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Cosmic Baryon Kinetics At Times After The Matter Recombination Analysed Via Its Velocity Moments

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Abstract

The interesting aspect that we are paying attention to in this article here is the variation of the kinetic distribution function of particles subject to the early Hubble expansion of the universe. Some generally made assumptions namely are not fulfilled here: Neither does their distribution function stay a Maxwellian as expected for the cosmic begin, nor does the density of these particles simply fall off as expected in a homogeneous universe with the cosmic scale $R = R(t)$ like $n(t) = n_0 (R_0/R)^3$. Instead we do show here, that it is quite complicated to understand, how cosmic gases like the first H-atoms, after recombination out of the plasma state of cosmic matter, do thermodynamically behave under the ongoing omni-directional Hubble-like expansion dynamics of the universe. This is because there is no trivial answer to the question, how cosmic gas atoms do in fact recognize the expansion of cosmic 3- space they are embedded in. Standard mainstream cosmology takes for granted that gas atoms do react polytropically or even adiabatically to cosmic volume changes, consequently assuming that they do get more and more tenuous and colder in accordance with gas- and thermo- dynamic expectations. However, one has to face the fact that cosmic gases at the recombination era are already nearly collisionless over scales of 10 AU. How then do they recognize cosmic volume changes under such conditions and how do they react to it kinetically? We derive in this article a kinetic transport equation which describes the evolution of the gas distribution function $f(t, v)$ in cosmic time t and velocity space of v. This resulting partial differential equation does not allow for a solution in form a separation of the two variables t and v, but instead one obtains that $f(v, t)$ is non-Maxwellian with its two lowest moments, i.e density $n(t)$ and the pressure $P(t)$, as pure functions of cosmic time t. Then we show that using kappa-like distribution functions $f(t, v) = f^{\kappa}(t, v)$ for the cosmic gas we can derive such functions as function of their velocity moments, i.e. as pure functions of cosmic time. It means we understand the kinetic evolution of the cosmic gas by understanding the evolution in cosmic time of their moments $n^{\kappa}(t)$ and $P^{\kappa}(t)$ with $K = K(t)$.

1. A Short Review On Cosmic Matter Recombination

It is generally assumed that before the phase of matter recombination cosmic matter and radiation were in perfect thermodynamic equilibrium, implying that protons and electrons in this evolutionary phase are described by Maxwell distributions $f(v, t_0) = Max(v, T_0)$ and photons are distributed according to a Planckian black body spectrum for the common temperature $T_0 = T_0(t_0)$. A deeper look, however, into the kinetic theory of the physical processes close to and just after the recombination phase of electrons and protons, makes it evident that in a homologously expanding universe, like in a Friedman-Lemaître universe, the baryon distribution function can not be expected to maintain its Maxwellian shape, since its most relevant velocity moments, i.e. the density and the temperature, vary in an unexpected nonclassical, non-adiabatic and non-isentropic manner (see Fahr, 2021). As consequence of that the entropy of H-atoms decreases at this phase of the cosmic expansion, in fact it changes with cosmic time in contrast to the standard thermodynamical gas behaviour.

Let us pay a brief look on the phase of this early cosmic electron - proton recombinations, perhaps thought to have occured at about 400000 years after the so-called Big-Bang, when the temperatures T of the cosmic plasma dropped to below 4000K (see e.g. Partridge, 1965, or later Fahr and Loch, 1991, Fahr and Zoennchen, 2009). It is assumed that at this phase electrons and protons are dynamically and physically tightly coupled to each other, since undergoing strong and frequent mutual interactions, both by Coulomb collisions and by Compton collisions with photons. Under such prerequisites a pure thermodynamical equilibrium state seems to be guaranteed, in fact implying that protons and electrons are distributed in velocity-space according to a Maxwellian velocity distribution, and photons maintain a Planckian blackbody spectrum in frequency. However, looking on this relevant point more in detail gives evidence that these assumptions are hardly fulfilled even in this early period, mainly because photons and particles react very differently to the cosmic expansion.

