 1) and n – {S03n}(4x – 1) = {Spn2}(4x + 1). n + {S03n}(4x + 1) = {Spn2}(4x + 1) and n + {S03n}(4x – 1) = {Spn1}(4x – 1).

Two Examples of n – {S03n} and n + {S03n}
80 = 2 x 40.
40 – 3 = 37 such that 3 is 4x – 1 and 37 4x + 1.
40 + 3 = 43 such that 3 is 4x – 1 and 43 is 4x – 1
In this case 80 = 37 + 43 and 37 and 43 are equidistant at 40.
40 – 33 = 7 such that 33 is 4x + 1 ad 7 is 4x – 1.
40 + 33 = 73 such that 33 is 4x + 1 and 73 is 4x + 1
In this 80 = 7 + 73 ad 7 and 73 are equidistant at 40.

To get all the possible sums we calculate with both {SO3n} and {Sopc} which means all odds except prime factors of the tested number. We thus calculate n - {Sopc} and n + {Sopc} together with n – {SO3n} and n + {SO3n}. But note that even if we do not have them all we can get them by using the available sums E = P + P' = (P ± 4n)+ (P' ± 4n) = P'' + P''' and so on. In other words E = P + P' = (P + 4n) + (P' – 4n) = P'' + P''' ; or E = P + P' = (P - 4n)+ (P' + 4n) = P'' + P'''.

Example. 80 = 2 x 40.
40 – **3** = **37** ; 40 – **7** = 33 ; 40 – **9** = 31 ; 40 – **11** = 29 ; 40 – **13** = 27 ; 40 – **17** = 23 ; 40 – **19** = 21 ; 40 – **21** = **19** ; 40 – **23** = 27 ; 40 – **27** = **13** ; 40 – **29** = 11 ; 40 – **31** = 9 ; 40 – **33** = **7** ; 40 – **37** = 3.
40 + **3** = **43** ; 40 + **7** = 47 ; 40 + **9** = 49 ; 40 + **11** = 51 ; 40 + **13** = 53 ; 40 + **17** = 57 ; 40 + **19** =59 ; 40 + **21** = **61** ; 40 + **23** = 63; 40 + **27** = **67** ; 40 + **29** = 69 ; 40 + **31** = 71 ; 40 + **33** = **73** ; 40 + **37** = 77.
We have 80 = 37 + 43 = 19 + 61 = 13 + 67 = 7 + 73. In all cases **80 = P(4x + 1) + P'(4x – 1) x > 0**

3a1–2. n is even and is 4x + 2

In this case n – {SO3n}(4x + 1) = {Spn2}(4x + 1) and n – {SO3n}(4x – 1) = {Spn1}(4x – 1)}. n + {SO3n}(4x + 1) = {Spn1}(4x – 1) and n + {SO3n}(4x – 1) = {Spn1}(4x + 1).

Example 100 = 2 x 50 such that 50 is 4x + 2.
50 – 9 = 41 such that both 9 and 41 are 4x + 1.
50 + 9 = 59 such that 9 is 4x + 1 and 59 is 4x – 1.
100 = 41 + 59 such that 41 and 59 are equidistant at 50.
50 – 39 = 11 such that both 11 and 39 are 4x – 1.
50 + 39 = 89 such that 39 is 4x – 1 and 89 is 4x + 1.
100 = 11 + 89 such that 11 and 89 are equidistant at 50.

Here are all the sums of two primes by n - {Sopc} and n + {Sopc} together with n – {SO3n} and n + {SO3n}.

50 – 3 = **47** ; 50 – 7 = 43 ; 50 – 9 = **41** ; 50 – 11 = 39 ; 50 – 13 = 37 ; 50 – 17 = 33 ; 50 – 19 = 31 ; 50 – 21 = **29** ; 50 – 23 = 27 ; 50 – 27 = 23 ; 50 – 29 = 21 ; 50 – 31 = 19 ; 50 – 33 = **17** ; 50 – 37 = 13 ; 50 – 39 = **11** ; 50 – 41 = 9 ; 50 – 43 = 7 ; 50 – 47 = 3.

50 + 3 = **53** ; 50 + 7 = 57 ; 50 + 9 = **59** ; 50 + 11 = 61 ; 50 + 13 = 63 ; 50 + 17 = 67 ; 50 + 19 = 69 ; 50 + 21 = **71** ; 50 + 23 = 73 ; 50 + 27 = 77 ; 50 + 29 = 79 ; 50 + 31 = 81 ; 50 + 33 = **83** ; 50 + 37 = 87 ; 50 + 39 = **89** ; 50 + 41 = 91 ; 50 + 43 = 93 ; 50 + 47 = 97.

We have 100 = 47 + 53 = 41 + 59 = 29 + 71 = 17 + 83 = 11 + 89 = 3 + 97.

