ISSN: 2639-0108

Research Article

Advances in Theoretical & Computational Physics

Decoding the Mystery of Wave Function Collapse

Ping Wang[1,2*]

[1]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*

[2]*College of Physics Sciences, University of Chinese, Academy of Sciences, Beijing 100049, China*

[*]Corresponding Author
Ping Wang, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China.

Submitted: 2024, Sep 11; **Accepted:** 2024, Oct 17; **Published:** 2024, Oct 30

Citation: Wang, P. (2024). Decoding the Mystery of Wave Function Collapse. *Adv Theo Comp Phy, 7*(4), 01-08.

Abstract

We provide a reasonable explanation for measurement where no wave function in the sense of quantum mechanics is collapsed. The wave functions of the particles in the measuring device should be considered together with the measured particle as one system. The measurement is actually a physical process, which makes the wave functions of different paths orthogonal. The collapsed wave function is just an equivalent one to explain certain physical phenomena. There is no physical transition or process from the wave function in quantum mechanics to the equivalent one. Our understanding of measurement also provides reasonable explanations for both EPR paradox and the experiments of delayed quantum eraser.

The physical phenomena of microscopic systems, such as complementarity, interference, uncertainty relation, violation of Bell inequality [1–3], etc, are completely different from the classical images of macroscopic systems. These characteristics can be well described and explained by quantum mechanics. Although quantum mechanics is a successful and complete theory for microscopic systems, we still lack a self-consistent explanation for measurement within the framework of quantum theory. Quantum entanglement [4–6], EPR paradox [7–10], delayed quantum eraser [11–13], etc, have been hot topics for many years which are all related to measurement or so-called wave function collapse.

In order to better understand the measurement and wave function collapse, let's start with the double-slit interference experiment. When a particle reaches the double slits at time t_1, the wave function of the particle can be written as

$$\psi(\vec{x},t_1)=\frac{1}{\sqrt{2}}\left[\psi_1(\vec{x},t_1)+\psi_2(\vec{x},t_1)\right], \tag{1}$$

where $\psi_1(\vec{x},t_1)$ and $\psi_2(\vec{x},t_1)$ are the normalized wave functions. They are nonzero only when $\vec{x}$ is located in the corresponding left and right slits. The particle will appear at a certain point on the screen behind the double slits with a certain probability. One can get the momentum dependence of $\psi(\vec{x},t_1)$ with the Fourier transformation as

$$\psi(\vec{x},t_1)=\int\frac{d^3p}{(2\pi)^3}\Psi(\vec{p})e^{-i\vec{p}\cdot\vec{x}+ip_0t_1}, \tag{2}$$

where $\Psi(\vec{p})$ is a sinc function. Therefore, the probability of the particle appearing on the screen in the $\vec{p}$ direction is

$$f = \frac{1}{2}|\Psi(\vec{p})e^{-i\vec{p}\cdot(\vec{x}-\vec{x}_1)+ip_0t} + \Psi(\vec{p})e^{-i\vec{p}\cdot(\vec{x}-\vec{x}_2)+ip_0t}|^2 = |\Psi(\vec{p})|^2 + |\Psi(\vec{p})|^2\cos(\vec{p}\cdot(\vec{x}_1-\vec{x}_2)), \quad (3)$$

where $\vec{x}_1$ and $\vec{x}_2$ are the positions of the two slits, respectively. When $L \gg a$, $\vec{p}\cdot(\vec{x}_1-\vec{x}_2)$ can be written as $\frac{2\pi ad}{\lambda L}$, where a, L, and d are the distances between the two slits, between the two slits and the screen, and between the point on the screen in the $\vec{p}$ direction and the center of the screen, respectively. $\lambda = 2\pi/|\vec{p}|$ is the wavelength of the particle. It is clear the last term in Eq. (3) is the interference term.