Photons generally are cooling due to permanently being cosmologically redshifted (see e.g. Peacock, 1999, Goenner, 1996, Fahr and Heyl, 2017). In contrast particles are not directly feeling the expansion of the universe, they are more or less freely moving through cosmic space, unless they feel the expansion adiabatically by mediation of the changing thermodynamic gas conditions via numerous Coulomb collisions with other particles.

Over distances D where the cosmic gas atoms can be considered as collision-free, i.e. for $D \leq \lambda_c$ (with λ_c denoting the actual mean free path with respect to elastic collisions), they will not feel the expansion at all. Only beyond, at distances $D > \lambda_c$, those atoms with velocities larger than $v \geq \lambda_c \bullet$ H (i.e. the critical Hubble drift!) are touching the so-called "collisional wall" of their cosmic environment and will start recognizing the cosmic expansion by the fact that the local bulk velocity of particles has changed, while others with $v \leq \lambda_c \bullet H$ are not touching this wall, i.e. not recognizing this bulk velocity change. Hereby the expansion of the universe is described by the Hubble parameter with $H = \dot{R}/R$, where R denotes the scale of the universe, and its derivative with respect to cosmic time t. Or expressing it in other words, if one expands the walls of a collision-free gas with a supersonic velocity $V \gg v_s$, then this gas will not recognize at all the expansion, only the few particles out of the gas distribution function with velocities $v > v_s$ can interact with the wall and thus can react "adiabatically" by returning to the system with accordingly reduced energy.

Furthermore an additional problem occurs, since Coulomb collisions redistributing velocities among particles and reconstituting the distribution function have a specific property which seriously aggravates things in this context. Namely the fact that Coulomb collision cross sections are strongly dependent on the relative velocity w of the colliding particles since in case of Coulomb collisions being proportional to $(1/w^4)$ (see Spitzer, 1956). This has the consequence that high-velocity particles are much less collision-dominated compared to low-velocity ones. The latter ones even behave as collison-free at supercritically large velocities $v > v_c$, not cooling at all. While the low-velocity branch of the distribution may still cool adiabatically, like a collision-dominated gas would do. Thus this low-velocity branch of the distribution feels and reacts to the cosmic expansion in an adiabatic form, the high-velocity branch in contrast behaves collision-free and hence changes in a different, yet at this moment here unspecified form.

This violates the concept of a joint equilibrium temperature and of a resulting mono-Maxwellian velocity distribution function. It means that there may be a critical evolutionary phase of the universe as consequence of different forms of cooling in the low- and high-velocity branches of the particle velocity distribution function. Such a situation does not permit the endurance of a Maxwellian distribution to later cosmic times. Hence we shall now look into this interesting evolutionary expansion phase a bit deeper and try to draw some first conclusions concerning the cosmic gas behaviour in the post-recombination era. We shall also demonstrate here that the realistic behaviour of cosmic gases during this phase and later depends on the specific form of the Hubble expansion of the universe, especially when at these days an accelerated expansion phase is discussed, will strongly influence the thermodynamics of the cosmic gas , creating - so-called "over-Maxwellian"- depletions of high velocity particles, i.e. distributions with strongly suppressed high-velocity particles. Just such types of functions with over-cooled tails we shall try to describe in the forthcoming sections of this paper.

2. Derivation of the kinetic transport equation for cosmic gases

We start out from the generally accepted assumption in modern cosmology, that during the collision-dominated phase of the cosmic matter evolution, just before the time of matter recombination, matter and radiation, due to frequent energy exchange processes between electrons and photons, as beween electrons and protons, are in complete thermodynamic equilibrium, i.e. the temperatures of matter and radiation are identical: i.e. $T_m = T_\nu = T_0$. In the following cosmic evolution this equilibrium will not survive, however, NLTE- perturbations will come up, as had already been emphasized in the section above and earlier by Fahr and Loch (1991). The upcoming part of the paper shall demonstrate now that, even if a Maxwellian distribution would actually prevail at the entrance to the collision-free cosmic expansion phase, it would not persist to times there after. Just after the recombination phase when electrons and protons recombine to H-atoms, and photons start propagating through cosmic space practically without further interaction with particles, the thermodynamic contact between matter and radiation furtheron hence is abolished or, to say it differently, is practically switched off. This is one reason why the initial Maxwellian atom distribution function would not persist in the universe during the ongoing collision-free expansion. On the other hand, however, the isolated fate of the radiation field does not lead to a non-equilibrium, nonthermal NLTE radiation field. This is because cosmic redshift cooling of a Planck spectrum again leads to a Planck spectrum, however with a reduced temperature, as has been shown by Fahr and Zoennchen (2009) or Fahr and Sokaliwska (2015).