100 = PN(4x + 1) + PN(4x – 1) x > 0

Here is another example with all the sums of two primes by n - {Sopc} and n + {Sopc} together with n – {SO3n} and n + {SO3n}.

Example 140 = 4 x 35 and 140 : 2 = 70 and 70 = 4 x 17 + 2 (4x + 2).
In this case n = (4x + 1) + (4x + 1) or n = (4x – 1) + (4x – 1) ⟷ n – (4x + 1) = 4x + 1 and n – (4x – 1) = 4x – 1. Exampe 70 – 5(4x + 1) = 65(4x + 1) and 70 – 7(4x – 1) = 63(4x – 1).

70 – 3 = **67** ; 70 – 5 = 65 ; 70 – 7 = 63 ; 70 – 9 = **61** ; 70 – 11 = 59 ; 70 – 13 = 57 ; 70 – 15 = 55 ; 70 – 17 = 53 ; 70 – 19 = 51 ; 70 – 21 = 49 ; 70 – 23 = 47 ; 70 – 25 = 45 ; 70 – 27 = **43** ; 70 – 29 = 41 ; 70 – 31 = 49 ; 70 – 33 = **37** ; 70 – 35 = 35 ; 70 – 37 = 33 ; 70 – 39 = **31** ; 70 – 41 = 29 ; 70 – 43 = 27 ; 70 – 45 = 25 ; 70 – 47 = 23 ; 70 – 49 = 21 ; 70 – 51 = 19 ; 70 – 53 = 17 ; 70 – 55 = 25 ; 70 – 57 = **13** ; 70 – 59 = 11 ; 70 – 61 = 9 ; 70 – 63 = 7 ; 70 – 65 = 5 ; 70 – 67 = 3

70 + 3 = **73** ; 70 + 5 = 75 ; 70 + 7 = 77 ; 70 + 9 = **79** ; 70 + 11 = 81 ; 70 + 13 = 83 ; 70 + 17 = 87 ; 70 + 19 = 89 ; 70 + 23 = 93 ; 70 + 25 = 95 ; 70 + 27 = **97** ; 70 + 29 = 99 ; 70 + 31 = 101 ; 70 + 33 = **103** ; 70 + 35 = 105 ; 70 + 37 = 107 ; 70 + 39 = **109** ; 70 + 41 = 111 ; 70 + 43 = 113 ; 70 + 47 = 117 ; 70 + 49 = 119 ; 70 + 53 = 123; 70 + 55 = 125 ; 70 + 57 = **127** ; 70 + 59 = 129 ; 70 + 61 = 131 ; 70 + 65 = 135 ; 70 + 67 = 137.

We have 140 = 67 + 73 = 61 + 79 = 43 + 97 = 37 + 103 = 31 + 109 = 13 + 127 = 3 + 137.

Conclusion: An even E = 4n such that n is even and whether n = 4x or n = 4x + 2 E = 4n = P(4x + 1) + P'(4x – 1) x > 0

3a1–3. n is odd and is 4x + 1or 4x – 1

Iit is impossible to get an even 2n which is 4x from 2 x (Odd numbers) that are either 4x + 1 or 4 x – 1. Because 2 x (4x + 1) ≠ 4x and 2 x (4x – 1) ≠ 4x.

3a2. The even 2n is 4x + 2

3a2–1. First Method

In this case n ≠ 4x and either n = 4 x + 1 or n = 4x – 1. However, an even is either 4x or 4x + 2. So n cannot be even in this case and is abosolutely odd. In this case we have two cases either 4x + 2 = (4x + 1) + (4x + 1) or 4x + 2 = (4x – 1) + (4x – 1) → n = 4x + 1 or n = 4x – 1. • If n = 4x – 1 then n – (4x) = 4x – 1 and n + (4x) = 4x – 1.

Example 70 = 2 x 35
70 = 4 x 17 + 2 = 4x + 2.
35 = 36 – 1 = 4x – 1.
35 – 4 = 31 – 4 = 27 – 4 = **23** – 4 = 19 – 4 = 15 – 4 = **11** – 4 = 7 – = 3 (31 ; 27 ; 23 ; 19 ; 15 ; 11 ; 7 ; 3 are all 4x – 1).
35 + 4 = 39 + 4 = 43 + 4 = **47** + 4 = 51 + 4 = 55 + 4 = **59** + 4 = 63 + 4 = 67 (39 ; 43 ; 47 ; 51 ; 55 ; 59 ; 63 and 67 are all 4x – 1).
Therefore 70 = 23 + 47 ; 70 = 11 + 59 knowing that 23 et 47 ; 11 and 59 are all equidistant at 35 thus satisfying the GSC.

• If n = 4x + 1 then n – (4x)= 4x + 1 and n + (4x) = 4x + 1.

