

Journal of Mathematical Techniques and Computational Mathematics

Considerations on the Collatz Conjecture

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Submitted: 2024, Aug 19; **Accepted:** 2024, Sep 10; **Published:** 2024, Oct 23

Citation: Loris, A. (2024). Considerations on the Collatz Conjecture. *J Math Techniques Comput Math, 3*(10), 01-26.

Abstract

Part of the scientific community has spent considerable time and resources to somehow validate Collatz's conjecture, countless efforts have achieved considerable progress in this direction, but this conjecture lacked definitive confirmation that choosing an odd number any xi ∈ ℕ[∗], we will obtain $\frac{x(i+1)}{x(i+1)} = 3x_i + 1$, this being an even number $x_{(i+1)}$, divide it if the *same by the number two (successively) until another odd number* ∈ *ℕ*[∗] *is obtained, the process xⁿ = 3x(n−1) + 1 and divisions by two until the result is a number equal to 1. This work presents deductions, algorithms and equations that corroborate this proposition, supporting this perception and conclusion that Collatz's conjecture points to the final cycle* $4 \rightarrow 2 \rightarrow 1$ *.*

Keywords: Collatz Conjecture, Chaotic Dynamics, Limit Cycle, Periodic Orbit, Principle of Mathematical Induction, Python and 'R' Language

1. Introduction

The direct approach that seeks proofs of convergence to conjunction has proven undecidable, at least any algorithm based on formal logic has been only partially successful, that is, there is currently no algorithm that definitively proves such a conclusion, probably with the advent of and algorithms and quantum computers it is possible to model and prove such a conjecture [1]. The use of transfinite numbers (\aleph_0 , \aleph_1 , ...) as well as the sets they represent allows a trend analysis when $xi \to \infty$ being $x_i \in \mathbb{N} \to |\mathbb{N}| = \aleph_0$. The present approach, based on processes and simulations obtained with the aid of public domain software, aims to conduct part of the research towards obtaining a proof that Collatz conjecture has a final cycle restricted to the sequence $4\rightarrow 2\rightarrow 1$.

In the body of this article, results are presented that, based on the principle of induction, when $x_i \to \infty$ point to the cycle $4 \to 2 \to 1$. Using simple tools and a programming language accessible to the general public, in some cases abusing "brute force" in the solution of the same algorithms. This work focuses on presenting the conjecture and its behavior in a succinct manner, taking into account the boundary conditions. Statistical and programmatic approaches that surround the Natural numbers will be explored in a very simple way. Finally, an 'alternative' form of the conjecture will be presented in addition to the programs used in this search [2].

It would not be reasonable to omit that the scientific community in a certain way advises to stay away from such a conjecture given the fact that the mathematical resources for solving such a problem are still unknown (or have not been listed) [3,4].

It is also worth mentioning that the Collatz conjecture has aroused enormous interest, especially among the young community that usually ventures into this wonderful world of Mathematics, in which sometimes due attention is not given to common and trivial statements, such as this simple example: $x_i \in \mathbb{N}^{\geq 1} = \{2, 3, 4, 5...\}$, it can be said that for any $x_i \in \mathbb{N}^{\geq 1}$, x_i will always have as a divisor one or more prime numbers (Fundamental Theorem of Arithmetic \rightarrow direct consequence of the factorization of integers > 1) [4].

1.1 Background

This article uses algorithms developed in Python and the 'R' language. When presented, they will be duly notified, as well as their relevance [5]. Initially, the environment used for developing the codes in 'R' is presented, and later in Python (this language and environment being preferably used in this work). Remember that the programming interface in 'R' also supports programming in Python. When possible, both solutions, i.e. in 'R' and Python, will be presented, thus allowing the reader to choose the environment that is most suitable for them.

1.1.1 Installing PyCharm

It is assumed that the reader has previously installed the Python language on his/her machine, if he/she has not done so, the following link provides the subsidy for this [6]: https://python.org.br/ (in Portuguese) https://wiki.python.org/moin/BeginnersGuide/Download (in English) There you will be directed to the available solutions and platforms. Then install the PyCharm environment (IDE) from the link:

https://www.jetbrains.com/pt-br/pycharm/download/

https://www.jetbrains.com/pt-br/pycharm/download/

Select your platform and download the Community version. https://livro.com/1-instalacao.html

1.1.2 Installing Rstudio

A good guide to download and install the 'R' environment can be found at: The good gance to do which during the first line of convictment to load (install) the state of code installed (install) the state of code installed (install) the library (package) the library (package) the library (package There, follow the necessary steps for your platform. *numbers* which are functions for various functions contains the function α and α displayers the function α

1.1.3 Testing the Installations

Starting with the 'R' environment, open RStudio and create a new file (R script) with the following code: Starting with the 'R' environment, open RStudio and create a new file (R script) $\frac{1}{\sqrt{1}}$

```
1 # Função Collatz presente na biblioteca numbers
2 library (numbers)<br>2 credition to the main (7)3 \vert \cdot \text{collatz}(\mathbf{7})\frac{1}{\sqrt{2}}
```
Figure 1: Code: collatz_1.R

After saving the file with the name collatz_1.R and 'running' it, you should get the following output [1]: 7, 22, 11, 34, 17, 52, 26, $13, 40, 20, 10, 5, 16, 8, 4, 2, 1$ the file with the fial

The first line of code instructs the environment to load (install) $\frac{1}{2}$ all programs will be made available various requested to the author and will be available in $\frac{1}{2}$ contains: the function Collatz displaying the sequence for: $x_i =$ the library (package) numbers which among its various functions 7 [7]. ϵ of code instructs the environm $\frac{1}{1}$ sequence $\frac{1}{1}$ including $\frac{1}{2}$

the file with the name collatz_1.R and 'running' it, Testing the PyCharm + Python environment. Open PyCharm A, 2, 1 in addition to the main.py file, include another Collatz_Files.py,
insert the following code: (see appendix C), create a new project called Python_Collatz, insert the following code:

```
1 \# Implementação vetorial
2 | \text{def} \text{collatz\_seq(x)}:
 3 \mid seq = [x]
 4 \mid if x < 1:
5 return []
6 while x > 1:
7 if x \% 2 = 0:
8 x = x \frac{7}{2}9 else:
10 x = 3 * x + 1\begin{array}{c|c} \hline 11 & \multicolumn{1}{l}{{\small\text{seq}}} \text{.append(x)} & \# \text{ Inclui} \text{~resultado} \text{~na~sequência} \end{array}12 return seq
\begin{array}{ll}\n\text{w rule } \mathbf{x} > 1; \\
\text{if } \mathbf{x} \times \mathbf{Z} > 0.\n\end{array}\begin{array}{|c|c|c|c|c|}\n\hline\n8 & x = x \end{array} // 2
 \begin{array}{c|c}\n\text{if } x = 3 \times x + 1 \\
\hline\n\end{array}
```
not limiting for reading and analyzing the article. The article the article α

[7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1] **Figure 2: Code: collatz_seq()**

 $[7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1]$ later modified in (2). The main program main.py will be seen in due time (appendix C), activate the Col- latz_Files.py tab and run the file using the Run File in Python Console option (access with the right mouse button on the function), access the console and activate the collatz $seq(7)$ function, you should get the following output:

ne (appendix The program collatz_1.R (Figure1) and the function collatz_ **Example 10** main console option (access with the right by equation (1) [8]. Once the environment is installed and α and eq(7) function, you should get the following output: functions related to the Collatz sequence presented in (1) and Ratz Files.py tab and run the file using $\text{seq}(x)$ (Figure 2) are versions of the Collatz sequence defined ϵ in 1 yulon Console option (access with the right by equation (1) [6]. Once the environment is instance and α n on the function), access the console and activate certified, the next items will consider the developm later modified in (2).

$$
x_{i+1} = f(x_i) = \begin{cases} \frac{x_i}{2^1} & \text{is } x_i \text{ é par} \\ 3 \times x_i + 1 \text{ is } x_i \text{ é impar} \end{cases} \quad (1a)
$$

Note that the exponent of the number 2 (two) being 1 (one) implies a single division per 'step' or 'cycle', that is, for successive iterations when x is even, only one division per cycle, a fact that will be adapted to 2^{ρ} later, where a single cycle may include more than one division by 2 (two). $\frac{1}{2}$ (two).

2. Expanding the Collatz Conjecture

2. Expanding the Conatz Conjecture
Consider the Collatz sequence shown below for $x1 = 7$:

consider the Conatz sequence shown below for $x_1 = r$:
collatz_d(7) = [7, 22, 11, 34, 17, 52, 13, 40, 5, 16, 1] The Python code for the function collatz_d() can be seen in Figure 3, the results correspond respectively to the values $[x_1][x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}]$, in other words for this sequence five odd numbers are identified xi (before the last $x11 = 1$ and, five even numbers, that is [9]:

$$
x_i \in N_1 = [7, 11, 17, 13, 5] = [x1, x3, x5, x7, x9] | i = \{1, 3, 5, 7, 9\}
$$

The values of $x_i \in \mathbb{N}$ being odd will be multiplied by 3 and added to the unit $(3x_i + 1)$, this operation will result in a necessarily even represent of $x_i \in Y_i$ being odd with be inatalyzed by 5 and dated to the direction, $(x_i \in Y_i)$, and operated \sum_i E \sum_i being odd will be multiplied by 3 and added to the unit $(3x_i + 1)$, this operation will result in a necessarily $\frac{1}{2}$ being odd will be multiplied by 3 and added to the unit $(3x + 1)$, this operation will result in a necessarily \sum_{i} \sum_{j} being odd will be inditiplied by 5 and added to the different $(\sum_{i} |T_{i}|)$, this operation will result in a necessarily will be divided (when even) by a nower of 2 they are: The values of $x \in \mathbb{N}$, being odd will be multiplied by 3 and added to the unit $(3x + 1)$, this operatio

 $x_i \in N_p = [22, 34, 52, 40, 16] = [x2, x4, x6, x8, x10] | i = \{2, 4, 6, 8, 10\}$ Often only $x_i \in \mathbb{N}^*$ or $x_i \in \mathbb{N}_I \cup \mathbb{N}_P$, remembering that indexes *i odd* numbers represent odd numbers, and *i even numbers* represent even numbers (for the result obtained by the function *collatz d*(7)). Adjusting the equations in (1) with the necessary modifications we obtain:

$$
collatz_d(x) = \begin{cases} \frac{x}{2^{\rho}} & \text{: se } x \text{ é par, e } \rho \in \mathbb{N}^* \\ 3 * x + 1 & \text{: se } x \text{ é impar} \end{cases}
$$
 (2)

where:

13 return seq

$$
x_2 = 3 * x_1 + 1
$$
 being $x_1 = 7$, $\implies x_2 = 22$
\n $x_3 = \frac{3*x_1+1}{2^1}$ ou $\frac{x_2}{2^1}$, resulting in $\implies x_3 = \frac{22}{2^1} = 11$
\n...