If we put Wilson cloud chambers just behind the two slits, when the particle enters the chamber, its motion trajectory will provide which-path information. It is supposed the wave function of Eq. (1) collapses instantly into

$$\psi_c(\vec{x},t_1) = \psi_1(\vec{x},t_1) \quad \text{or} \quad \psi_c(\vec{x},t_1) = \psi_2(\vec{x},t_1). \quad (4)$$

Certainly, we have difficulties here. On the one hand, quantum mechanics enables us to make predictions for the wave function evolution in principle. On the other hand, we cannot predict the collapsed wave function and the trajectory of the particle even in principle.

Now we deal with the difficulties in the framework of quantum theory. The crucial point is that we should treat the incident particle and the particles in the cloud chambers together as one system. The initial wave function of the system at time t_1 is

$$\begin{aligned}\psi^i(\vec{x},t_1) &= \frac{1}{\sqrt{2}}\left[\ {}_1^i(\vec{x},t_1) + \psi_2^i(\vec{x},t_1)\right]\psi^i(\vec{x}_{Li},...,\vec{x}_{Ln},t_1)\psi^i(\vec{x}_{R1},...,\vec{x}_{Rn},t_1)\\ &= \frac{1}{\sqrt{2}}\ {}_1^i(\vec{x},t_1)\psi^i(\vec{x}_{L1},...,\vec{x}_{Ln},t_1)\psi^i(\vec{x}_{R1},...,\vec{x}_{Rn},t_1)\\ &\quad+\frac{1}{\sqrt{2}}\ {}_2^i(\vec{x},t_1)\psi^i(\vec{x}_{L1},...,\vec{x}_{Ln},t_1)\psi^i(\vec{x}_{R1},...,\vec{x}_{Rn},t_1), \end{aligned} \quad (5)$$

where $\psi^i(\vec{x}_{L1},...,\vec{x}_{Ln},t_1)$ and $\psi^i(\vec{x}_{R1},...,\vec{x}_{Rn},t_1)$ are the normalized wave functions for the particles in the left and right chambers, respectively. After the incident particle entering the chamber (measurement), the final wave function at time t_2 when it left the chamber can be generally written as

$$\begin{aligned}\psi^f(\vec{x},t_2) &= \frac{1}{\sqrt{2}}\psi_1^f(\vec{x},t_2)\psi^f(\vec{x}_{L1},...,\vec{x}_{Ln},t_2)\psi^i(\vec{x}_{R1},...,\vec{x}_{Rn},t_2)\\ &\quad+\frac{1}{\sqrt{2}}\psi_2^f(\vec{x},t_2)\psi^i(\vec{x}_{L1},...,\vec{x}_{Ln},t_2)\psi^f(\vec{x}_{R1},...,\vec{x}_{Rn},t_2). \end{aligned} \quad (6)$$

The above final wave function means the incident particle has the same probability to interact with the particles in the left and right chambers. The incident particle will appear on the screen in the $\vec{p}$ direction with the probability

$$f = |\Psi^f(\vec{p})|^2 + |\Psi^f(\vec{p})|^2 \left[\cos(\vec{p}\cdot(\vec{x}_1 - \vec{x}_2))\mathrm{Re}(W) + \sin(\vec{p}\cdot(\vec{x}_1 - \vec{x}_2))\mathrm{Im}(W)\right], \qquad (7)$$

where W is expressed as

$$W = \int d^3x_{L1}...d^3x_{Ln}d^3x_{R1}...d^3x_{Rn}$$
$$\times\ ^{f*}(\vec{x}_{L1},...,\vec{x}_{Ln},t_2)\psi^i(\vec{x}_{L1},...,\vec{x}_{Ln},t_2)\psi^{i*}(\vec{x}_{R1},...,\vec{x}_{Rn},t_2)\psi^f(\vec{x}_{R1},...,\vec{x}_{Rn},t_2). \quad (8)$$