Example 122 = 2 x 61 122 = 4 x 30 + 2 61 = 4 x 15 + 1. 61 – 4 = 57 – 4 = 53 – 4 = 49 – 4 = 45 – 4 = 41 – 4 = 37 – 4 = 33 – 4 = 29 – 4 = 25 – 4 = 21 – 4 = 17 – 4 = **13** – 4 = 9 – 4 = 5 (57 ; 53 ; 49 ; 45 ; 41 ; 37 ; 33 ; 29 ; 25 ; 21 ; 17 ; 13 ; 9 ; and 5 are all 4x + 1). 61 + 4 = 65 + 4 = 69 + 4 = 73 + 4 = 77 + 4 = 81 + 4 = 85 + 4 = 89 + 4 = 93 + 4 = 97 + 4 = 101 + 4 = 105 + 4 = **109** + 4 = 113 + 4 = 117 (65 ; 69 ; 73 ; 77 ; 81 ; 85 ; 89 ; 93 ; 97 ; 101 ; 105 ; 109 ; 113 ; 117 are all 4x + 1). 122 = 61 + 61 ; 122 = 13 + 109 such that 13 and 109 are equidistant at 61.

• The limitation of this method is that it does not give all possible sums of two Ps of the number being tested. But note that even if we do not have them all we can get them by using the available sums this way E = P + P' = (P ± 4n)+ (P' ± 4n) = P'' + P'''.

3a2–2. Second Method

The other method is to calculate n – Se2n(n = 1) and n + Se2n(n = 1)

Example 70 = 2 x 35 ; 70 = 4 x 17 + 2 = 4x + 2 and 35 = 36 – 1 = 4x – 1. 35 – 2 = 33 – 2 = 31 – 2 = 29 – 2 = 27 – 2 = 25 – 2 = 23 – 2 = 21 – 2 = 19 – 2 = 17 and so on. 35 + 2 = 37 + 2 = 39 + 2 = 41 + 2 = 43 + 2 = 45 + 2 = 47 + 2 = 49 + 2 = 51 + 2 = 53 and so on. We will have 70 = 3 + 67 = 11 + 59 = 17 + 53 = 23 + 47 = 29 + 41 and so 70 = (4x – 1) + (4x – 1) or 70 = (4x + 1) + (4x + 1). Example 122 = 2 x 61 ; 122 = 4 x 30 + 2 and 61 = 4 x 15 + 1. 61 – 2 = 59 ; 61 – 4 = 57 ; 61 – 6 = 55 ; 61 – 8 = 53 ; 61 – 10 = 51 ; 61 – 12 = 49 ; 61 – 14 = 47 and so on. 61 + 2 = 63 + 2 = 65 + 2= 67 + 2 = 69 + 2 = 71 + 2 = 73 + 2 = 75 + 2 = 77 + 2 = 79 + 2 = 81 and so on. We will have 122 = 61 + 61 = 43 + 79 = 19 + 103 = 13 + 109. And so 122 = P(4x + 1) + P'(4x + 1) or 122 = P(4x - 1) + P'(4x - 1).

• The great advantage of this method is that it gives all possible sums of two Ps of the number being tested. Indeed, (4x – 1) – 2 = 4x – 3 = 4x + 1 while (4x – 1) + 2 = 4x + 1. Whereas (4x + 1) – 2 = 4x – 1 and (4x + 1) + 2 = 4x + 3 = 4x – 1. On the other hand either 4x – 1 (± 4n) = 4x – 1 ; or 4x + 1 (± 4n) = 4x + 1.

3. Conclusion: An even 2n which is 4x + 2 or E(4x+2) = P(4x – 1) + P'(4x – 1) ; or E(4x+2) = P(4x + 1) + P'(4x + 1).

3b. The even 2n is 3n and is 4x

In this case n = (4x – 1) + (4x + 1) or n = (4x + 1) + (4x – 1).

Because 2n is 3n, n is 3n and to get primes we have to calculate n – {SOpc} = {Spn1} or C and n + {SOpc} = {Spn2} or C. In this case n – {Spn1} = {Spn2} and n – {SPn2} = {Spn1}. Note that we exclude prime factors of the tested number and {SO3n}. The major difference between evens that are 3n and non3n is that we exclude {SO3n} from the former and we cannot calculate n – {SO3n} = 3n and n + {SO3n} = 3n which are not primes.