.
.
i *x* $\frac{1}{2}$, $\frac{1}{2}$, ··· *x*¹¹ = *^x*¹⁰ ²⁴ , resulting in (this is the last term) ⁼[⇒] *^x*¹¹ ⁼ ¹⁶ $x_{11} = \frac{x_{10}}{2^4}$, resulting in (this is the last term) $\implies x_{11} = \frac{16}{2^4} = 1$

In the equation (2) and the following code: Collatz sequence seen in the equation (2) and the following code: Collatz sequence seen in the equation (2) and the following code:

```
1 # Implementação impar/par
                  2 \text{ def collatz}_d(x):
                  3 \mid seq = []
                  4 while True :
4 while True :
                  5 \mid seq. append (x)6 if x = 1:
                  7 break
7 break
                  8 \mid if \ge \% 2:
                  9 x = x * 3 + 110 else:
                 11 | while (x \times 2) = 0:
                 12 \quad | \quad x = x \quad // \quad 213 return seq
13 return seq
2 def collatz_d (x) :
4 while True :
\begin{array}{|c|c|c|c|c|}\n\hline\n3 & \phantom{0} & \phantom{0}\n\end{array}\begin{array}{c|c} 4 & \end{array}\begin{array}{c|c} 5 & \end{array}\sim 6\frac{1}{2} \frac{1}{1}19 x
\overline{10} \overline{20}
```
Fig. 3 – Code: *collatz_d()* **Figure 3: Code:** *collatz_d***()**

Once the values of xi are known, as per the previous example (collatz_ $d(7)$ where $x_1 = 7$) the ratio between the successive xi is calculated as follow

 $\rho_i = \frac{x_{(2i+1)}}{x_{(2i)}}$, where: ρ_i **x**7 **=** (3 ∗ *x*⁵ + 1) ∗ *ρ*3 + 1) ∗ *ρ*3 + 1) ∗ *ρ* $\rho_i = \frac{\overline{X_i(x_i)}}{x_{(2i)}}, \text{ where: } \rho_i$ *x*¹¹ = (3 ∗ *x*⁹ + 1) ∗ *ρ*⁵ Generalizing we have: *ρⁱ* = *^x*(2*i*+1) *^x*(2*i*) , where: *ρⁱ* ²¹ *,* ¹ ²¹ *,* ¹ ²² *,* ¹ ²³ *,* ¹ ²⁴]=[¹ 2 *,* 1 2 *,* 1 4 *,* 1 ⁸ *,* ¹ ¹⁶]. ¹⁰= [¹ Generalizing we have: $\rho_i = \frac{x_{(2i+1)}}{x_{(2i)}},$ where: $\rho_i^{10} = \left[\frac{1}{2^1}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}\right] = \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right]$. Rewriting the equations in (2):

 $x_3 = (3 * x_1 + 1) * \rho_1$ *x*₃ $(x - x_1 + 1)$ *r*₁ $(x_1 + x_2)$ $x_7 = (3 * x_5 + 1) * \rho_3$ *x*₁ $(3 + x - 1) + c$ $\begin{pmatrix} 0 & x^2 & -y & -y \\ y & z^2 & z^2 & z^2 \end{pmatrix}$ $x_{11} = (3 * x_9 + 1) * \rho_5$ $x_5 = (3 * x_3 + 1) * \rho_2$ $x_9 = (3 * x_7 + 1) * \rho_4$

 $R_{\rm eff}$ in (2): $R_{\rm eff}$ in (2): $R_{\rm eff}$ in (2): $R_{\rm eff}$ in (2): $R_{\rm eff}$

the following equation: or with the appropriate substitutions: *x*¹¹ $\frac{1}{2}$ $\$

$$
x_{11} = (3 * ((3 * ((3 * ((3 * ((3 * ((3 * x1 + 1) * \rho1) + 1) * \rho2) + 1) * \rho3) + 1) * \rho4) + 1) * \rho5
$$

The final term (in this case) $x_{11} = 1$, with the appropriate operations we will obtain the following equation

$$
1 = x_1 * 3^5 * \prod_{i=1}^5 (\rho_i) + 3^4 * \prod_{i=1}^5 (\rho_i) + 3^3 * \prod_{i=2}^5 (\rho_i) + \cdots + 3^0 * \prod_{i=5}^5 (\rho_i)
$$
(3)

The equation (3) can be rewritten as follows: $T_{\rm eff}$ can be rewritten as follows: σ $3₁(c)$ can see the Collatz Constant c

$$
x_1 * 3^I * \prod_{i=1}^I (\rho_i) + 3^{I-1} * \prod_{i=1}^I (\rho_i) + \underbrace{\sum_{j=I}^I (3^j * \left[\prod_{i=(I-j)}^I (\rho_i) \right]}_{(C)} + \underbrace{3^0 * (\rho_I)}_{(D)} = x_n \qquad (4)
$$

i=1 an orbit defined by \overline{c} e numerical sequence *i*=*i*=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*)^{*i*}=(*I*) \c{c} $1 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 2$ (A) (B) (C)

ions (3) and (4) it is assumed that the Collatz sequence always ends at the number $x_n = 1$ (one)(final cycle 4 \rightarrow 2

ve obtain an orbit defined by the numerical sequence $1 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 1$ In the equations (3) and (4) it is assumed that the Collatz sequence always ends at the number $x_n = 1$ (one)(final cycle $4\rightarrow 2\rightarrow 1$), In the equations (3) and (4) it is assumed that the Collatz sequence always ends at the number $x_n = 1$ (one)(final cycle 4 \rightarrow 2-
rigorously we obtain an orbit defined by the numerical sequence $1\rightarrow4\rightarrow2\rightarrow1\rightarrow4\rightarrow2\rightarrow1\rightarrow4\rightarrow2\rightarrow$ ve obtain an orbit defined by the numerical sequence $1 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 1$ 3 and (4) it is assumed that the Collatz sequence alw $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{2}$ In the equations (3) and (4) it is assumed that the Colla The equation presented in (4) will be broken down into its four parts (*A*)*,*(*B*)*,*(*C*) h *h*(*A*) it is assumed that the Collatz sequence always ends at the number $x_n = 1$ (one)(final cycle 4 \rightarrow 2 *x* cobtain an orbit defin

The *head* (*A*) =⇒ *h*(*xi*) stands out from the equation (4) where the initial term *xⁱ*

3. Considerations on the Collatz Conjecture
The equation presented in (4) will be broken down into its four parts (A) (B) (C) and (D) for the purpose of study **iz Conjecture**
ill be broken down inte (*ρi*) *j*=(*I*−2) *a*^{*j*} ∗*(B) (C)* and *i*=(*I*−*j*) 3. Considerations on the Collatz Conjecture
The equation presented in (4) will be broken down into its four parts (A), (B), (C) and (D) for the purpose of study. = *xⁿ* (4) $\frac{1}{2}$ (D) for the numbers of study

3.1 Head of the Collatz Conjecture (A) *D i*d of the Collatz C 3.1 Head of the Collatz Conjecture (*A*) $\sum_{n=1}^{\infty}$ *k* $\sum_{n=1}^{\infty}$ *k* $\sum_{n=1}^{\infty}$ *z* $\sum_{n=1}^{\infty}$ *k* $\sum_{n=1}^{\infty}$ *z* $\sum_{n=1}^{\infty}$ *z* $\sum_{n=1}^{\infty}$ *z* $\sum_{n=1}^{\infty}$ *z* $\sum_{n=1}^{\infty}$ *z* $\sum_{n=1}^$

The *head* (A) = \Rightarrow h (x_i) stands out from the equation (4) where the initial term $x_i = x_1$ can be seen: \mathcal{V} *i*=14 equation (4) where the initial term $x_i = x_1$ can The *head* $(A) \implies h(x_i)$ stands $\frac{f_{\text{nom}}}{f_{\text{nom}}}$ the equation $\left(\Lambda\right)$ \ldots μ moin the equation (4) where the (*ad* of the Collatz Conjecture (**A**)
iead (*A*) =⇒ *h* (*x*) stands out from the equation (4) where the initial term *x* $\sum_{i=1}^{n}$ *i*=1 \Rightarrow *h*(x_i) stands out from the equation (4) when **atz Conjecture (A)**
 \mathbf{r}_i) stands out from the equation (4) where the initial term $x_i = x_i$ can be seen: $j \Rightarrow h(x_i)$ stands out : Head of the Collatz Conjecture (A)
 $\text{the } \text{head}(A) \implies h(x_i) \text{ stands out from the equation (4) where the initial term } x_i = x_i \text{ can be seen:}$ The head $(A) \implies h(x_i)$ stands out from the equation (4) where the initial term $x_i = x_1$ can be seen: the function *r_collatz(7)* (shown in Fig. 4 below) will provide the necessary subsidies.

$$
(A) \Longrightarrow h(x_1) = x_1 * 3^I * \prod_{i=1}^I (\rho_i)
$$
\n
$$
(5)
$$

below) will provide the necessary subsidies. When executing the function $r_{collatz}(7)$ the answer is \rightarrow [16,5,11] which corresponds to [C,I,P], that is, 16 Cycles (operations) before obtaining 1(one), with 5 being Odd and 11 being Even [11] in that to calculate $h(x_1)$ it is necessary to know the values of I and $\prod_{i=1}^{t}(\rho_i)$, the function r_{col} collatz(7) (shown in Figure 4 It can be seen that to calculate $h(x_1)$ it is necessary to know the values of I and $\prod_{i=1}^{I}(\rho_i)$, the function $r_collatz(7)$ (shown in Figure 4 to [C,I,P], that is, 16 Cycles (operations) before obtaining 1(one), with 5 being Odd and 11 being Even [11] being Even¹¹