If the initial and final wave functions of the particles in the left chamber $\psi^i(\vec{x}_{L1}, ..., \vec{x}_{Ln}, t_2)$ and $\psi^f(\vec{x}_{L1}, ..., \vec{x}_{Ln}, t_2)$, or in the right chamber $\psi^i(\vec{x}_{R1}, ..., \vec{x}_{Rn}, t_2)$ and $\psi^f(\vec{x}_{R1}, ..., \vec{x}_{Rn}, t_2)$, are orthogonal, the second term in Eq. (7) which includes the real and imaginary parts of W will be zero. As a result, the interference fringes on the screen will disappear. One can see that the particle still passes through both chambers even though there is no interference between the two paths.

Therefore, when there is no measurement, the particle has to pass through the two slits simultaneously in order to generate interference. The wave function is uniquely determined. When the particle is measured, we could have two kinds of wave functions which can describe the same phenomenon. One wave function Eq. (6) can be derived using time evolution of quantum theory in principle and the other is the collapsed wave function Eq. (4) which corresponds to the outcome of an individual observation. We should emphasis that the “classical” collapsed wave function is just an equivalent one to describe certain physical phenomenon which cannot be calculated with quantum theory. The initial wave function will never evolve into the equivalent one. In this sense, there is no process called wave function collapse.

The measurement in quantum mechanics had been studied by Zurek with density matrix ρ [14]. After the measurement, the off-diagonal terms of ρ vanish, which means the system evolves from coherent superposition state to a mixture of eigenstates. With the diagonal density matrix, it was supposed that the state of the measured system collapsed to one of its eigenstates. As a result, “reduction of the wave packet”, postulated by von Neumann to explain definiteness of an outcome of an individual observation, can be explained [14, 15]. It is still an open question whether collapse is a physical process [16–18]. We have effectively reproduced the decoherence result using wave function directly rather than introducing density matrix. However, in our opinion, the final wave function obtained from quantum theory would not collapse to one of the eigenstates when the particle is measured. In this classical limit, there is no transition or process from the wave function determined by quantum theory to the equivalent one.

One can imagine the cloud chambers consist of a sealed environment containing unsaturatedrather than supersaturated vapor of water. In this case, the initial and final wave functions in the chambers are not orthogonal. With the increasing of the saturation of water vapor, the fringes become more and more blurry. And finally, with the appearance of the motion trajectory in the chamber with supersaturated vapor of water, the interference fringes disappear. In the process of continuous changes of water vapor density, the value of the integral W varies from 1 to 0. It is clear the wave function is always expressed as Eq. (6) which can be calculated with quantum theory in principle. There is no wave function collapse even though the which-path information is provided. The collapsed wave function is just an equivalent one to describe the physical phenomenon of disappearing fringes. Similar as in the noninterference case, when the saturation of water vapor in the chambers continuously increases, one can also have the collapsed wave function as

$$\psi_c(\vec{x},t) = A\psi_1(\vec{x},t) + \sqrt{1-A^2}\psi_2(\vec{x},t) \quad \text{or} \quad \psi_c(\vec{x},t) = \sqrt{1-A^2}\psi_1(\vec{x},t) + A\psi_2(\vec{x},t), \tag{9}$$

where $0 \leq A \leq 1/\sqrt{2}$. $A = 0$ when the which-path information is known and $A = 1/\sqrt{2}$ when there is no cloud chamber. With the increasing saturation, A changes from $1/\sqrt{2}$ to 0, causing a continuous loss of interference. With this collapsed wave function, the probability of the particle appearing on the screen in the $\vec{p}$ direction is

$$f = |\Psi(\vec{p})|^2 + 2A\sqrt{1-A^2}|\Psi(\vec{p})|^2\cos(\vec{p}\cdot(\vec{x_1}-\vec{x_2})). \tag{10}$$