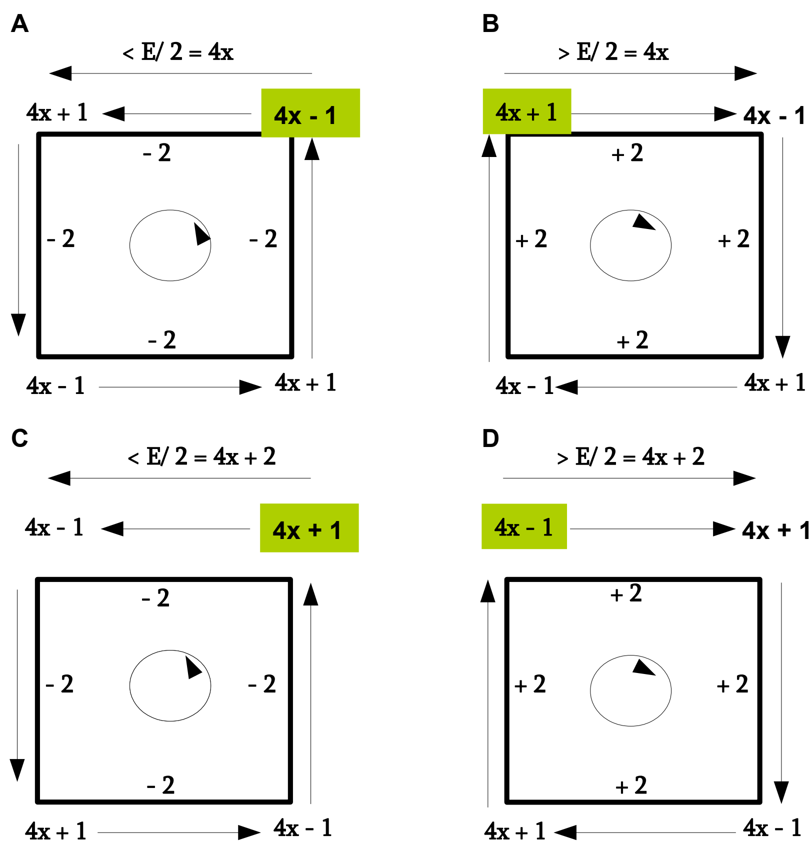
Example. 96 = 4 x 24 ; 96 is 3n and 96 : 2 = 48 = 4x which is 3n. Calculate 48 – {Sopc} by excluding {SO3n} . 48 – **5** = 43 ; 48 – **7** = 41 ; 48 – **11** = 37 ; 48 – **13** = 35 ; 48 – **17** = 31 ; 48 – **19** = 29 ; 48 – **23** = 25 ; 48 – **25** = 23 : 48 – **29** = 19 ; 48 – **31** = 17 ; 48 – **35** = 13 ; 48 – **37** = 11 ; ; 48 – **41** = 7 ; 48 – **43** = 5. Note 48 – 5 (4x + 1) = 43 (4x – 1) ; and 48 – 7(4x – 1) = 41(4x + 1). 48 + **5** = 53 ; 48 + **7** = 55 ; 48 + **11** = 59 ; 48 + **13** = 61 ; 48 + **17** = 65 ; 48 + **19** = 67 ; 48 + **23** = 71 ; 48 + **25** = 73 : 48 + **29** = 77 ; 48 + **31** = 79 ; 48 + **35** = 83 ; 48 + **37** = 85 ; ; 48 + **41** = 89 ; 48 + **43** = 91. We have 96 = 43 + 53 = 37 + 59 = 29 + 67 = 23 + 73 = 17 + 79 = 13 + 83 = 7 + 89. We therefore have all possible sums of two primes.

For the rest the same rules apply to both 2n that is 3n or not after excluding {SO3n} with the former (3n).

4. Demonstration of Goldbach's Strong conjecture following Equations 4x ± 1 and gaps of 4n.

1. First, let's clarify the relationship between odd numbers and 4x ± 1. The 4x ± 1 equations always run in a 4-step loop involving 4 successive odd numbers. For example, 3 5 7 9 11 13 15 17 we have 3 is 4x - 1; 5 is 4x + 1; 7 is 4x - 1; and 9 (4x + 1) (the first loop) and we have 11 is 4x - 1; 13 is 4x + 1; 15 is 4x - 1 and 17 is 4x + 1 (the second loop) and so on to infinity. On the other hand, there are 5 types of odd numbers ending in the unit digit 1; 3; 5; 7 and 9. Composite numbers end with any of these 5 unit digits, but a prime number never ends with 5 (except 5 itself), which is one of the reasons why composite numbers outnumber primes (in addition to multiples of 3 which are also composites). Recall that E is any even number ≥ 4 and E/2 is any integer ≥ 2. Figure 1 shows how even numbers are formed by the equations 4x ± 1. An even number E = 2n (E/2 = n) is either 4x or 4x + 2. In the first case, E/2 = 4x or E/2 = 4x + 2. In the second case, E/2 = 4x - 1 or E/2 = 4x + 1. This means that when an even number is 4x, it will be the sum of two primes or two composite numbers, one of which is 4x - 1 and the other 4x + 1. Whereas when it is 4x + 2 ; it will be the sum of two primes or two composite numbers, both of which are at the same time 4x - 1 or 4x + 1. Remember that an odd number 4x - 1 or 4x + 1 is either prime or composite. Neither can be predicted by this equation, unless primality tests or factorization tests are performed.