 $\frac{d}{dx}(7)$ the function $f(x)$ function $f(x)$ function $\frac{d}{dx}(7)$ the answer is $\frac{d}{dx}(7)$ to $\frac{d}{dx}(7)$ the answer is $\frac{d}{dx}(7)$ $\frac{1}{2}$ contact (*f*) also informs that [*f*] \rightarrow (equation (*f*) \rightarrow *f*), nowever $\frac{1}{2}$ contact (*f*) also not provide the variates of p
if the total number of pairs [Pl = 11, so we can calculate πI , $\left(\circ \right$ being Even¹¹ *forms that* $[I] = 5$ (equation (5) $I = 5$), however $r_{collatz}(7)$ does not provide the values of ρ_r , but pairs [P] = 11, so we can calculate $\prod_{i=1}^{I}(\rho_i) = \prod_{i=1}^{P}(\frac{1}{2}) = 2^{-P}$ or: *r*_{*collatz*(7) also informs that $[I] = 5$ (equation (5) $I = 5$), however *r_{_collatz*(7) does not provide the values of ρ}} if the total number of pairs $[P] = 11$, so we can calculate $\prod_{i=1}^{I}(\rho_i) = \prod_{i=1}^{P}$ The function $r_collatz(7)$ also informs that [*I*] indicates the The function *r_collatz*(7) also informs that $[I] = 5$ (equation (5) $I = 5$), however *r_collatz*(7) does not provide the values of ρ _{*i*}, but indicates that the total number of pairs [P] = 11, so we can calculate $\prod_{i=1}^{I}(\rho_i) = \prod_{i=1}^{P}(\frac{1}{2}) = 2^{-P}$ or: $The func$ icates that the total number of pairs [P] = 11, so we can calculate $\prod_{i=1}^{I}(\rho_i) = \prod_{i=1}^{P}$ $\sum_{i=1}^{\infty}$ *r* callatz(7) also informs that $[I] = 5$ (equation (5) $I = 5$) however r colle *i*¹ $\prod_{i=1}^{n} (p_i) = \prod_{i=1}^{n} (\frac{1}{2}) = 2$ −

$$
\prod_{i=1}^{I}(\rho_i) = 2^{-P} \longrightarrow \prod_{i=1}^{5}(\rho_i) = 2^{-11}
$$

In fact, from the function *collatz_d*(7) seen in figure 3, we obtained the sequence [7, 22,11, 34, 17, 52, 13, 40, 5, 16, 1] and from the 11, 34, 17, 52, 13, 40, 5, 16, 1] and from the generic formula *ρⁱ* = *^x*2*i*+1 generic formula $\rho_i = \frac{x_{2i+1}}{x_{2i}}$ (presented after figure 3): 11, 34, 17, 52, 13, 40, 5, 16, 1] and from the generic formula *ρⁱ* = *^x*2*i*+1 In fact, from the function *collatz_d*(7) seen in figure 3, we obtained the sequence [7, 22,11, 34, 17, 52, 13, 40, 5, 16, 1] and from the generic formula $\rho_i = \frac{x_{2i+1}}{x_{2i}}$ (presented after figure 3):

²¹ [∗] ¹

ⁱ=1(*ρi*)=[¹

$$
\Pi_{i=1}^{5}(\rho_i) = \left[\frac{1}{2^1} * \frac{1}{2^1} * \frac{1}{2^2} * \frac{1}{2^3} * \frac{1}{2^4}\right] = 2^{-11}
$$

²¹ [∗] ¹

²² [∗] ¹

²³ [∗] ¹

²⁴]=2−¹¹

Which allows us to calculate the *head* (A) of Collatz (7) :

$$
h(x_1) = x_1 * 3^I * \prod_{i=1}^I (\rho_i) \rightarrow h(7) = 7 * 3^5 * \frac{1}{2^{11}} \simeq 0.83056640625
$$

3.1.1 Head of the Collatz Conjecture: How are the Powers of 2

 $I = 0$ and $P = M$, we arrive at the following expression $\Rightarrow h(x_i) = 2^M * 3^0 * \rho |\rho = \frac{1}{2^M}$, in short $h(x_i) = 1$ in these cases, confirming
the validity of the equation described in (5) for $\forall x = 2^M$ 3.1.1 Head of the Collatz Conjecture: How are the Powers of 2
Consider a number $M \in \mathbb{N}^* = \{1, 2, 3, 4, 5, ...\}$ and the integer 2^M when expressed in binary will have only one of the bits with the value one, being divisible by two "M" times until the number is obtained one as the final answer (in binary the bit whose value is value one, being divisible by two "M" times until the number is obtained one as the final value one, being divisible by two M times until the number is obtained one as the final answer (in binary the oft whose value is one will be shifted to the right "M" times). By reevaluating the equation presented in (5) a the validity of the equation described in (5) for $\forall x_i = 2^M$. Γ similar way, the term (Γ) see term (4) can be calculated: It can be easily verified that 3 ∗ *x*¹ ∗ (*B*)=(*A*).

Considering (B) in binary will have only one of the bits with the value of the value of the value of the value of two "*M*" μ " μ " **3.2 Neck of the Collatz Conjecture (B)**

3.2 Neck of the Collatz Conjecture (B)
In a similar way, the term (B) seen in the equation (4) can be calculated: [7, **22**, 11, **34**, 17, **52**, **26**, 13, **40**, **20**, **10**, 5, **16**, **8**, **4**, **2**, 1], in which the following stand out the even 1*d* **1**.039550781250 $\frac{3}{3}$ and term (*D*) seem in the equation (+) can be calculate

$$
(B) \Longrightarrow 3^{I-1} * \prod_{i=1}^{I} (\rho_i) \rightarrow 3^4 * \frac{1}{2^{11}} \simeq 0.03955078125
$$

It can be easily verified that $3 * x_1 * (B) = (A)$.

3.3 Tail of the Collatz Conjecture (D)

3.3 Tail of the Collatz Conjecture (D)
Continually, the term (D) is calculated according to equation (4), the term (C) will be calculated later: *D*_{*l*}**n** $\frac{1}{2}$ *d*_{*i*} $\frac{1}{2}$ *d*

$$
(D) \Longrightarrow t(x_i) = 3^0 * \rho_n \quad \to \quad t(7) = 1 * \frac{1}{2^4} \simeq 0.0625
$$

to the final sequence $16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ within which the cycle $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$ is included, that is, from 16 to reach 1 it is t_1 and t_2 + 8 $\frac{1}{2}$ + 2 $\frac{1}{2}$ + 2 $\frac{1}{2}$ + 2 $\frac{1}{2}$ + 1 $\frac{1}{2}$ + 1 is an analyze $\frac{1}{2}$ $\frac{1}{2}$ and $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ + 1 \frac It can be seen that the term (*D*) is constant (for $x_i \ge 3$ and $x_i \ne 2^M$) as it corresponds 3.4 Collatz Conjecture Body (C) \sim divided by 24. ϵ is, from 16 to ϵ is divided by 24.1 it is divided by 24.1 i

Γ can besteur Pad_X (C) **3.4 Collatz Conjecture Body (C)**

3.4 Collatz Conjecture Body (C)
In equation (4), we can see that we need to know the values of x_1 , I and ρ_i , remembering that $\rho_i = \frac{1}{\gamma_i}$, the following program In equation (4), we can see that we need to know the values of x_1 , T and ρ_p , remembering that $\rho_i = \gamma_i$, the following program
fragment, shown in Figure 4, presents the function r_collatz(x₁) which will provide su function *r_collatz(x*1*)*¹² which will provide such values: *^r*_*collatz*(*x*1) −→ [*C, I, P*].

```
1 def r _collatz (num) :
                 P = 0,<br>
I<sub>1</sub> and n<sup>1</sup>, I and p<sup>1</sup>, I<sup>2</sup>, I
 \begin{array}{c|c}\n\hline\n4 \\
5\n\end{array} \begin{array}{c}\n1 - 0 \\
C = 0\n\end{array}\begin{array}{c|c} \n\text{O} & \text{C} = \text{0} \\
\text{resp} = \text{np} \cdot \text{array} ([\text{C}, \text{I}, \text{P}])\n\end{array}\begin{array}{c|c} 6 & \text{resp.} = \text{np. array } ([C, 1, P]) \\ \hline \text{# verification se nBIN eh impact se sim continua, caso par RSH \\ \end{array}\begin{array}{c} 8 \text{ }\text{min} = \text{converte} \text{ (num)} \\ 9 \text{ }\text{while }\text{nBIN}[-1:] == '0': \text{ } \text{ } \# \text{ bit a direita menos sig}. \end{array}\begin{bmatrix} 3 \\ 10 \end{bmatrix} while \begin{bmatrix} 1 \\ 1 \end{bmatrix} and \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} and \begin{bmatrix} 1 \\ 1 \end{bmatrix} and \begin{bmatrix} 1 \\ 1 \end{bmatrix} and \begin{bmatrix} 1 \\\begin{array}{c} 0 \\ 1 \end{array}\begin{array}{c|c}\n1 & \cdots & 1 \\
\hline\n0 & 0 & 0\n\end{array}\begin{array}{c|c}\n2 & \cdots & 0 \\
\vdots & \vdots & \vdots\n\end{array}\begin{array}{c|c|c|c|c|c} \hline \text{5} & \text{m} & \text{m} & \text{m} \\ \hline \text{6} & \text{m} & \text{m} & \text{m} & \text{m} \\ \hline \text{7} & \text{m} & \text{m} & \text{m} & \text{m} \end{array}7 \frac{4}{5} # 150 enquality s equation s e simple s e simple simulation s e simulation 
  \begin{array}{c} 8 \text{ m} \\ 2 \text{ m} \\ 2 \text{ m} \end{array} while \begin{array}{c} \text{em}(\text{m}) \times 1. \\ \text{cm}(\text{m}) \times 1. \end{array}\begin{bmatrix} 0 \\ 2 \end{bmatrix} if \begin{bmatrix} 1 \text{IDIN}[-1!] & -1 \end{bmatrix} == \begin{bmatrix} 1 \end{bmatrix} : # en impair
10 nBIN = add binary_nums (nBIN, 1)
\begin{array}{c|c}\n 10 & \text{m} \\
 \hline\n 10 & \text{m}\n\end{array}\begin{array}{c|c}\n 12 \\
 \hline\n 20\n \end{array}\begin{array}{c|c}\n 20 & \text{nBIN} = \text{add\_binary\_nums} (\text{nBIN\_t}, \text{nBIN}) \\
 \hline\n 1 & \text{if } 1 \leq x \leq 1\n \end{array}\begin{array}{ccc} 21 & & & 1 \ + & 1 & & \\ 20 & & & 1 \end{array}\begin{array}{ccc} 22 & \multicolumn{2}{c}{} &amp\begin{array}{c|cccc}\n 23 \\
 \hline\n 04\n \end{array} \begin{array}{c|cccc}\n 1 & \text{+} \\
 & & & & & & & & \text{+} \\
 & & & & & & & & & & \text{24 eise:<br>or
125 nBIN = nBIN [-1]\begin{array}{c|c}\n 20 \\
 \hline\n 07\n \end{array}20 \bigcup_{n=1}^{\infty} \bigcup_{n=1}^{\infty} , nBIN \bigcup_{n=1}^{\infty} , 
28 \quad \text{resp} = [\text{C}, \text{ I}, \text{ P}]29 return resp
                 In equation (4), we can see that we need to know the values of x1, I and ρi,
 1 def r_collatz(num):<br>2 # funcao retorna array [Total de Ciclos ,Impares ,Pares]
 \begin{array}{c|c} \text{3} \\ \text{4} \end{array} \begin{array}{c} \text{P} = \text{0} \\ \text{I} = \text{0} \end{array}8 \mid \text{nBIN} = \text{converte}(\text{num})1 \vert \qquad P \neq 12 | C  \leftarrow 1
13 |       # criamos  nBIN_t   somamos  1  a  nBIN  e  RSH,
14 \# isto enquanto len (nBIN) > 1
15 | while len (
16 | if nBI
17 nBIN_t = nBIN
19 \mid \text{mBIN} = \text{nBIN}[-1]21 | I \neq 1182 c \rightarrow 1 \rightarrow23 P += 1 \# estou x3 + 1 e dividindo por 2
24 | else:
26 | P += 1
27 P \qquad \qquad \text{C} \neq 13 P = 0
 4 I = 0
11 P += 1
12 C += 1
15 | while len (nBIN) > 1: # existem bits a serem processados
16 if nBIN[-1:] = '1': # eh import22 \qquad \qquad C \ \mathrel{+}= 224 else
27 | C \neq 12 # funcao retorna array [ Total de Ciclos , Impares , Pares ]
                                                                                                               \arcs , \text{Pares} ]
                                                                                                               7 # verifica s e nBIN eh impar s e sim continua , caso par RSH
                                                                                                              10^{10} nBin 10^{11}\begin{array}{ll} \texttt{211} & \texttt{212} \\ \texttt{211} & \texttt{212} \end{array}.15 while len (nBIN ) > 1: # existem bits a serem processados
                                                                                                              18 nBIN = add_binary_nums (nBIN ) = add_binary_nums (nBIN )
                                                                                                              n processados
                                                                                                              20 nBIN \sim add binary nums ( n ) and n ) and n , nBIN \sim and \sim and \sim23 P += 1 estou x3 + 1 e dividindo por 2 estou x3 + 1 e dividindo por 2 estou x3 + 1 e dividindo por 2 estou x
                                                                                                              24 else : 24 
                                                                                                              \overline{J}
```
Figure 4: Code: *r_collatz***()**