The above equation is comparable with Eq. (7) only if $\text{Im}(W)$ is zero and

$$\begin{aligned} 2A\sqrt{1-A^2} = &\int d^3x_{L1}...d^3x_{Ln}d^3x_{R1}...d^3x_{Rn} \\ \times\ &{}^{f*}(\vec{x}_{L1},...,\vec{x}_{Ln},t_2)\psi^i(\vec{x}_{L1},...,\vec{x}_{Ln},t_2)\psi^{i*}(\vec{x}_{R1},...,\vec{x}_{Rn},t_2)\psi^f(\vec{x}_{R1},...,\vec{x}_{Rn},t_2). \end{aligned} \tag{11}$$

Therefore, the collapsed wave function is not exactly equivalent to the one obtained from quantum theory when the wave functions of the particles in the chambers are not orthogonal. The interference disappears when which-path information is obtained and it could reap-pear when which-path information is erased. This can be easily understood because the particle always passes through the two paths simultaneously and quantum eraser is to make the wave functions of the two paths non-orthogonal. Since 1982, quantum eraser behavior has been reported in many experiments [19–25]. The schematic diagram of the experimental setup was shown in Fig. 2 in Ref. [23]. If we do not detect the entangled photons $\varphi(\vec{x}, t)$ and $\phi(\vec{x}, t)$, the wave function of the photon at detector D_0 is

$$\psi(\vec{x}, t) = \frac{1}{\sqrt{2}} \left[\psi_1(\vec{x}_{D_0}, t)\varphi(\vec{x}, t) + \psi_2(\vec{x}_{D_0}, t)\phi(\vec{x}, t)\right]. \tag{12}$$

The probability of the photon at D_0 is

$$f = |\Psi(\vec{p})|^2 + |\Psi(\vec{p})|^2 \left[\cos(\vec{p} \cdot (\vec{x}_1 - \vec{x}_2))\mathrm{Re}(W_{\mathrm{et}}) + \sin(\vec{p} \cdot (\vec{x}_1 - \vec{x}_2))\mathrm{Im}(W_{\mathrm{et}})\right], \tag{13}$$

where W_{et} is the wave function integral of the entangled photons

$$W_{\mathrm{et}} = \int d^3x \varphi^*(\vec{x}, t)\phi(\vec{x}, t). \tag{14}$$

The intensity of interference is determined by the magnitude of the integration value W_{et}. If we make a joint detection at D_0 and D_1 for the entangled photon pairs, the joint wave function will be

$$\psi_{D_0D_1} = \frac{1}{\sqrt{2}} \left[\psi_1(\vec{x}_{D_0}, t)\varphi(\vec{x}_{D_1}, t_1) + \psi_2(\vec{x}_{D_0}, t)\phi(\vec{x}_{D_1}, t_1)\right]. \tag{15}$$

The corresponding joint probability of photons is

$$f_{D_0D_1} = |\Psi(\vec{p}_{D_0})|^2|\Psi(\vec{p}_{D_1})|^2 + |\Psi(\vec{p}_{D_0})|^2|\Psi(\vec{p}_{D_1})|^2\cos((\vec{p}_{D_0} + \vec{p}_{D_1}) \cdot (\vec{x}_1 - \vec{x}_2)). \tag{16}$$

If there is no optical path difference between the entangled photons at D_1, $f_{D_0D_1}$ becomes

$$f_{D_0D_1} = 2|\Psi(\vec{p}_{D_0})|^2|\Psi(\vec{p}_{D_1})|^2\cos^2(\vec{p}_{D_0.} \cdot (\vec{x}_1 - \vec{x}_2)/2). \tag{17}$$

The joint probability $f_{D_0D_2}$ has the same expression as $f_{D_0D_1}$ if we do not consider the π phase shift. Similarly, the joint wave function at D_0 and D_3 is

$$\psi_{D_0D_3} = \frac{1}{\sqrt{2}} \left[\psi_1(\vec{x}_{D_0}, t)\varphi(\vec{x}_{D_3}, t_3) + \psi_2(\vec{x}_{D_0}, t)\phi(\vec{x}_{D_3}, t_3)\right]. \tag{18}$$