2. Figure 2 shows all possible 4x ± 1 loops that accur before and after E/2. Be E = 4x and so E/2 = 4x (Panels 2A + 2B) or E/2 = 4x + 2 (Panels 2C + 2D). For instance the first odd number that preceded E/2 = 4x is 4x – 1 and the one that follows it is 4x + 1 and to go from one to another, we substract or add 2. In the case where E = 4x + 2 we have either E/2 = 4x + 1 or E/2 = 4x – 1. Figure 2E + 2F shows the loops that occur in the former case and Figure 2G + 2H shows the latter case. These are the loops that always occur, and if E tends to infinity and E/2 tends to infinity, the loops repeat ad infinitum. The greater the even number, the greater the number of loops, and the greater the total number of possible sums of two prime numbers. A loop requires 8 numbers, 4 of them odd in their natural order; so to find the approximate total number of loops for an even number E, simply calculate E/8. To find the one before or after E/2, calculate E/16. Note that you can have half-loops when the number is not a multiple of 4. For example, the even number 10 has two half-loops < and > 5. We have 1 2 3 4 5 6 7 8 9 10 and the first halfloop is 3 - 1 (to exclude 1) and the second is 7 - 9. Knowing that 3 is 4x - 1 and 7 is 4x - 1 so 3 + 7 = 4x - 2 which is also 4x + 2 and in fact 10 is either 4x + 2 or 4x – 2. A major rule postulates that if two prime numbers P and P' are such that P + P' = E, then P and P' occupy the same positions in two symmetrical loops, one of which is < E/2 and the other > E/2. This applies to any even numbers 4x and 4x + 2 enough large to have at least two loops.



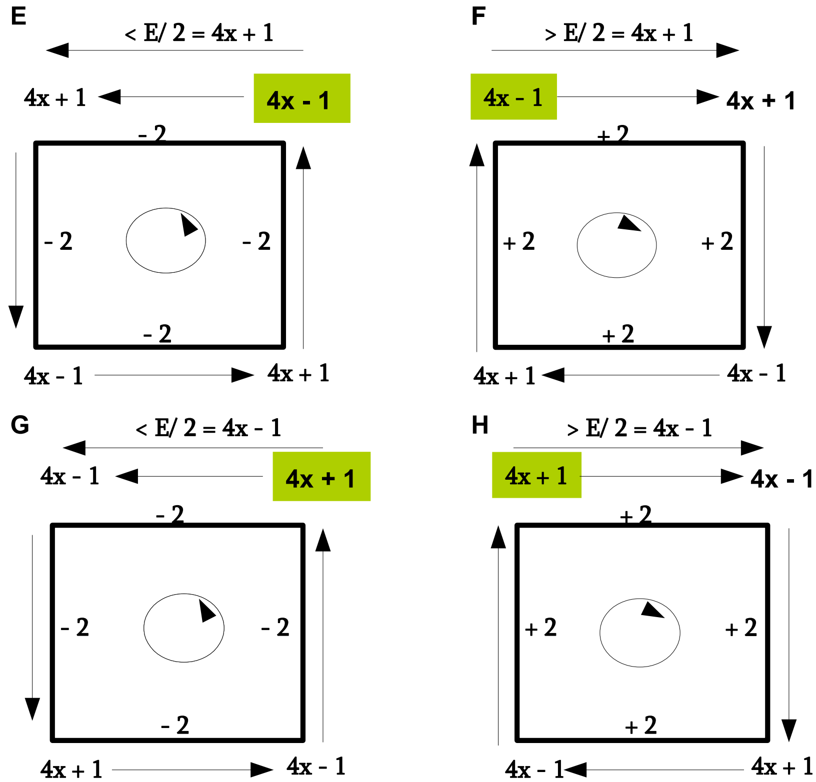


Figure 2: Odd umbers P or C that are 4x – 1 or 4x + 1 appear in loops of 8 numbers that repeat in the same way from E/2 to 0 and from E/2 to E.

This loop takes place every 8 integers from E/2 in the direction towards 0 and in the opposite direction from E/2 towards E. The two loops of 8 < E/2 and > E/2 are symmetrical. The two loops of 8 are shown for E = 4x even numbers with either E/ 2 = 4x (A + B) or E/2 = 4x + 2 (C + D). This is true to infinity. In the figure only odd numbers are shown separated of a gap = 2. Case of Evens E = 4x + 2 and therefore E/2 = 4x + 1 (Panels E and F) or E/2 = 4x - 1 (Panels G and H).