26 P + 26 P

Fig. 4 – Code: *r_collatz()*

¹² The function *r_collatz(x*1*)* and others that follow depend on several other functions and adjustments in the

In the item (C) of the equation (4) considering the Collatz sequence relative to the number 7, that is, using the data of $r_{collatz}(7) \rightarrow [16, 16]$ 5, 11] ([C,I,P]), where $I = 5$ we obtain: *j*=(*I*−2) obtain:

In the item (*C*) of the equation (4) considering the Collatz sequence relative to the

$$
\underbrace{\sum_{j=(I-2)}^{1} \left(3^{j} * \left[\prod_{i=(I-j)}^{I} (\rho_i)\right]\right)}_{(C)}
$$
\n
$$
\underbrace{3^3 * \prod_{i=2}^{5} (\rho_i) + 3^2 * \prod_{i=3}^{5} (\rho_i) + 3^1 * \prod_{i=4}^{5} (\rho_i)}_{(\beta)} \underbrace{(\rho_i)}_{(\beta)}
$$

t was shown that $ρ$ ^{*i*} = [$\frac{1}{21}$, $\frac{1}{21}$, $\frac{1}{22}$, $\frac{1}{23}$, $\frac{1}{24}$]. In this way we calculate (*α*), (*β*) and (*δ*): t was shown that $\rho_i = \left[\frac{1}{2^1},\right]$ $\frac{1}{2^1}$ 1 $\frac{1}{2^2}$, $\frac{1}{2}$ $\frac{1}{2^4}$. In this way we ve cal aulate⊔
≈ (α) , (β) and (δ) : Previously it was shown that $\rho_i = \left[\frac{1}{2^1}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}\right]$. In this way we calculate (α) , (β) and (δ) :

$$
\underbrace{3^3 * [\frac{1}{2^1} * \frac{1}{2^2} * \frac{1}{2^3} * \frac{1}{2^4}]}_{(\alpha)} + \underbrace{3^2 * [\frac{1}{2^2} * \frac{1}{2^3} * \frac{1}{2^4}]}_{(\beta)} + \underbrace{3^1 * [\frac{1}{2^3} * \frac{1}{2^4}]}_{(\delta)}
$$

$$
\underbrace{0.0263671875}_{(\alpha)} + \underbrace{0.017578125}_{(\beta)} + \underbrace{0.0234375}_{(\delta)} = \underbrace{0.0673828125}_{(C)}
$$

Recovering the previous results we will have: **3.5 Complete Collatz Conjecture**

3.5 Complete Collatz Conjecture
Recovering the previous results we will have:

 $3.6640625, (B) \approx 0.03955078125, (C) \approx 0.0673828125, (D) \approx 0.0673828125$ Recovering the previous results we will have:
 $(A) \approx 0.83056640625$, $(B) \approx 0.03955078125$, $(C) \approx 0.0673828125$, $(D) \approx 0.0625$

presented in the program in Figure 5: A terms of the equation (4) we will finally have: $(A) + (B) + (C) + (D) = 1$ As can be seen from the function *abco* Adding the terms of the equation (4) we will finally have: $(A) + (B) + (C) + (D) = 1$ As can be seen from the function *abcd*('*7*') recorded in the process in Figure 5:

 $abcd('7') \rightarrow ('7', 0.8305664062499994, 0.039550781249999986, 0.0673828125, 0.0625, 0.999999999999994).$

1 def abcd(num): #Retorna os valores $(A)A$, $(B)B$, $(C)C$ e $(D)D$
2 [Ci I P gama] = r collatz1(num) $3 \text{ gamas} = \text{np. array (gamma)}$ *abcd('7')* → ('7', 0.8305664062499994, 0.039550781249999986, 0.0673828125, 0.0625, 0.9999999999999994). 1 def abcd (num) : #Retorna o s valores (A)A, (B)B, (C)C e (D)D 5 D = 1/(2∗ ∗ gamas[−1]) *abcd('7')* → ('7', 0.8305664062499994, 0.039550781249999986, 0.0673828125, 0.0625, 0.9999999999999994). $\begin{array}{c|c} b & C = 0 \\ 7 & A = 2 \end{array}$ $* (math.log2(int(num)) + math.log2(3) * I - P)$ $8 \quad \text{for J in range (I-2,0,-1)}$: 3 gamas = np . a r r ay (gama) 5 D = 1/(2∗ ∗ gamas[−1]) 9 sgama = 1 4 B= 2 ∗ ∗ (((I −1)∗math . lo g 2 (3))−P) 10 \vert for i in range (I-J-1, I): 11 sgama = sgama * $(1/(2**\text{gamma}[i]))$ 2 | C $\begin{array}{lll} \text{12} & \text{C} = \text{C} + (3**\text{J}) * \text{gamma} \ \text{4} & \# \text{qq Xi} = ((2**\text{e})-1) & \text{divisivel (int) por 3 somente tera um impar, I, o proprio} \end{array}$ 8 for Julian 1990 in range (I −2,0, −2,0 14 $\qquad \qquad \uparrow \qquad \qquad +$ sendo assim $(A) > 0$, $(B) > 0$, $(C) =$ 15 if $I = 1$; $16 \quad | \quad D = 0$ $\begin{array}{c|c} \n 10 & D = 0 \\
 17 & \text{return } (\text{num}, \text{ A}, \text{ B}, \text{ C}, \text{ D}, \text{ A+B+C+D}) \n \end{array}$ $2 \text{ } C = 0$ 0 f 12 | $C = \overline{C} + (3**J) * \text{sgama}$ 10 for in 10 for it is in 10 for it in 10 for it is in 10 for it in 10 for it is in 10 for it in 10 for it is i
Definition in 10 for it is in 10 for it in 10 for it is in 10 for it in 10 for it is in 10 for it is in 10 fo $16 \vert \nabla = 0$ $2 \begin{bmatrix} \text{Ci} & \text{I} & \text{P} \\ \text{c} & \text{a} & \text{A} \end{bmatrix} = \text{r}_\text{collatz1}(\text{num})$ $\overline{B} = 2 * * ((I - 1) * \text{math} \cdot \log 2(3)) - P)$ 14 \parallel # sendo assim (A) > 0, (B) > 0, (C) = 0 e (D) = 0 $\begin{array}{c|c}\n16 & D = 0 \\
17 & \text{return } (\text{num.})\n\end{array}$ $\begin{array}{c} \begin{array}{c} \text{if } \\ \text{if } \\ \text{if } \\ \text{if } \\ \end{array} \end{array}$ 10 $\begin{array}{c|c}\n 13 & \# & \text{qq}\n \end{array}$ 9 sgama = 1 17 return (num, A, B, C, D, A+B+C+D)

14 # sendo assim (A) > 0 , (B) > 0 , (C) = 0 e (D) = 0 14 # sendo assim (A) > 0 , (B) > 0 , (C) = 0 e (D) = 0 **Figure 5: Código:** *abcd***()**

13 # qq Xi = ((2∗ ∗ e) −1) divisivel (int) por 3 somente tera um impar , I , o proprio

 $n_{\rm b}$ from table 1 were te Several numbers from table 1 were tested with the function $abcd(x_i)$ and returned the sum $A + B + C + D \approx 1$

 14.4 \pm 0 \pm

x_1	\mathcal{C}		$\mathbf P$	$\sim I/P\%$	$\sim x_1 * \frac{3^I}{2^I}$	$\sim A+B+C+D^6$
9	19	6	13	46.15	0.8009	
97	118	43	75	57.33	0.8428	
871	178	65	113	57.52	0.8639	
6 171	261	96	165	58.18	0.8395	
77 031	350	129	221	58.37	0.8084	
837 799	524	195	329	59.27	0.8373	
8 400 511	685	256	429	59.67	0.8423	
63 728 127	949	357	592	60.30	0.8450	
670 617 279	986	370	616	60.06	0.8450	
9 780 657 630	1132	425	707	60.11	0.8683	
75 128 138 247	1228	461	767	60.10	0.8683	
989 345 275 647	1348	506	842	60.09	0.8942	
Big $num1^7$	10466	3455	7011	49.28	0.8472	
NN4 ⁸	36780	12293	24487	50.20	0.8078	

Table 1: $x_1 \rightarrow [C, I, P]$

presented in the function $abcd(x_i)$, shown in penultimate column of this table, in several tests it is shown that $h(x_1) = x_1 * \frac{3^T}{2^F}$ tends to be >0.7, a fact that will be addressed later [13]. in the equation (5), being equal to the term 'A' presented in the function *abcd(xi)*, shown Note that the column x_1 * corresponds to the Collatz function *head* presented in the equation (5), being equal to the term 'A' to be ≥ 0.7 , a fact that will be addressed later [13].