To enlucidate this point let us first consider a collision-free particle population in an expanding, spatially symmetric Robertson-Walker universe. Hereby it is clear that due to the cosmological principle and, connected with it, the requirement of spatial homogeneity, also the velocity distribution function of the particles must be specifically symmetric and isotropic in v, and independent on the local cosmic place x. Thus it must be of the following general form

$$f(v,t) = n(t) \bullet \bar{f}(v,t) \quad \#$$

where $n(t)$ denotes the time-variable, cosmic density, only depending on the worldtime t, and $\bar{f}(v, t)$ is the normalized, time-dependent, isotropic velocity distribution function with the property: $\int \bar{f}(v,t) d^3v = 1$.

If we now do take into account that particles, moving freely with their velocity v into their $\vec{v}$-associated directions over a distance l, at their new place of reappearance have to restitute the actual cosmic distribution there, despite the differential Hubble flow and the explicit time-dependence of f, then a locally prevailing co-variant distribution function $f(v', t')$ must exist with the property that the two associated functions $f(v', t')$ and $f(v, t)$ are related to each other in a Liouville-conform way (see e.g. Cercigniani, 1988, Landau-Lifshitz, 1990). To quantify this request needs some special care, since particles that are freely moving in a homologously expanding Hubble universe, do in this specific case at their motions not conserve their associated phasespace volumes $d^6\Phi = d^3vd^3x$ as they usually do in gas dynamics, since in a homologously expanding cosmic space no particle Lagrangian $L(v, x)$ does exist, as usually does in gas dynamics, and thus no Hamiltonian canonical relations of their dynamical coordinates v and x are valid.

As consequence Liouville's theorem (see e.g Chapman and Cowling, 1952) does not require that the two associated differential 6D-phase space volumes - $d^6\Phi$ and $d^6\Phi'$ - are identical, but that the conjugated differential phase space densities are identical to guarantee the cosmic particle number conservation. This is expressed by the following relation:

$$f'(v',t')d^3v'd^3x' = f(v,t)d^3vd^3x \tag{1}$$

When arriving at the place x', these particles, after passage over a distance l need to be incorporated into a particle population which has a relative Hubble drift with respect to the original place of the particle given by $v_H = l \cdot H$, co-aligned with $\vec{v}$. Thus the original particle velocity v registered at the new place x' appears locally tuned down to $v' = v - l \cdot H$, since at the present place x', deplaced from the original place x by the increment l, all velocities have to be judged with respect to the new local reference frame (standard of rest) with its differential Hubble drift of $(l \cdot H)$ with respect to the particle's origin x.

If all of that is taken into account, it can be shown (see Fahr, 2021) that one finally is lead to the following kinetic transport equation:

$$\frac{\partial f}{\partial t} = vH(t) \cdot (\frac{\partial f}{\partial v}) - H(t) \cdot f \tag{2}$$

which should enable one to derive the resulting distribution function as function of the velocity v and of the cosmic time t. Of course it is assumed hereby that the Hubble parameter H $= H(t)$ is known as function of the world time t (as e.g given by Perlmutter, Aldering, Goldhabe et al., 1999, or Perlmutter, 2003). As it was shown already by Fahr (2021), the above kinetic transport equation does not allow for a solution in the form of a separation of variables , i.e. putting $f(v, t) = f_t(t) \cdot f_v(v)$, but one rather needs a different, non-straightforward method of finding a kinetic solution of this above transport equation (2) which now is done next here.