Since the photon emitted from the second slit cannot arrive at D_3 which means $\phi(\vec{x}_{D_3}, t_3) = 0$, the corresponding joint probability $f_{D_0D_3}$ is

$$f_{D_0D_3} = \frac{1}{2}|\Psi(\vec{p}_{D_0})|^2|\Psi(\vec{p}_{D_3})|^2. \tag{19}$$

Therefore, there is no interference at detector D_0 in this case. Our results are consistent with those in Ref. [23].

With the clear understanding of the measurement in quantum mechanics, we can have a reasonable explanation for EPR paradox [7, 8]. For example, the two entangled spin-1/2 particles form a spin-0 state. The initial wave function of a spin-0 system is

$$\psi^i(S=0) = \frac{1}{\sqrt{2}}\left[\psi_1(+u)\psi_2(-u) - \psi_1(-u)\psi_2(+u)\right], \tag{20}$$

where $+u$ and $-u$ are for the positive and negative spins along any axis. The two particles could be far away from each other. If we assume the wave function is collapsed into $\psi_1(+u)\psi_2(-u)$ or $\psi_1(-u)\psi_2(+u)$ when we measure the spin of one particle, it is hard to explain why the measurement on one particle can instantaneously affect the other far away particle. It was concluded that quantum mechanics did not provide a complete description of reality and there must be some local hidden variables accounting for the behavior of entangled particles. Though Bell inequality tests [26–35] have supported the theory of quantum mechanics, and not the hypothesis of local hidden variables, we still need to understand the EPR paradox as well as the so-called nonlocal behavior of quantum theory.

In our opinion, when we measure the spin of one particle, say particle 2, the wave function of the final system including all the interacting particles turns into

$$\psi^f = \frac{1}{\sqrt{2}}\left[\psi_1(+u)\psi_2(-u)\varphi(\vec{x}_1,...,\vec{x}_n,t) - \psi_1(-u)\psi_2(+u)\phi(\vec{x}_1,...,\vec{x}_n,t)\right], \tag{21}$$

where $\phi(\vec{x}_1,...,\vec{x}_n,t)$ and $\varphi(\vec{x}_1,...,\vec{x}_n,t)$ are the wave functions of the particles in the measuring device after interacting with particle 2 with spin $+u$ and $-u$, respectively. If the wave function $\varphi(\vec{x}_1,...,\vec{x}_n,t)$ and $\phi(\vec{x}_1,...,\vec{x}_n,t)$ are orthogonal, i.e.

$$\int d^3x_1...d^3x_n\varphi^*(\vec{x}_1,...,\vec{x}_n,t)\phi(\vec{x}_1,...,\vec{x}_n,t) = 0, \tag{22}$$

there is no interference between the first and second terms of Eq. (21). It is therefore equivalent to regard the particle's spin is fixed to be either $+u$ or $-u$. From the wave function of Eq. (21), one can see the spin of particle 1 would not be changed when the spin of particle 2 was measured, i.e., there was no information transferred instantly from particle 2 to particle 1. Certainly, the wave function in quantum mechanics can only tell us particle 1 has 50%

probability of spin $+u$ and 50% probability of spin $-u$. The final wave function Eq. (21) can be obtained in principle with quantum theory. Similar as in the double-slit case, the "classical" collapsed wave function $\psi_1(+u)\psi_2(-u)$ or $\psi_1(-u)\psi_2(+u)$ is just an equivalent one where the spin of particle 1 and particle 2 are both fixed. This equivalent wave function can not be predicted. There is no physical "process" from the initial wave function of Eq. (20) to the equivalent collapsed one, i.e., there is no instant change of the spin of particle 1 when the spin of particle 2 is measured. Therefore, EPR paradox can be easily understood without assuming the hidden variables or nonlocal behavior.