4. Exploring Some Sequences This chapter explores a restricted set of data that, after being processed, and allowed, allowed, allowed, allowed, allowed, and allowed, and allowed, and allowed, and allowed, allowed, allowed, allowed, allowed, and allow

the sum *A* + *B* + *C* + *D* ≈ 1

in perfect of the column of the several tests is shown that *h***(***x***¹) =** *x***¹***N***¹ ∗ 3***I***¹ ∗ 3** Computers have routinely tested the Collatz Conjecture for increasingly larger numbers (see issue NN4 note 8 above), using powerful machines and improved algorithms that indicate that the Collatz Conjecture 'apparently' ends in its cycle $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$. In the previous item it was shown that for $\forall x_i = 2^M, M \in \mathbb{N}$ the final cycle will always be $1 \to 4 \to 2 \to 1$.

This chapter explores a restricted set of data that, after being processed, allowed some graphs to be drawn and some considerations to be made about them. Because they are restricted (data and graphs), it is clear that they can and should be improved as the tests to be carried out advance. be carried out advance.

4.1 Limit Cycle or Orbit Previously it was considered that the final cycle for the Collatz Conjecture is

Previously it was considered that the final cycle for the Collatz Conjecture is $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$, a brief explanation of the concept of limit cycle [7] or Orbit is in order. Consider the following transformation: Consider the following transformation:

$$
f^n(x_i) = x_{(i+n)} \tag{6}
$$

Since $f''(x_i)$ is the process of transforming the variable x_i , implying one or more times the application of the Collatz Conjecture initially on the variable $x_i \in \mathbb{N}^*$, it is not just the possibility of applying a singular function, as the stages may involve multiple steps \int_{a}^{b} (x_i) denotes that there are several steps, with n being the steps that correspond to increases and decreases. initially on the variable $x_i \in \mathbb{N}^*$, it is not just the possibility of applying a singular function, as the stages may involve multiple steps
where the variable x_i will increase and subsequently decrease, as seen i index *n* in $f^n(x)$ denotes that there are several steps, with n being the steps that correspond to increases and decreases.

A limit cycle or Orbit is considered in mathematical operations systems that present the occurrence of the fact that $f^n(x_i) = x_i$ being x_i $\in \mathbb{N}^*$, in this way the existence of a cycle (may be repetitive) or Orbit (may be periodic) within the sequence is verified, it is observed that any limit cycle that may exist in the Collatz Sequence where $x_i \neq 1$, $x_i \neq 2$, $x_i \neq 4 | x_i \in \mathbb{N}$ leads to the collapse of the Conjecture, as it will inevitably not reach the $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$ orbit, since it was 'captured by another orbit' [9]. 61782879907300386230346233746806307761944850543497850771536758125478963250468647363750480359016178719876557983546 49187234516275795946527548120364557821371 (composto por 1506 dígitos).

of the Collatz Conjecture, using the definition found in equation (2) and used in the construction of the *collatz* function, it is possible to verify the existence of (*probably just*) a 'limit' cycle where $x(i) = f^{n}(x_i)$. In the case of the Collatz Conjecture, using the definition found in equation (2) and used in the construction of the *collatz* $d(x)$

From the equation (2) we can group the 'growth' and 'decay' operation of the system into a single equation (for the sequence $1 \rightarrow$ $3s$ 61782879907300386230346233746806307761944850543497850771536758125478963250468647363750480359016178719876557983546 $n = 1$, the 1 orbit α + 2 α + $4 \rightarrow 2 \rightarrow 1$), namely:

11

$$
\frac{3 * x_i + 1}{2^{\gamma}} = x_{(i+n)} \qquad \text{(remembering that } x_i \text{ is odd)} \tag{7}
$$

With the necessary adjustments where $x_i = x_{(i+n)}$ is obtained from the equation *x*_{*i*} = x_i is obtained from the equation (22 x_i = 3) x_i = x^2 $\frac{dy}{dx}$ and $\frac{dy}{dx}$ is obtained from the equation If the necessary adjustments where $x_i - x_{(i+n)}$ is obtained from the equation *^xⁱ* ⁼ *^x*(*i*+*n*) = 1, thus the expression (2² [−] 3) = 1 evidencing the existence of a 'limit' cycle With the necessary adjustments where $x_i = x_{(i+n)}$ is obtained from the equation $\sum_{i=1}^{n}$ is obtained from the equation

1

$$
\frac{1}{(2^{\gamma}-3)} = x_i \qquad \text{(remembering that } x_i, x_{(i+n)}, \gamma \in \mathbb{N}^* \text{)} \tag{8}
$$

that the only values that solve the equation (8) are $y = 2$ and $x = x = 1$ thus the expression $(2^2 - 3) = 1$ evider that the only values that solve the equation (8) are $\gamma = 2$ and $x_i = x_{(i+n)} = 1$, thus the expression $(2^2 - 3) = 1$ evidents in the California behavior of the solution of the solution of the solution of the solution of the the existence of a 'limit' cycle in the Collatz Conjecture, which will be seen later at the end of section 4.1.2. It is easy to see that the only values that solve the equation (8) are $\gamma = 2$ and $x_i = x_{(i+n)} = 1$, thus the expression $(2^2 - 3) = 1$ evidencing
the existence of a ilimit, exclo in the Collatz Conjecture, which will be see It was previously shown that if there are other limit cycles (or bits) with α

⁹ A limit cycle is a closed trajectory, as demonstrated by **Monterio**[7](pg 224), thus we can infer that the points

4.1.1 Periodic Orbit \ddot{c} Collatz Conjecture, which will be seen later at the end of section 4.1.2.1.2. Collatz Conjecture other than the final cycle 1 → 4 → 2 → 1 it is false. Strictly speaking, $t + 1$ and the conjecture), one for Ω

 5.5 return 5.5 return 5.5

ously shown that if there are other fimite cycles (orbits) within the Collatz Conjecture other than the final cycle T
is false. Strictly speaking, the system in (2) cannot be treated as a single procedure function (linear) of considering the two operations in just one algebraic equation is sought. The candidate equation is presented below, which will
certainly facilitate the study of the orbits: certainly facilitate the study of the orbits: the Orbit possible approaches (use of the conjecture), one for even numbers and the other for odd numbers. In this way, an alternative way It was previously shown that if there are other limit cycles (orbits) within the Collatz Conjecture other than the final cycle $1 \rightarrow 4$
 $\rightarrow 2 \rightarrow 1$ it is false. Strictly speaking the system in (2) cannot be treated as a sin \rightarrow 2 \rightarrow 1 it is false. Strictly speaking, the system in (2) cannot be treated as a single procedure function (linear) since there are two $\overline{}$ (9)

$$
x_{i+1} = [(3 * x_i + 1) * (1 - \cos^2(x_i * \frac{\pi}{2}))] + [\frac{x_i}{2} * \cos^2(x_i * \frac{\pi}{2})]
$$
(9)

²)] becomes

odd numbers. In the puttern of next \mathbf{F}_{GUPX} of constant interference the function (or each) collate $d(x)$ n coded above 10 in Python as per Figure 6 below, it operates identically to the function (or code) *collatz_a*(x_i). α coded above TO in Python as per Figure 6 below, \sim \sim The equation coded above 10 in Python as per Figure 6 below, it operates identically to the function (or code) *collatz* $d(x_i)$.

2 def collatz_ang (x) :

1 # Implementa ção fun ção Collatz *x*^{*j*} = $\frac{1}{1}$: T while $x > 1$:
 $\#x = ((3 * x + 1)*(1 - int((np.cos(np, pi/2*x))**)*) + x*((int((np.cos(np, pi/2*x)))$ $x = \frac{3*(2)(2)}{8}$
 $x = \frac{3*(2)(2)}{8}$ $\begin{array}{c|c}\n\hline\n\end{array}$ $\begin{array}{c}\n\hline\n\end{array}$ $\begin{array}{c}\n\hline\nm\end{array}$ $\begin{array}{c}\n\hline\n\end{array}$ $\begin{array}{c}\n\hline\n\end{array}$ $\begin{array}{c}\n\hline\n\end{array}$ $\begin{array}{c}\n\hline\n\end{array}$ $\begin{array}{c}\n\hline\n\end{array}$ $\begin{array}{c}\n\hline\n\end{array}$ $\begin{array}{c}\n\hline\n\end{array}$ $\begin{array}{c}\n\hline\n\end{array}$ $9 \mid$ $seq.append(x)$ 10 return seq 4 i <u>f x i f</u> $\frac{2}{3}$ def collatz_ang (x) : $\begin{array}{c} 2 & 0 & -1 \\ 4 & 1 & -1 \end{array}$ $\begin{array}{c|c}\n 4 & \text{if } x < 1:\n \end{array}$ $6 \quad \text{while } x > 1$: $\begin{array}{c|c|c|c|c|c|c|c} \hline 0 & \text{where } x > 1. & \text{if } (3 * x + 1) * (1 - \text{int} ((np \cdot \cos(np \cdot pi/2*x)) **2))) + x * ((\text{int} ((np \cdot \cos(np \cdot pi/2*x))) & & \text{if } (3 * x + 1) * (1 - \text{int} ((np \cdot \cos(np \cdot pi/2*x))) & & \text{if } (3 * x + 1) * (1 - \text{int} ((np \cdot \cos(np \cdot pi/2*x))) & & \text{if } (3 * x + 1) * (1 - \text{int} ((np \cdot \cos(np \cdot pi/2*x))) & & \text{if$ $\binom{10}{2}$ $\binom{10}{2}$ $\binom{2*1}{2}$ $\binom{2*2}{2}$ $\frac{\text{appona}}{2}$ $3 \mid$ $seq = [x]$ $\begin{array}{c|c|c|c} 5 & \text{return} & \begin{bmatrix} 1 & \end{bmatrix} \end{array}$ $(*2)$) $/2)$ $\begin{array}{c|c|c|c|c} \n4 & & \text{if } x < 1. \n\end{array}$ $\overline{7}$ returns sequence $\overline{7}$ $\begin{array}{c|c|c|c} \text{5} & \text{return} & \text{1} \end{array}$ 7 #x = ((3 ∗ x +1)∗(1− int ((np . cos (np . pi /2∗x)) ∗ ∗2))) + x ∗ ((int ((np . cos (np . pi /2∗x) $\text{np} \cdot \text{cos} \left(\text{np} \cdot \text{p1} / 2 \cdot \text{x} \right) + \text{p2}$ $(x + 1) * (1 - r)$ Fig. 6 – Code: *collatz_ang*(*xi*)

 $$ null and the operation is just division by two, i.e. *xi*+1 = *^xⁱ* $2 \times \mu$