3. Cosmic Kappa-functions

One way which may prove to be promising here, is to think of kappa-functions as the underlying distribution functions at cosmic times after the matter recombination. These latter functions a priori have the advantage of covering all kinetic function phenomena spanned between pure power law functions and pure Maxwellian functions (see e.g. Lazar, Fichtner, Yoon, 2016, Fahr and Fichtner, 2021) which have to be expected at times after matter recombination in the universe ($t \geq t_0$!) . Let us therefore now have a look on this latter type of functions with respect to their applicability on this problem in cosmology.

Starting from an isotropic kappa-distribution in the frame of the cosmic bulk motion which latter has to dissappear anyway in a Robertson-Walker universe (i.e. due to the cosmological principle requiring full 3D- space symmetry!). Locally specific bulk motions would evidently violate this cosmological principle. These types of required functions are generally given in the following form (see e.g. Lazar, Fichtner and Yoon, 2016)

$$f_\kappa(v) = \frac{n}{\pi^{3/2}\kappa^{3/2}\Theta^3}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}[1+\frac{v^2}{\kappa\Theta^2}]^{-(\kappa+1)} \tag{3}$$

Here n denotes the particle density, κ and Θ denote two independent, typical kappa-function parameters, and $\Gamma = \Gamma(z)$ means the well known mathematical Gamma-function for the argument z. The above distribution function $f_\kappa(v)$ is typical for deviations from the normally expected thermodynamical, collision-dominated equilibrium situations which latter would be characterized by a Maxwellian distribution and would automatically be contained in the upper function family by the case $\kappa \rightarrow \infty$.

Calculating now on the basis of the above distribution function $f_\kappa(v)$ the associated pressure moment P_κ, by carrying out the necessary velocity-space integration, then leads to the following expression (see e.g. Heerikhuisen et al., 2008, Fahr and Fichtner, 2021):

$$P_\kappa = \frac{4\pi m}{3}\int_0^\infty f_\kappa(v)v^4dv = \frac{m}{2}n\Theta^2\frac{\kappa}{\kappa-3/2}$$

with m denoting the particle mass. This then shows, however, that kappa distributions with kappa-function parameters κ and Θ nevertheless do lead to the same pressure moment P_κ (i.e. "isobaric" functions!), – if! the κ - associated parameter Θ (i.e. the "thermal" spread of the function) is a specific function of κ, i.e. $\Theta = \Theta(\kappa)$, and if! this function $\Theta(\kappa)$ is given through the following relation:

$$\Theta^2(\kappa) = 2P_\kappa \frac{\kappa - 3/2}{mn\kappa} = \Theta^2_{\kappa,M} \frac{\kappa - 3/2}{\kappa}$$

This then opens up another possibility, or if prefered an other way around, one namely can keep P_κ as a function parameter of the distribution function and can express Θ as function of the remaining other function parameters κ, n_κ, P_κ in the form:

$$\Theta^2(\kappa, n, P) = 2P_\kappa \frac{\kappa - 3/2}{mn\kappa} = \frac{2P_\kappa}{mn_\kappa} \frac{\kappa - 3/2}{\kappa}$$

This for instance is generally practised in writing Maxwellians $Max(v)$ as functions of their two velocity moments $nMax$ and $TMax = PMax/(KnMax)$ in the form:

$$Max(v) = n_{Max} \frac{1}{(\pi T_{Max})^{3/2}} \exp[-\frac{mv^2}{KT_{Max}}]$$

In this sense, analogue to the above example, the above kappa-type distribution function (3) could as well be expressed through its parameter κ and the function moments n_κ and P_κ in the form:

$$f_\kappa(v) = \frac{n_\kappa}{\pi^{3/2}\kappa^{3/2}(\frac{2P_\kappa}{mn_\kappa}\frac{\kappa-3/2}{\kappa})^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} [1 + \frac{mn_\kappa v^2}{2P_\kappa(\kappa-3/2)}]^{-(\kappa+1)} \quad (4)$$

which for $\kappa \to \infty$ reproduces the upper Maxwellian $Max(v)$.