3. An even number denoted E ends in 0; 2; 4; 6; and 8 units digits. In the rest of this section, even numbers will be denoted as NX with X the unit digit. An even number ending in 0 is therefore denoted E = N0. In this case, E/2 = N0 (Figure 3A + 3B) or N/2 = N5 (Figure 3C + 3D). Figure 3 then shows the unit digits of the odd numbers within the loops < E/2 and > E/2. Two odd numbers occupying symmetrical positions must be added together to obtain the unit digit of the even number. In the case of E = N0 and E/2 = N5, we can see that the 4 odd numbers in symmetrical loops < E/2 and > E/2 add up for E = N0. For example 9 + 1; 7 + 3; 1 + 9 and 7 + 3, which shows that N0 even numbers have a high probability of being formed by the addition of two prime numbers. Note that the difference between odd numbers within the loop is 2 or 4. In fact, 4 is used to exclude 5, which cannot be the unit figure of a prime number. However, in some cases, the P number 5 is re-used. We have the same result with E = N0 ad E/2 = N5 (Figure 3C and 3D). In the case of E = N2 and E/2 = N1 (Figure 3E + 3F), we have the possible sums of two primes ending with the unit digits 9 + 3; 3 + 9; and 1+ 1. In the case of E = N2 and E/2 = N1 we have the possible sums of two primes ending with the unit digits 9 + 3; 3 + 9; and 1 + 1. The remainder of Figure 3, i.e. Figure 3G-L, gives the cases of evens ending with the unit digits 4; 6 and 8. With evens ending 6, it was necessary to use the unit digit 5 (evens if the corresponding number is not prime) to get all possible sums of 6 including two primes 3 + 3 ; 7 + 9 and 9 + 7.
Note: that this loop system allows us to predict the positions of the two potential equidistant primes P < E/2 and P' > E/2 such that E = P + P'. In addition, the unit digits is also a good too to determine if they can add up to form an even with a given unit digit.