7 #x = ((3 ∗ x +1)∗(1− int ((np . cos (np . pi /2∗x)) ∗ ∗2))) + x ∗ ((int ((np . cos (np . pi /2∗x) Note that for even values of x_i the term $[(3 * x_i + 1) * (1 - \cos^2(x_i * \pi/2))]$ becomes null and the operation is just division by two, . However, if x is odd, the term $[\frac{x_i}{2} * cos^2(r_i * \frac{\pi}{2})]$ is nullified, leaving only the result $x = (3 * x + 1)$. Fig. i.e. $x_{i+1} = \frac{x_i}{2}$. However, if x_i is odd, the term $\left[\frac{x_i}{2} * cos^2(x_i * \frac{\pi}{2})\right]$ is nullified, leaving only the result $x_{i+1} = (3 * x_i + 1)$. From the equation (9) it is possible to study the periodic orbit and its resp equation (9) it is possible to study the periodic orbit and its respective equilibrium points, considering the following sequence:

$$
x_2 = f(x_1)
$$

\n
$$
x_3 = f(x_2) = f^2(x_1)
$$

\n...
\n
$$
x_n = f(x_{n-1}) \quad \text{ou} \quad x_n = f^{(n-1)}(x_1)
$$

\n
$$
x_{n+1} = f(x_n) \quad \text{ou} \quad x_{n+1} = f^{(n)}(x_1)
$$

 $\frac{1}{\sqrt{1-\frac{1}{2}}}$

thit is established when $\mathbf{y} = f(\mathbf{0}(\mathbf{y}))$ that is $\mathbf{y} = \mathbf{y}$. According to Montevia 71 to know the characteristics of an a to know its eigenvalue λ_n which corresponds *x*² = *f*(*x*1) a blished when $x_i = f^{(n)}(x_i)$, that is, $x_{n+m} = x_n$. Actively also the product A perform of the statement when $x_i - y \sim (x_i)$, that is, $x_{n+m} - x_n$. According to Monterform to the entrancements of an order it
is necessary to know its eigenvalue λ , which corresponds to the product of each eigenvalue ¹⁰ The round function was used in the code because the arithmetic precision used can generate residual values A periodic orbit is established when $x_i = y_i^N$, x_i , and is, $x_{n+m} = x_n$. According to momental ℓ if the value of ℓ as follows: $\mathbf{f}^{(n)}(\mathbf{x})$ that is $\mathbf{x} = \mathbf{x}$. According to Montario 71 to know the abargeteristics of an orbit it A periodic orbit is established when $x_i = f^{(n)}(x_i)$, that is, $x_{n+m} = x_n$. According to Monterio [7] to know the characteristics of an orbit it as follows: t_{max} as t_{max} as t_{max} $f(x_n)$ ou $x_{n+1} = f^{(n)}(x_1)$
it is established when $x_i = f^{(n)}(x_i)$, that is, $x_{n+m} = x_n$. According to Monterio[7] to know the characteristics of an only its eigenvalue λ , which corresponds to the product of each eigen

$$
\lambda^{11} = \frac{df^{(n)}(x_i)}{dx}|_{x_i^*} = \frac{df(x)}{dx}|_{x_1^*} \quad \frac{df(x)}{dx}|_{x_2^*} \quad \dots \quad \frac{df(x)}{dx}|_{x_i^*}
$$

By deriving the equation (9) we obtain:

$$
\frac{df(x)}{dx} = \frac{1}{2}[(-5(\cos^2(x\frac{\pi}{2}))) + (\pi(5x+2)(\sec(x\frac{\pi}{2}))(\cos(x\frac{\pi}{2}))) + 6]
$$
(10)
ues Comput Math, 2024
Volume 3 | Issue 10

It can be seen that the expression $(\operatorname{sen}(x\frac{\pi}{2})\cos(x\frac{\pi}{2}))$ will always be zero for $x_i \in \mathbb{N}^*$ consequently cancelling the term that multiplies, in this way for the proposed purposes it is possible to calculate λ_j through the following expression: $\lambda_j = \frac{df(x_i)}{dx}|_{x_i^*}$ or
executing to equation (10) modified to $x \in \mathbb{N}$. according to equation (10) modified to $x_i \in \mathbb{N}^*$: equation (10) modified to *xⁱ* ∈ N∗: *cough the followir π* 2 multiplies, in this way for the proposed purposes it is possible to calculate λ_j through the following expression: $\lambda_j = \frac{df(x_i)}{dx}|_{x_i^*}$ or It can be seen that the expression $(sen(x_2^{\pi})cos(x_2^{\pi}))$ *ⁱ* or according to ²)*cos*(*x^π* ²)) will always be zero for *xⁱ* ∈ N[∗] $c(x\frac{\pi}{2})cos(x\frac{\pi}{2})$ will always be zero for $x_i \in \mathbb{N}^*$ consequently cancelling the term that equation (10) modified to *xⁱ* ∈ N∗:

consequently cancelling the term that multiplies, in this way for the proposed purpose \mathcal{L}_max

$$
\lambda_i^{12} = \frac{1}{2} [(-5(\cos^2(x_i^* \frac{\pi}{2}))) + 6]
$$
\n(11)

²)) will always be zero for *xⁱ* ∈ N[∗]

²)*cos*(*x^π*

It can be seen that the expression (*sen*(*x^π*

vn periodic orbit $4 \rightarrow 2 \rightarrow 1 \rightarrow 4$ we proceed to calculate the eigenvalue λ for this orbit: the known periodic orbit $4 \rightarrow 2 \rightarrow 1 \rightarrow 4$ we proceed to calculate the eigenvalue λ for this orbit: Considering the known periodic orbit $4 \rightarrow 2 \rightarrow 1 \rightarrow 4$ we proceed to calculate the eigenvalue *i* \rightarrow 4 we proceed to calculate the eigenvalue λ for this orbit: Considering the known periodic orbit $4 \rightarrow 2 \rightarrow 1 \rightarrow 4$ we proceed to calculate the eigenvalue λ for this orbit:

$$
\lambda = \lambda_{(x^*=4)} \times \lambda_{(x^*=2)} \times \lambda_{(x^*=1)}
$$
 or $\lambda = \frac{1}{2} \times \frac{1}{2} \times 3 = \frac{3}{4}$

Also according to Monteno_[1] as $x > 1$ the orbit of period 5 (4 \rightarrow 2 \rightarrow 1 \rightarrow 4) is stable, which allows us to state that such a cycle repeats indefinitely once any of the points belonging to the orbit is reached. $\sum_{i=1}^{n}$ is stable, which allows us to state that such a cycle repeats in $\sum_{i=1}^{n}$ Also according to Monteiro[7] as λ < 1 the orbit of period 3 (4 \rightarrow 2 \rightarrow 1 \rightarrow 4) is stable, which allows us to state that such a cycle repeats indefinitely once any of the points belonging to the orbit is reached.

The sequence produced by the Collatz Conjecture for $\forall x_i \in \mathbb{N}^*$ being xi < 268[10] will end upon reaching the fundamental orbit 4 \rightarrow The set of the set of the sequence of λ and λ and λ are λ and λ The sequence produced by the Conatz Conjecture for $v x_i \in N^*$ being $x_i > 200$ [10] will end upon reaching the fundamental orbit 4 \rightarrow
2 \rightarrow 1 since this is stable, we will see in the next subsections if there are other Here are some observations about the orbit $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$ $2 \rightarrow 1$ since this is stable, we will see in the next subsections if there are other possible orbits in the Collatz Conjecture.
Here are some observations about the orbit $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$

 $\sum_{i=1}^{n} a_i$ $\emph{grafo 1 - ciclo fundamental}$

grafio 1 - civil 1 - civil all the third elements of the third elem The Collatz conjecture as seen in (1) is formed by the first three prime numbers 1, 2, 3 $\in \mathbb{N}^*$, with the main pole being the number result will be continuously divided by TWO (while even). Note that the number ONE is the smallest that can be 'calculated' by the conjecture after several divisions by TWO. The number FOUR $(4 = 3 \times 1 + 1)$ constitutes the ONE, this independent term having the function of making any odd number pre- viously multiplied by THREE even, the even divided by TWO generates the third element of the same orbit, this in turn 'ends' in ONE. 11 This definition for *λ* is identical to the book cited in *[7]* of the book cited in **[7]** of the book cited in **[7]** the conjecture after several divisions by TWO. The number FOUR $(4 = 3 \times 1 + 1)$, constitutes the other pole of the orbit that when

¹² Notar que as derivadas da Conjectura de Collatz conforme sistema (1) são as mesmas obtidas por esta equação. **4.1.2 Limiting Case** $x_i \to \infty$ for analysis purposes *x*¹ = 295147905179352825857 the program *r*_*collatz*(*x*1) provides as

 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ is identical to the book control to the book cited in $\frac{1}{2}$ is identical to the book control in the book cited in $\frac{1}{2}$ is the book control to the book control in the book c Note that the initial cycle will be $3 \times x_1 + 1$, the independent term ONE is much smaller than the product $3 \times x_1$, $(1 \ll (3 \times x_1))$, $\lim_{t \to \infty} \frac{1}{t} \to \infty$, that is, is a very large number 15, making $\lambda_1 = 2(00)$ + 1 for analysis purpose r condiz (x_1) provides as answer [502, 191, 571] namely, Cycles – 502, Odd (fising) – 191 and Even (family) –
2 initial axels will be $2 \times x_1 + 1$, the independent term ONE is much smaller than the moduat $2 \times y_1$, $(1 \le \$ the program $r_collatz(x_1)$ provides as answer [562, 191, 371] namely, Cycles = 562, Odd (rising) = 191 and Even (falling) = 371. 4.1.2 Limiting Case $x_i \to \infty$
Considering that $x_i \to \infty$, that is, is a very large number 13, making $x_1 = 2(68) + 1$ for analysis purposes x1 = 295147905179352825857 and can be neglected14 for study purposes, in this way the conjecture will be simplified to according to the following sequence:

$$
x_2 \simeq \frac{3}{2^{\gamma_1}} \times x_1
$$

\n
$$
x_3 \simeq \frac{3}{2^{\gamma_2}} \times x_2 \text{ or } x_3 = \frac{3}{2^{\gamma_2}} \times \frac{3}{2^{\gamma_1}} \times x_1 \text{ in short: } x_3 = \frac{3^2}{2^{(\gamma_2 + \gamma_1)}} \times x_1
$$

\n...
\n
$$
x_n \simeq \frac{3^{n-1}}{2^{(\sum_{i=1}^{n-1} \gamma_i)}} \times x_1 \text{ as seen previously in the equation (5):}
$$

\n
$$
x_n \simeq x_1 * 3^I * \prod_{i=1}^I (\rho_i) \text{ or even } x_n \simeq \frac{x_1 * 3^I}{2^P}
$$

Starting from $x_1 = 295147905179352825857$ we obtain: $x_n \simeq \frac{x_1 * 3^{191}}{2^{371}}$

2*P*

By evolving the previous expression we obtain:

*x*ⁿ ≈ 2(191∗log2(3)−371)

 $x_n \simeq x_1 \times 2^{(191 * \log_2(3) - 371)}$

 (A) -

*x*ⁿ *x*¹ *X*² *I* ∞ 3*I* ∗ 3*I* ∗ 3*I*

...