Now it turns out from a recent paper (Fahr, 2021) that, prior to the knowledge of the distribution function $f_\kappa(v)$ itself, one can show that the moments of the above function, starting from the kinetic transport equation (2) for gases in an expanding universe, can be found without having at first available the solution of this kinetic transport equation. Namely from the corresponding moment transport equations of this equation (see Fahr, 2021) the moments $n_\kappa(t)$ and $P_\kappa(t)$ can be derived, and with the Hubble parameter $H_0 = (R_0)\,\dot{R}/R_0$ (the problem of treating the Hubble parameter as a constant will be discussed in the next section), lead to the following results for the time-dependence of these moments can be found as given in the following form:

$$n_\kappa = n_{\kappa 0} \exp[-4H_0(t - t_0)] \quad (5)$$

and:

$$P_\kappa(t) = P_{\kappa 0} exp[-6H_0(t - t_0)] \quad (6)$$

This requires prior to solving equation (1) that the kinetic distribution function, whatever form it has, has to obey the following fact:

$$\frac{P_\kappa(t)}{n_\kappa(t)} = \frac{P_{\kappa 0}}{n_{\kappa 0}} \exp[-2H_0(t - t_0)]$$

If we now take this knowledge and introduce it into the upper form of a cosmic kappa-function, we then obtain the following form for it:

$$f_\kappa(v,t) = \frac{n_{\kappa 0}\exp[-4H_0(t-t_0)]}{\pi^{3/2}\kappa^{3/2}[\frac{2P_{\kappa 0}}{mn_{\kappa 0}}\exp[-2H_0(t-t_0)]\frac{\kappa-3/2}{\kappa}]^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \cdot [1 + \frac{mn_{\kappa o}v^2}{2P_{\kappa 0}\exp[-2H_0(t-t_0)](\kappa-3/2)}]^{-(\kappa+1)}$$

or after some evident mathematical rearrangements:

$$f_\kappa(v,t) = \frac{n_{\kappa 0}\exp[-H_o(t-t_0)]}{\pi^{3/2}\kappa^{3/2}[\frac{2P_{\kappa 0}}{mn_{\kappa 0}}\frac{\kappa-3/2}{\kappa}]^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \cdot [1 + \frac{\frac{1}{2}mv^2}{(P_{\kappa 0}/n_{\kappa 0})\exp[-2H_0(t-t_0)](\kappa-3/2)}]^{-(\kappa+1)} \quad (7)$$

We now introduce the following quantity, - one could call it: the mean thermal particle energy E_0 at the cosmic time $t = t_0$ - :

$$E_0 = \frac{P_{\kappa 0}}{n_{\kappa 0}} = \frac{n_{\kappa 0} k T_{\kappa 0}}{n_{\kappa 0}}$$

and, when doing so, obtain the upper distribution function (7) in the following form:

$$f_\kappa(v,t) = \frac{n_{\kappa 0}\exp[-H_0(t-t_0)]}{\pi^{3/2}\kappa^{3/2}[\frac{2E_0}{m}\frac{\kappa-3/2}{\kappa}]^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}[1+\frac{\frac{\frac{1}{2}mv^2}{E_0}\exp[2H_0(t-t_0)]}{(\kappa-3/2)}]^{-(\kappa+1)} \quad (8)$$

or when expressing for practical reasons the mean thermal energy by $E_0 = (1/2)mv_0^2$ one obtains:

$$f_\kappa(v,t) = \frac{n_{\kappa 0}\exp[-H_0(t-t_0)]}{\pi^{3/2}\kappa^{3/2}v_0^3[\frac{\kappa-3/2}{\kappa}]^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}[1+\frac{\frac{v^2}{v_0^2}\exp[2H_0(t-t_0)]}{(\kappa-3/2)}]^{-(\kappa+1)} \quad (9)$$