In summary, the process of measurement is a physical process which can be described by quantum theory in principle. From the case of double-slit interference, we can see the incident particle always passes through the double slits with the same probability. The disappearance of interference fringes on the screen is because the wave functions of the particles in the measurement system are orthogonal, resulting in a zero interference term. The so-called collapsed wave function is just an equivalent one which cannot be predicted or calculated from quantum theory. Based on our understanding of measurement, we can also provide a reasonable explanation for the EPR paradox. When measuring particle 1, the wave function of particle 2 in the sense of quantum mechanics is not changed. Similarly, the quantum eraser experiments are also easily explained by including all the entangled particles in the system. The joint probability is determined by the interference of both photon pairs. The explanations of the measurement in quantum theory in this manuscript are very helpful to understand many quantum phenomena.

Acknowledgments
This work is supported by the National Natural Science Foundation of China under Grant No. 11975241.

References

1. Bell, J. S. (1964). Physics, Physique. *Fizika, 1*, 195-200.
2. Fine, A. (1982). Hidden variables, joint probability, and the Bell inequalities. *Physical Review Letters, 48*(5), 291.
3. Braunstein, S. L., & Caves, C. M. (1990). Wringing out better Bell inequalities. *Annals of Physics, 202*(1), 22-56.
4. Joshi, M. K., Kokail, C., van Bijnen, R., Kranzl, F., Zache, T. V., Blatt, R., ... & Zoller, P. (2023). Exploring large-scale entanglement in quantum simulation. *Nature, 624*(7992), 539-544.
5. Zhong, Y., Chang, H. S., Bienfait, A., Dumur, É., Chou, M. H., Conner, C. R., ... & Cleland, A. N. (2021). Deterministic multi-qubit entanglement in a quantum network. *Nature, 590*(7847), 571-575.
6. Bienfait, A., Satzinger, K. J., Zhong, Y. P., Chang, H. S., Chou, M. H., Conner, C. R., ... & Cleland, A. N. (2019). Phonon-mediated quantum state transfer and remote qubit entanglement. *Science, 364*(6438), 368-371.
7. Einstein, A., Podolsky, B., & Rosen, N. (1935). EinsteinPodolskyRosen. *Phys. Rev., 47*, 777.
8. Bohm, D., & Aharonov, Y. (1957). Discussion of experimental proof for the paradox of Einstein, Rosen, and Podolsky. *Physical Review, 108*(4), 1070.
9. Lucia, U., & Grisolia, G. (2022). Thermodynamic definition of time: Considerations on the EPR paradox. *Mathematics, 10*(15), 2711.
10. Bricmont, J., Goldstein, S., & Hemmick, D. (2022). From EPR-Schrödinger paradox to nonlocality based on perfect correlations. *Foundations of Physics, 52*(3), 53.
11. Chiou, D. W. (2023). Delayed-choice quantum erasers and the Einstein-Podolsky-Rosen paradox. *International Journal*