 $x_n \simeq (2^{(68)} + 1) \times 2^{(-68.27216236225917)} \simeq 0.8280774634724271$

ⁱ=1(*ρi*) or even *^xⁿ* [≃] *^x*1∗3*^I*

The same result can be verified through the function:

*abcd(*2⁶⁸ + 1) = (*x*1*, A, B, C, A* + *B* + *C* + *D)* which presents the following output: $abcd(268 + 1) = (x1, A, B, C, A + B + C + D)$ which presents the following output:

(295147905179352825857, 0.8280774634724271, 9.352118592531982e-22, 0.10942253652757464, 0.0625, 1.00000000000000018)

As previously mentioned, 13 numbers \leq 268 were experimentally tested for the Collatz Conjecture and all of them invariably ended in the cycle $4\rightarrow2\rightarrow1$, empirically demonstrating the non-existence of another repeating cycle other than the trivial, therefore if any other cycle exists, it must have as its origin numbers greater than 268 and of course have its fixed points all greater than 268, $(\text{xk} > 268, x_k \in \mathbb{N}^*)$, however it was found that such a statement $(x_k = x_n \approx 0.8280774634724271)$ shows that when $x_i = 2(68) + 1$ is obtained, $x_n \le 1$ is obtained, thus evidencing the decrease of the 'series'.

In addition and based on the equation (4) and its alternative form 15 below equation (12), assuming the existence of xk a fixed point of a repetitive cycle we obtain the following equation (13):

$$
x_k * 3^I * \prod_{i=1}^I (\rho_i) + \underbrace{\sum_{j=I-1}^0 \left(3^j * \left[\prod_{i=(I-j)}^I (\rho_i) \right] \right)}_{(B)+(C)+(D)} = x_k
$$
(12)
(A) = Ψx_k , (B) + (C) + (D) = Ω $\Psi x_k + \Omega = x_k$

h terms by x_k results in: $x_k^2(\Psi - 1) + \Omega x_k = 0$. $\cdots \omega_k$ $\left(\begin{array}{cc} x & 1 \end{array} \right)$ | $\omega \omega_k$ – 0. *"As of 2020, the conjecture has been checked by computer for all starting values up to* 10²⁰*"* [2] Multiplying both terms by x_k results in: $x_k^2(\Psi - 1) + \Omega x_k = 0$. \mathcal{L} *x*_k results in: \mathcal{L} results in: \mathcal{L} Multiplying both terms by x_k results in: $x_k^2(\Psi - 1) + \Omega x_k = 0$. $-$ 1*xk*, (*D*) + (*C*)

Therefore, in addition to the trivial root $x_k = 0$ the other root will be (this valid one): Therefore, in addition to the trivial root $x_k = 0$ the other root will be (this valid one):

$$
x_k = \Omega\left(-\frac{1}{\Psi - 1}\right), \quad \text{remembering that } -\frac{1}{\Psi - 1} = \frac{1}{1 - \Psi} \text{ we obtain:}
$$
\n
$$
\Psi x_k + \Omega = x_k \quad \text{ou} \quad x_k = \frac{\Omega}{1 - \Psi} \tag{13}
$$

bers greater than 2⁶⁸*,*(*x*[∗]

ⁱ in

ⁱ in

*x*² it is also observed that in accordance with wh Which can also be obtained from $\psi_{x_k} + \Omega = x_k$, it is also observed that in accordance with what was seen previously 16 0 < Ψ < 1, and $\Omega \ge 0$, consequently the terms (A), (B), (C) and (D) are all positive. equently the terms (A) , (B) , (C) and (D) are all positive. $\Omega \ge 0$, consequently the terms (A), (B) , (C) and (D) are all positive. which can also be obtained 1
 $Q \ge 0$, consequently the term

2) we obtain that $w = 3^l * \Pi^l$ (a) or even $\psi = \frac{3^l}{2^l}$ seen previously we know that $0 < \Psi < 1$ in this wa when (12) we obtain that $\psi = 3 \times 11$ ψ_i of even $\psi = 2^i$, seen previously, we know that $0 \le 1 \le 1$, in this was $0 \le 2^{i}$, ψ_i and $0 \le 2^{i}$, (2), for it to be true (that is: valid for bie 1, also verifying that according to $x \to \infty$ we have that $\psi = \frac{1}{2}$ $\to 0$ (without ever being zero). uation (12) we obtain that $w = 3^l * I$ $\frac{F}{F}$ < 1, adopting the base two we rewrite it in the following way $0 < 2^{\frac{1}{(4\log(2\epsilon)^{3/2})}}$ / < 1 (where P represents site be true (that is: valid for $\forall x \in \mathbb{N}^*$) we have that $(I * \log(3) - P) < 0$ or even, $I * \log(3$ previously, we know that 0 *<* Ψ *<* 1, in this way we can write that: 0 *<* ³*^I* ²*^P <* 1, adopting the base two we rewrite it in the following way 0 *<* 2(*I*∗log2(3)−*P*) *<* 1 (where P $\frac{d}{dx}$ (10) $1/(1-x)$ $\frac{d}{dx}$ $\frac{d}{dx}$ $\frac{3}{x}$ $\frac{3}{x}$ $\frac{1}{x}$ $\frac{1}{x}$ $\frac{d}{dx}$ (10) $\frac{d}{dx}$ (1) represents simple division of the base two we rewrite it in the following way $0 < 2^{(k \log(2\beta)-P)} < 1$ (where P represents simple $0 < 2^{(k \log(2\beta)-P)} < 1$ (where P represents simple p) (*I* $\frac{1}{2^p}$ \leq 1, also ping the state the meteorities in the sensing may $\frac{1}{2^p}$ \leq 11 (meteor represents sm), for it to be true (that is: valid for $\forall x_i \in \mathbb{N}^*$) we have that $(I * log_2(3) - P) < 0$ or even, en in table 1, also verifying that according to $x^* \to \infty$ we have that $\Psi = \frac{3^I}{2^P} \to 0$ (without ever being zero). From the equation (12) we obtain that $\psi = 3' * 11'_{i=1}(\rho_i)$ or even $\psi = 2^p$, seen
can write that: $0 < \frac{3^p}{2} < 1$ adopting the base two we rewrite it in the followir divisions by 2), for it to be true (that is: valid for $\forall x \in \mathbb{N}^*$) we have that $(I * \text{!})$ and the base two we rewrite it is the form $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ ($\sum_{i=1}^{n}$) $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ represents simple divisions by 2), for it to be true (that is: valid for ∀*xⁱ* ∈ N∗) we have that From the equation (12) we obtain that $\psi = 3^I * \Pi_{i=1}^I(\rho_i)$ or even $\psi = \frac{9}{2^P}$, seen previously, we know that $0 < \Psi < 1$, in this way we can write that: $0 < \frac{3^p}{2^p} < 1$, adopting the base two we rewrite it in the following way $0 < 2^{(\mu \log(3)-P)} < 1$ (where P represents simple divisions by 2), for it to be true (that is: valid for $\forall x_i \in \mathbb{N}^*$) we have that $(I * log_2(3) - P) < 0$ or even, $I * log_2(3) < P 17$. This limit can also be seen in table 1, also verifying that according to $x^* \to \infty$ we have that $\Psi = \frac{3^2}{2^2} \to 0$ (without ever being zero). From the equation (12) we obtain that $\psi = 3^{l} * \prod_{i=1}^{l} (\rho_i)$ or even $\psi = \frac{3^{l}}{2^{p}}$, seen can write that $0 < \frac{3^I}{2^P} < 1$, adopting the base two we rewrite it in the following divisions by 2), for it to be true (that is: valid for $\forall x_i \in \mathbb{N}^*$) we have that $(I * log$ can also be seen in table 1, also verifying that according to $x^* \to \infty$ we have that *f* From the equation (12) we o $\text{can write that: } 0 < \frac{3^I}{2^P} < 1,$ divisions by 2), for it to be to represents on also be seen in table 1, a

ⁱ), which take *x*[∗]

ⁱ), which take *x*[∗]

an increasing way up to the maximum value of the maximum value of the maximum value of the maximum value of th

grafo 2 - any closed cycle

Exists it must have as fixed points x^*_{i} numbers greater than 2^{68} ($x^*_{i} > 2^{68}$, $x^*_{i} \in \mathbb{N}^*$) can be seen that in this cycle there 'exists' amaximum value $x^*_{i} = x^*_{max}$ being the result of the transformations $f^k(x^*)$, which take x^* in an increasing way up to the maximum value of the cycle. Similarly, the transformations $f^{k+j}(x_{max}) = x_{n-1}$ and finally $f^n(x_{n-1}) = x_i$, lead to a decrease in the maximum value Consider the figure to the side (graph 2) which represents any closed (repetitive) cy- cle, as suggested previously if such a cycle x^*_{max} . It is clear that the 'largest divisions by two' are obtained by such transformations f^{k+j} and f^n from the values $\frac{1}{2^{p_i}}$, where each value of ρ is part of the vector ρ_i .

series of values for $\Omega = (B) + (C) + (D)$ does not include the initial term x_i , depending only on the values of **I** and **P**. Still according to equation (12) the term Ω tends towards 'small' finite values when compared to x_i (> 2⁶⁸), it can be seen that the

For demonstration purposes, consider the Sequence formed by 10 ascents and 10 descents presented below for the number 57, r_conaiz1(37) = [32, 10, 22], [2, 1, 2, 2, 4, 1, 1, 2, 3, 4] 18
collatz_d(57) = [57, 172, 43, 130, 65, 196, 49, 148, 37, 112, 7, 22, 11, 34, 17, 52, 13, 40, 5, 16, 1] ror demonstration purposes, consider the sequence formed by To assems and To descense presented below for the namber 37,
remembering as observed for Lagarias [6] the cycles of ascents $(3x_i + 1)$ are the same number as tho r_collatz1(57) = [32, 10, 22], [2, 1, 2, 2, 4, 1, 1, 2, 3, 4] 18
 $\frac{11}{2}$ (67) = [57, 172, 42, 120, 65, 106, 40, 140, 27, 112, 7, 22, 11, 24, 17, 52, 12, 40, 5, 16, 11 the cycles of ascents (3*xⁱ* + 1) are the same number as those of descents (division by 2*^ρ*): the cycles of assume the cycles of ascents (3*x* + 1) are the same number remembering as observed for Lagarias (6) the evolus of ascents $(3x + 1)$ are the same number the cycles of ascents (3*xⁱ* + 1) are the same number as those of descents (division by 2*^ρ*):

Expanding the equation (12) using the previous values we have:

$$
\underbrace{\frac{x_k*3^{10}}{2^{22}}}_{(A)} + \underbrace{\frac{3^9}{2^{22}}}_{(B)} + \underbrace{\frac{3^8}{2^{20}} + \frac{3^7}{2^{19}} + \frac{3^6}{2^{17}} + \frac{3^5}{2^{15}} + \frac{3^4}{2^{11}} + \frac{3^3}{2^{10}} + \frac{3^2}{2^9} + \frac{3^1}{2^7}}_{(C)} + \underbrace{\frac{3^0}{2^4}}_{(D)}
$$

Using the function $abcd(57)(x_i = 57)$ we obtain:

$$
\underbrace{0.8024675846099856}_{(A)} + \underbrace{0.004692792892456051}_{(B)} + \underbrace{0.1303396224975586}_{(C)} + \underbrace{0.0625}_{(D)}
$$

where $\Omega = 0.19753241539001465$, as is evident $\Omega \ll x_i$.