Finally one obtains the differential velocity space density, with introduction of the normalized variable $w = v/v_0$, by the following expression:

$$f_\kappa(x,t)x^2dx = \frac{n_{\kappa 0}\exp[-H_0(t-t_0)]}{\pi^{3/2}[\kappa-3/2]^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \cdot [1+\frac{x^2\exp[2H_0(t-t_0)]}{(\kappa-3/2)}]^{-(\kappa+1)}w^2dw \quad (10)$$

The above function is essentially well defined concerning its v - and t - dependencies, - up to the missing knowledge on the time-dependence of the parameter $\kappa = \kappa(t)$. Assuming, however, the prevalence of a Maxwellian distribution at time $t = t_o$ would imply that $\kappa(t_0) = \kappa_0 \geq 10$, and then expecting for later cosmic times $t \geq t_0$ due to the Hubble-drift influence more low-velocity-loaded "over-Maxwellian ´ized" distributions should suggest that the κ - parameter perhaps continues to decrease according to $\kappa(t) = \kappa_0\exp[-H_\Lambda(t-t_0)]$. This then leads to the results shown in Figures 1 and 2.

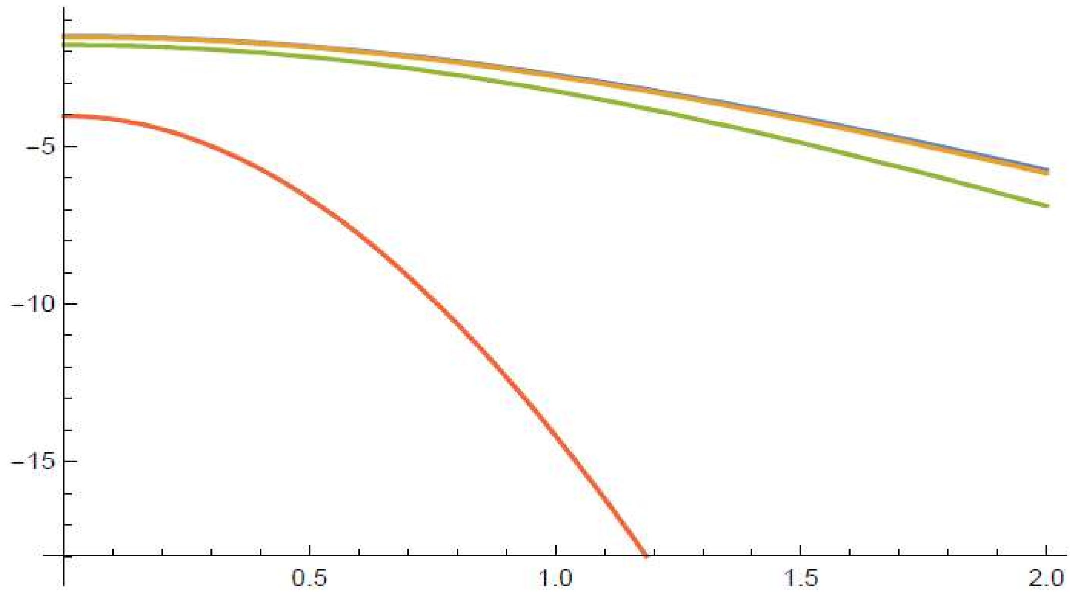

Figure 1: The cosmic baryon distribution function in the times t_1 = 1year, t_2 = 10years, and t_3 = 100years after the matter recombination at $t = t_0$ as function of the normalized velocity $w = v/v_0$.