of Theoretical Physics, 62(6), 120.
12. Kim, S., & Ham, B. S. (2023). Observations of the delayed-choice quantum eraser using coherent photons. Scientific Reports, 13(1), 9758.
13. Thenabadu, M., & Reid, M. D. (2022). Macroscopic delayed choice and retrocausality: Quantum eraser, Leggett-Garg, and dimension witness tests with cat states. *Physical Review A, 105*(6), 062209.
14. Zurek, W. H. (1982). Environment-induced superselection rules. *Physical review D, 26*(8), 1862.
15. Song, X. F., Liu, S., Chen, X. H., & Turek, Y. (2024). Theoretical investigation of the relations between quantum decoherence and weak-to-strong measurement transition. *arXiv preprint arXiv:2404.19488.*
16. Soulas, A. (2023). The measurement problem in the light of the theory of decoherence. arXiv preprint arXiv:2303.03465.
17. Bassi, A., Lochan, K., Satin, S., Singh, T. P., & Ulbricht, H. (2013). Models of wave-function collapse, underlying theories, and experimental tests. *Reviews of Modern Physics, 85*(2), 471-527.
18. Bedingham, D., Dürr, D., Ghirardi, G., Goldstein, S., Tumulka, R., & Zanghì, N. (2014). Matter density and relativistic models of wave function collapse. *Journal of Statistical Physics, 154*, 623-631.
19. Scully, M. O., & Drühl, K. (1982). Quantum eraser: A proposed photon correlation experiment concerning observation and" delayed choice" in quantum mechanics. *Physical Review A, 25*(4), 2208.
20. Zajonc, A. G., Wang, L. J., Zou, X. Y., & Mandel, L. (1991). Quantum eraser. *Nature, 353*(6344), 507-508.
21. Kwiat, P. G., Steinberg, A. M., & Chiao, R. Y. (1994). Three proposed ‘‘quantum erasers’’. Physical Review A, 49(1), 61.
22. Herzog, T. J., Kwiat, P. G., Weinfurter, H., & Zeilinger, A. (1995). Complementarity and the quantum eraser. *Physical Review Letters, 75*(17), 3034.
23. Kim, Y. (2024). Choice. In *Encyclopedia of Quality of Life and Well-Being Research* (pp. 989-991). Cham: Springer International Publishing.
24. Walborn, S. P., Cunha, M. T., Pádua, S., & Monken, C. H. (2002). Double-slit quantum eraser. *Physical Review A, 65*(3), 033818.
25. Maleki, Y., Liu, J., & Zubairy, M. S. (2021). Quantum eraser from duality-entanglement perspective. *Physical Review A, 104*(4), 042207.
26. Freedman, S. J., & Clauser, J. F. (1972). Experimental test of local hidden-variable theories. *Physical review letters, 28*(14), 938.
27. Aspect, A., Grangier, P., & Roger, G. (1981). Experimental tests of realistic local theories via Bell's theorem. *Physical review letters, 47*(7), 460.
28. Weihs, G., Jennewein, T., Simon, C., Weinfurter, H., & Zeilinger, A. (1998). Violation of Bell's inequality under strict Einstein locality conditions. *Physical Review Letters, 81*(23), 5039.
29. Pan, J. W., Bouwmeester, D., Daniell, M., Weinfurter, H., & Zeilinger, A. (2000). Experimental test of quantum nonlocality in three-photon Greenberger–Horne–Zeilinger entanglement. *Nature, 403*(6769), 515-519.
30. Rowe, M. A., Kielpinski, D., Meyer, V., Sackett, C. A., Itano, W. M., Monroe, C., & Wineland, D. J. (2001). Experimental violation of a Bell's inequality with efficient detection. *Nature, 409*(6822), 791-794.
31. Ansmann, M., Wang, H., Bialczak, R. C., Hofheinz, M., Lucero, E., Neeley, M., ... & Martinis, J. M. (2009). Violation of Bell's inequality in Josephson phase qubits. *Nature, 461*(7263), 504-506.
32. Christensen, B. G., McCusker, K. T., Altepeter, J. B., Calkins, B., Gerrits, T., Lita, A. E., ... & Kwiat, P. G. (2013). Detection-loophole-free test of quantum nonlocality, and applications. *Physical review letters, 111*(13), 130406.
33. Schmied, R., Bancal, J. D., Allard, B., Fadel, M., Scarani, V., Treutlein, P., & Sangouard, N. (2016). Bell correlations in a Bose-Einstein condensate. *Science, 352*(6284), 441-444.
34. BIG Bell Test Collaboration, *Nature 557* (2018) 7704, 212.
35. Storz, S., Schär, J., Kulikov, A., Magnard, P., Kurpiers, P., Lütolf, J., ... & Wallraff, A. (2023). Loophole-free Bell inequality violation with superconducting circuits. *Nature, 617*(7960), 265-270.