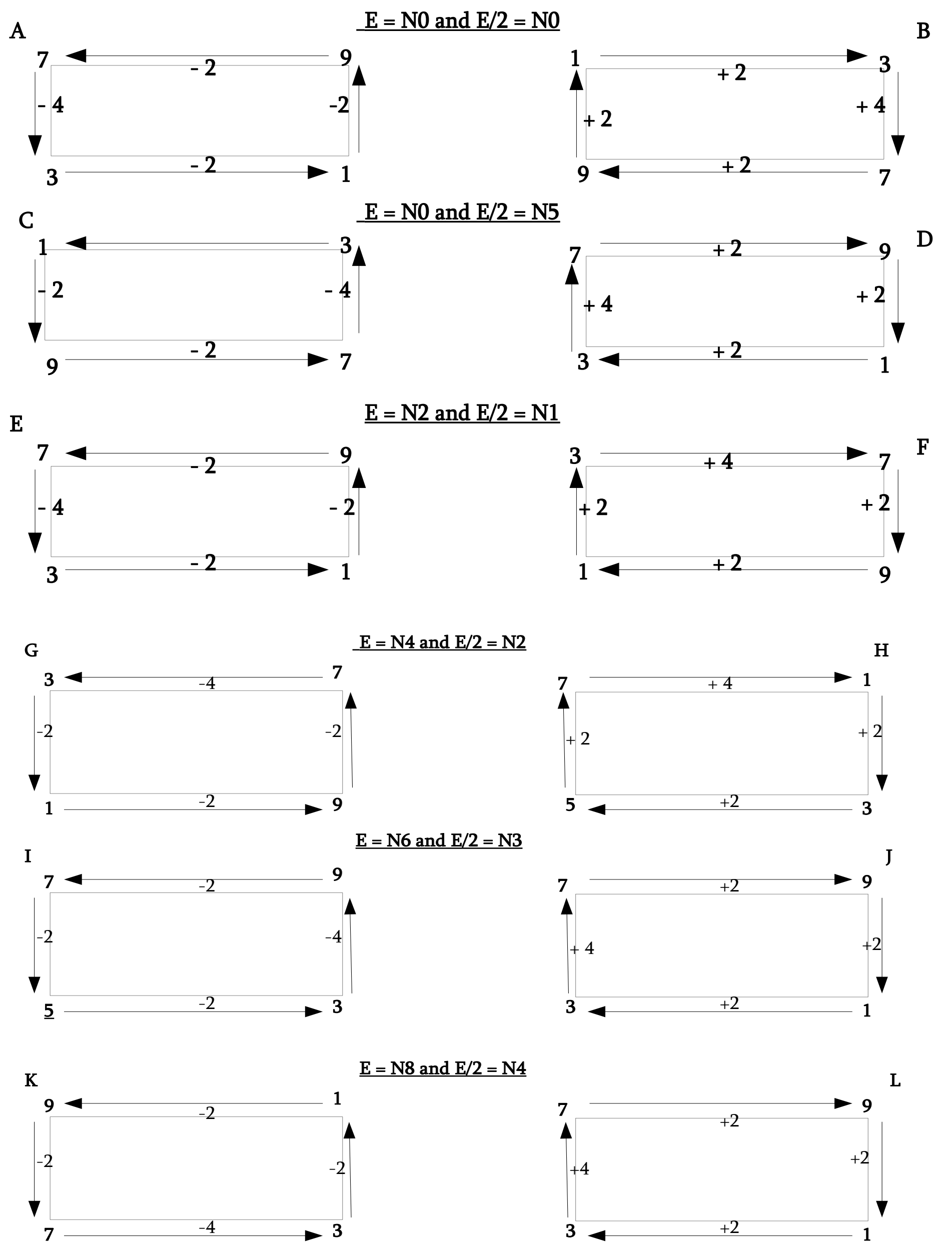


Figure 3A-F: Unit digits of primes (P) or composite (C) numbers < E/2 and > E/2 in a loop of 10 from 0 to E. If E tends to infinity, the loops of 10 repeat ad infinitum. The first number of the loop either < E/2 or E/2 is calculated by E/2 – n (n > 0) to obtain all possible sums of units digits that correspond to E. The number 5 is omittedwhen necessary because no prime number P ends with 5 except the number 5. The Gaps of 2 or 4 preceded by the sign - (< E/2) and + (>E/2) are used to find the numbers P which add up to give the unit figure of E in accordance with the GSC. E or E/2 are written as N followed by the unit digit. Panels A-B (E ending with 0 unit digit) ; Panels C-D (E ending with 0 unit digit) ; Panels E - F (E ending with 2 unit digit). Note that the sum of two odd numbers < E/2 and >E/2 equidistant from E/2 gives E by obeying the rules for addingtheir unit digits.

(Panels G-L). Unit digits of primes (P) or composite (C) numbers < E/2 and > E/2 in a loop of 10 from 0 to E. Panels G-H (E ending with 4 unit digit) ; panel I-J (E ending with 6 unit digit) ; panels K-L (E ending with 8 unit digit).

• Figure 4 shows that loops < E/2 and those > E/2 give three possible sums of E including E = C + C'; E = C + P; and E = P + P'. A second major rule is that these three forms of E-sums are inter-convertible or inter-deductible, so you can switch from one to the other and so we have E = C + C' E = C + P E = P + P'↔ ↔ . Figure 4 shows that by subtracting and adding 4n to both addition terms, we can convert one sum into another.

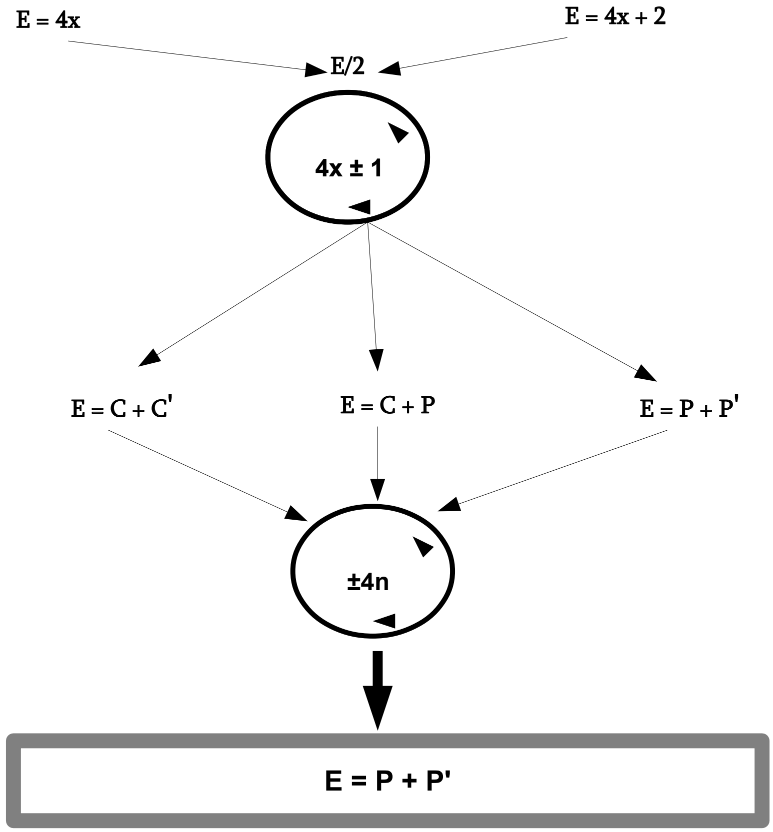


Figure 4: The deductive reasoning that starts with the even numbers E = 4x and E = 4x + 2 and leads to their conversion into the sum of two prime numbers P and P' such that E = P + P'.
Goldbach's strong conjecture is then demonstrated by this reasoning, proving that it is provable by a deductive chain calculus.