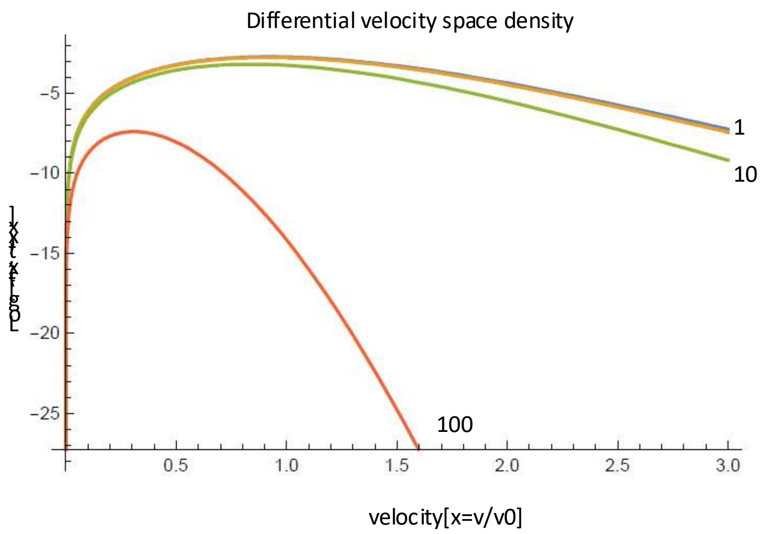

Figure 2: Differential velocity space density of the cosmic baryons at times

$t_1 = 1 year$, $t_2 = 10 years$, $t_3 = 100 years$ - **after the cosmic matter recombination at time t_0.**

In our Figures 1 and 2 we have assumed that the parameter κ attains a dependence on cosmic time according to $\kappa = \kappa_0 \exp[-H_0(t - t_0)]$ with $\kappa_0 = 10$, and it is shown, how within 1, 10, 100 years the cosmic distribution function would then change its velocity profile starting from a Maxwellian tending to more centrally piled-up, "over-Maxwellian", i.e. just the opposite to non-equilibrium, power law distributions.

The basis hereby in Figure 1 is a Hubble constant of H_0 = 70km/s/Mpc which is observationally confirmed for the present time. If this Hubble constant is used by us for the time after matter recombination $t \geq t_0$, it means and requires that the Hubble constant $H = H_0$ more or less should not have changed since these times till now - at first glance a rather astonishing and audacious assumption. - But, astonishingly enough, this is in fact a viable assumption as we are going to show here now at the end of this paper.

4. The Hubble parameter in the early universe

ForFriedman-Lemaître-Robertson-Walker cosmologies (FLRW) the Hubble parameter $H = \dot{R}/R$ can be given in form of the following differential equation (1. Friedman equation; e.g. see Goenner, 1996, Fahr, 2016):

$$H^2 = \frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}[\rho_B + \rho_D + \rho_v + \rho_\Lambda]$$

where G is Newton‘s gravitational constant, and ρ_B, ρ_D, ρ_v, ρ_Λ denote the equivalent cosmic mass densities of baryons, of dark matter, of photons, and of the vacuum energy. For the case that all of these quantities do count, then it is complicated to find a solution for H and $R(t)$ over all cosmic times, because ρ_B may vary proportional to R^{-3}, ρ_D most probably also according to R^{-3}, but ρ_v is generally thought to vary according to R^{-4} (see Goenner, 1996, or Fahr and Heyl, 2017, 2018). Amongst these quantities the cosmic vacuum energy density ρ_Λ is perhaps physically the least certain quantity, but if it is described with Einstein´s cosmological constant Λ, then it represents a positive, constant energy density, i.e its mass equivalent ρ_Λ hence would as well be a positive constant quantity.

From recent supernova SN1a observations (Permutter et al., 1999, Perlmutter, 2003, Riess et al., 1998, Schmidt et al., 1998) it has been concluded that at the present cosmic era and most probably already some times ago we were and are in an accelerated cosmic expansion phase of the universe, expressing the fact that ρ_Λ is the dominant quantity amongst the upper ingredients in the universe. If this could be taken as the truth even back to times of the matter recombination, then in fact we can assume that the above differential equation can be written in the much more simplified form:

$$H_\Lambda = \frac{\dot{R}}{R} = \sqrt{\frac{8\pi G}{3}[\rho_B + \rho_D + \rho_v + \rho_\Lambda]} \simeq \sqrt{\frac{8\pi G}{3}[\rho_\Lambda]} = const$$

in fact then describing the expansion of the universe by the expression:

$$R(t) = R_0 \exp[\sqrt{\frac{8\pi G}{3}[\rho_\Lambda]}\,(t - t_0)] = R_0 \exp[H_\Lambda(t - t_0)]$$

Taking the above result and reminding the result that we derived in the section before for the first moment of the baryon distribution function, i.e. the density $n_K(t)$, given by:

$$n_\kappa(t) = n_{\kappa 0} \exp[-4H_\Lambda(t - t_0)] = n_{\kappa 0} \cdot (R(t)/R_0)^{-4} \qquad \#$$

we obtain a somewhat astonishing result, meaning that in an acceleratedly expanding universe like the one with $H = H_A$ the local density is falling off with the inverse of the fourth power of the scale of the universe . This should mean that the total mass M_U of the universe is not constant, but decreasing like:

$$\frac{dM_U}{dt} = \frac{d}{dt}[\frac{4\pi}{3}R^3\rho] = \frac{4\pi}{3}[3\rho_0\frac{R_o^4}{R^2} - 4\rho_0\frac{R_0^4}{R^2}] = -\frac{4\pi}{3}\rho_0\frac{R_0^4}{R^2}$$

However, the reader must be warned, since the concept of a total mass M_U of the universe is by far not clearcut, it rather must be deeply discussed how precisely the meaning of M_U should defined. It turns out that it must be understood as the value of all masses "instantaneously or simultaneously" surrounding each arbitrary point in the FLRW- universe and its precise formulation leads to unexpected complications (see e.g. Overduin and Priester, 2001, Overduin and Fahr, 2001, Fahr and Heyl, 2006, 2007). So for instance in Fahr and Heyl (2007) it leads to the following expression

$$M_U(t) = 4\pi\rho(t)\int_0^{R_u}\frac{r^2}{\sqrt{1 - \frac{8\pi G}{rc^2}\rho(t)\int_0^r x^2 dx}}dr$$

and evaluates to:

$$M_U(t) \simeq \frac{c^2}{G}R_U(t)$$

expressing the fact that the "so-called" total mass of the universe has a "Machian character" (Mach, 1883) and increases with the size R_U of the universe. If therefore it could be concluded that each mass of a particle increases in the same way as the mass of the universe , then the mass density is again falling off with $\rho(t) = \rho_o \cdot (R_U(t)/R_{U0})^{-3}$ and no problem to worry remains.

5. Conclusions

In the aforegoing sections of this paper we have started from the kinetic transport equation (2) for the distribution function $f(v, t)$ of a baryon gas embedded in the cosmic FLRW space-time metrics of an expanding universe. We first could show that this differential equation does not allow for a solution by separation of the variables in the form $f(v, t) = f_v(v) \cdot f_t(t)$, but could demonstrate that the kinetic transport equation (2) allows to derive solutions for two of its velocity moments when underlying Kappa-functions, namely the baryon density $n_\kappa(t)$ and the baryon pressure $P_\kappa(t)$, prior to the solution of $f(v, t)$ itself. Based on this knowledge we have then presented the kinetic distribution function in form of a general isotropic kappa-function $f(v, t) = f_\kappa(v, \kappa(t), \Theta(t))$ which by use of its already known moments then can be written in the form $f(v, t) = n_K(t) \cdot f_\kappa(v, \kappa(t), P_\kappa(t))$. As we can show here, to overcome the Hubble drift between two reference points bridged per cosmic time dt by moving baryons in the expanding universe, high velocity branches of the distribution function are systematically suppressed, and the velocity spread of the distribution function decreases with increasing cosmic times t, a phenomenon which we may call "super-Maxwellisation". This is seen in Figures 1 and 2 showing the resulting distribution function for times $t_1 = 1 years$; $t_2 = 10 years$; $t_3 = 100 years$ after the time t_0 of the cosmic matter recombination. The cosmic particles with increasing cosmic times are systematically more concentrated at the low velocity region of velocity space which is also described by the temperature decrease with time according to the result derived from the moments:

$$kT_\kappa(t) = P_\kappa(t)/n_\kappa(t) = kT_0 \exp[-H_\Lambda(t - t_0)]$$

telling that in an expanding universe with a constant Hubble-constant H_A the cosmic gas temperatures $T_K(t)$ should permanently decrease and finally even fall down to the absolute zero-point.

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