As Table 1 shows, there's always a gap of 4n between two primes, either 4x - 1 or 4x + 1; but there's a variable gap of 2n between 4x - 1 and 4x + 1 primes. As is well known, there's a gap of 6 between the prime numbers 6x - 1 or between those 6x + 1, but a variable gap of 2n between the 6x - 1 and the 6x + 1 primes. Similarly, there is a gap of 4n between a composite odd number C = 4x – 1 and prime number P = 4x - 1 or a composite odd numbers C = 4x + 1 and prime number P = 4x + 1. There is though a variable 2n gaps between C 4x – 1 numbers and P 4x + 1 primes and reciprocally. Here are demonstrations of the inter-conversions E = C + C' ↔ E = C + P ↔ E = P + P'.
First, note that E = 4x or E = 4x + 2. In both cases, 4x ± 4n = 4X and (4x + 2) ± 4n = 4X + 2. Adding or subtracting 4n to or from an even number E does not change its initial writing to 4x or 4x + 2.

E = C + C' = (P ± 4n) + (P' ± 4n) → E ± 4n ± 4n = P + P' → E ± 4N = P + P'
If E = 4x → E ± 4N = 4x ± 4N = 4X → **4X = P + P'**.
If E = 4x + 2 → E ± 4N = (4x + 2) ± 4N → **4X + 2 = P + P'**.
E = P + C = (P'' ± 4n) + (P''' ± 4n) → E ± 4N = P'' + P'''
If E = 4x → E ± 4N = 4x ± 4N = P'' + P''' → **4X = P'' + P'''**.
If E = 4x + 2 → E ± 4N = (4x + 2) ± 4N = 4X + 2 = P'' + P''' → **4X + 2 = P'' + P'''**.
E = P + P' = (P'' ± 4n) + (P''' ± 4n) → E ± 4n ± 4n = P'' + P''' → E ± 4N = P'' + P'''
If E = 4x → E ± 4N = 4x ± 4N = P'' + P''' → **4X = P'' + P'''**.
If E = 4x + 2 → E ± 4N = (4x + 2) ± 4N = 4X + 2 = P'' + P''' → **4X + 2 = P'' + P'''**.

Note that there are gaps of 10n between C or P numbers which have the same unit digits and there are always gaps of type 2n between two prime numbers which follow one another. These gaps can also be used to convert a sum E = C + C' or E = C + P into E = P + P'. We have **E = (C ± 2n) + (C' ± 2n)** ↔ **E = (P ± 2n) + (C ± 2n)** ↔ **E = P + P'**. The most regular 2n gaps between two consecutive C or P numbers are 4n and 6n (or 10n between C and P having the same unit digits).

5. Discussion

The theorem « Between n and 2n and n > 6, there is at least one prime in 4k - 1 and at least one in 4k + 1 - Proven by Erdös. Example between 7 and 14 we have 7 = 4 x 2 - 1; 11 = 4 x 3 - 1; 13 = 4x3 + 1 » doesn't predict the position of all equidistant primes neither their uits digits. It is well known that primes can be 4x – 1 or 4x + 1 but what is needed is to predict their positions. This article shows that for each even E = 4x or E = 4x + 2 there are repeating loops of 8 numbers where the prime or composite numbers always occupy the same positions 4x – 1 ou 4x + 1 before or after E/2. If E = P + P'; P and P' must be in the same loop and in the same positions symmetrically before and after E/2. This finding is new and can be exploited to design new computer programs that reproduce these loops in order to note the positions of the primes and test their equidistance (being in the same loop before and after E/2 and occupying the same positions). This will make it possible to convert an even number into all possible sums of two primes; or even test other types of conjectures. This article offers a unique tool for investigating prime numbers in loops of 8 numbers < E/2 and > E/2.

The accuracy of the data in this article can be seen in the symmetry before and after E/2 and in the consistency between the different cases of numbers. Examining the unit digits of even numbers helps refine the search for equidistant primes whose two-by-two sums give E.

The article offers for the first time a deductive proof of Goldbach's strong conjecture based on the equivalence between the sums E = C + C' ↔ E = C + P ↔ E = P + P'. This inter-conversion depends on the gaps 4n between the primes P ; or between primes P and composite odd numbers C. It is certain that the prime numbers have a critical density in the set N of natural integers that makes this inter-conversion possible in all cases of even numbers. In fact composite or prime numbers are all 4x ± 1 (or 6x ± 1 [1-3]) which makes this interconversion always possible therefore this article demostrates Goldbach's strong conjecture. By using the 4x ± 1 equations in a novel way, this article finally offers a clue to the solution of the age-old Goldbach conjecture. The interconversion of possible sums of E is a valid demonstration and will certainly mark future research on this conjecture. What's more, the system of 4x ± 1 loops provides fertile ground for computer science aimed at deciphering the secret of prime numbers or studying their infinite distribution.

References

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