Taking into account the complexity of the vector $\rho i = \left[\frac{1}{2^2}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^2}, \frac{1}{2^4}, \frac{1}{2^1}, \frac{1}{2^1}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}\right]$, which are the divisors responsible Taking this account the complexity of the vector $p_1 = \frac{1}{2^2}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \frac{1}{2^4}, \frac{1}{2^4}, \frac{1}{2^5}, \frac{1}{2^6}, \frac{1}{2^7}, \frac{1}{2^8}, \frac{1}{2^9}, \frac{1}{2^9}, \frac{1}{2^9}, \frac{1}{$ therefore the study is carried out based on the average value between ascents and descents which, according to Lagarias [6], points to a value lower than the unit $\left(\longrightarrow \frac{3}{4}\right)$.

according to the unit (−→ 3).
Cordinate the unit (−→ 3), points to a value of the unit (−→ 3), points the unit (−→ 3). $\frac{1}{2}$ account 'as an average value' $\rho_i = \log_2(3)$ we obtain the following equation (14): *P* ⩾ *I* × log2(3), rewriting the equation (12) taking into account 'as an average value' Previously it was shown that the minimum limit between **I** and **P** is $log_2(3)$ or $P \ge I \times log_2(3)$, rewriting the equation (12) taking into *P* ⩾ *I* × log2(3), rewriting the equation (12) taking into account 'as an average value'

i.

I

²*^P* → 0 (without ever being zero).

$$
\underbrace{\frac{x_k * 3^I}{2^{(I*\log_2(3))}}}_{(A)} + \underbrace{\sum_{j=(I-1)}^0 \left(3^j * \prod_{i=(I-j)}^I 2^{-(I*\log_2(3))}\right)}_{(B)+(C)+(D)} = x_k
$$
\n(14)

evolving it we have:

$$
\frac{x_k * 3^{10}}{2^{(10*\log_2(3))}} + \frac{3^9}{2^{(10*\log_2(3))}} + \frac{3^8}{2^{(9*\log_2(3))}} + \dots + \frac{3^1}{2^{(2*\log_2(3))}} + \frac{3^0}{2^{(1*\log_2(3))}} = x_k
$$
\nor:
\n
$$
\underbrace{x_k}_{(A)} + \frac{1}{3} + \frac{1}{3} + \dots + \frac{1}{3}}_{(B)} + \frac{1}{3} \underbrace{\frac{1}{3}}_{(C)} \text{ or even: } \underbrace{x_k}_{(A)} + \sum_{j=1}^{I} \left(\frac{1}{3}\right)}_{(B)+(C)+(D)} = x_k
$$

Previously it was shown that the minimum limit between **I** and **P** is log2(3) or

 $\left(\frac{I}{3}\right)$, which expresses an inc in short: $x_k = x_k + (\frac{l}{3})$, which expresses an incoherent relationship, unless I were equal to zero, but as previously mentioned $I =$ equal to zero¹⁹, but as previously mentioned *I* = 10. 10 [19]. in short: $x_k = x_k + \left(\frac{I}{3}\right)$, which expresses an incoherent relationship, unless I were equ equal to $[19]$. $x_k + \left(\frac{I}{2}\right)$, which expresses an incoherent relationship, unless I were equal to equal to zero¹⁹, but as previously mentioned *I* = 10.

The function *equation* $12(x_i, type) = (x_i, \psi, A, \Omega, \frac{\Omega}{x_i}, x_n)$ (coded in Python) returns the following where $(tipo = 3 \implies \rho = j \times$ $\log_2(3)$):
(57, 1.0000000000000007, 57.000000000000036, 3.333333333333335, 0.058479532163742715, 60.3333333333333337) $log₂(3)$): $\log_{3}(3)$: The function *equation_12(x_i, type)*= $(x_i, \psi, A, \Omega, \frac{\Omega}{x_i}, x_n)$ (coded in Python) returns the

compatible with such a statement as $\Omega \ll x_i$ or x^* .

 $\frac{1}{\sqrt{2}}$ such that $\frac{1}{\sqrt{2}}$ such that $\frac{1}{\sqrt{2}}$ or $\frac{1}{\sqrt{2}}$ or orbit, we have: $x^* = \frac{\Omega}{1-\Psi}$, assuming $x^* \to \infty$: $\Psi^{20} = \frac{3^*}{2^2} \to 0$ (without reaching zero). 13 that: Suppose that \exists is an orbit in the Collatz conjecture such that $x_k = x^*$ is a fixed point of the orbit, we have: $x^* = \frac{\Omega}{\Omega}$, assuming $x^* \to \infty$: $\Psi^{20} = \frac{3^2}{\Omega^2} \to 0$ (without reaching zero), it results from the equation (13) that: Suppose that \exists is an orbit in the Collatz conjecture such that $x = x^*$ is a fixed point of the orbit, we have: $x^* = \frac{\Omega}{\Omega}$ assume Suppose that ∃ is an orbit in the Collatz conjecture suc point of the orbit, we have: $x^* = \frac{32}{1-\Psi}$, assuming $x^* \to \infty$ \therefore $\Psi^{20} = \frac{1}{2}$ uppose that \exists is an orbit in the Collatz conjecture such that $x_k = x^*$ is a fixed point of the orbit, we have: $x^* = \frac{\Omega}{1-\Psi}$, assuming $x^* \to \infty$ ∴ $\Psi^{20} = \frac{3^I}{2^P} \to 0$ (without reach Suppose that \exists is an orbit in the Collatz conjecture such that $x_k = x^*$ is a fixed point of the orbit, we have: $x^* = \frac{\Omega}{1-\Psi}$, assuming $x^* \to \infty$ ∴ $\Psi^{20} = \frac{3^I}{2^P} \to 0$ (without reflection (13) that: $\sum_{i=1}^{\infty}$ is the Collatz conjecture such that $x = x^*$ is a fixed t ∃ is an orbit in the Collatz conjecture such that $x_k = x^*$ is a fixed
orbit, we have: $x^* = \frac{\Omega}{1-\Sigma}$, assuming $x^* \to \infty$ ∴ $\Psi^{20} = \frac{3^I}{2^P} \to 0$ (without reaching zero), it results Figure 2. it reaches $x = \frac{1}{1-\Psi}$, dissuming $x \to \infty$. $\mathbf{r} =$ (13) that:

$$
\lim_{(x^*,\Psi)\to(\infty,0)} \left(\frac{\Omega}{1-\Psi}\right) = \Omega
$$

$$
\lim_{x^*\to\infty} (x^*) = \Omega
$$

N that satisfies the equation (13) , thus that $\frac{1}{x}$ and $\frac{1}{x}$ and $\frac{1}{x}$ is the equation (13), the equation (13), the equation (13), thus $\frac{1}{x}$, thus $\$ that $\frac{1}{x}$ are conclude that: However as seen previously $x^* \to \infty$ consequently $0 \le \Omega \le x^*$ it is again evident that \sharp any value of $x \ne 1$, $x^* \ne 2$, $x^* \ne 4 | x^*21 \in \mathbb{N}$ that satisfies the equation (13), thus that ∄ any value of *^x*[∗] ̸= 1*, x*[∗] ̸= 2*, x*[∗] ̸= 4 [|] *^x*∗²¹ [∈] ^N that satisfies the equation (13), thus

we conclude that:

In the Collatz Conjecture there is only one limit cycle formed by the stable points: $1 \rightarrow 4 \rightarrow 2 \rightarrow 1^{22}$.

5. Stochastic Models, Deterministic Process!

The Collatz sequence has been described in several texts under different names, one of which is: *Hailstone Numbers* [5], just as hailstones in clouds before being precipitated go through several 'ascents' and 'descents', the numbers jump from one place to another before reaching the final cycle $4 \rightarrow 2 \rightarrow 1^{23}$.

Série de Collatz: serie collatz log(753257,10,1)

before reaching the final cycle 4→2→1²³.

Figure 7: Collatz sequence for $x_1 = 753257$

Figure 7 presents an example when $x_1 = 753257$, you can see the Natural sequence, upper right graph, and next to it the log_{10} of the same sequence, below we see the sequence in a spider web graph and in detail the cycle $4\rightarrow 2\rightarrow 1$ [24,25]. Several attempts to understand the Collatz sequence through computer simulations point to the previously seen final cycle $4\rightarrow2\rightarrow1$,

It can be seen that when the cycles approach the end (in this case 110 iterations) the response tends towards the final cycle $4\rightarrow 2\rightarrow 1$, the negative coefficient (-0.0523) of the approximated line stands out in the logarithmic graph, which causes the successive values of x_i to decrease, the same normalized coefficient $10^{-0.0523} = 0.8865...$ shows that being less than ONE in the successive iterations the value of xi should decrease. In fact, the 'progression' factor of this 'apparent' series is on average less than unity, according to predictions made by Lagarias [6] thus converging in successive iterations to the final cycle $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$.

numbers; however, numbers $xi \in \mathbb{N} | x_i \to \infty$ allow for a complementary approach using probabilistic models, presenting significant results that point to this conclusion: *series* (α), independent term (α), independent term (α), in In item 4.1 it has been demonstrated that the sequence contains only one cycle $4\rightarrow 2\rightarrow 1$, which is sufficient proof as such for all

".. a basic probabilistic model of iterations of the function $3x + 1$ proposes that most trajectories for iterations $3x + 1$ have equal *numbers of even and odd iterations*" [6](translated by the author). differently from what was exposed we use two limit lines (in red) the upper one being the function 3 × *xⁱ* + 1

In the items 2 and 3.1 we saw that the number of ascents is identical to the number of descents, that is, the number of odd numbers is the same as the number of even numbers (equations (2) and (5)), Lagarias also presents this fact $[26]$.

The following items present in a simplified manner some subsidies linked to statis- tical processes that infer a similar conclusion.

6. Relationship between Even and Odd cycles

The following image contains four graphs generated from the function *nucleo1*() and *nucleo2*() present in the main code (main.py)

detailed in appendix A.

Figure 8: Graphs P x I

Fig. 8 – Graphs *P* x *I* given x_1 to reach *ONE*, while the internal lines (in red) are obtained as the best (linear) approximation to the cited points; the two external lines (in green) limit the values within a specific region that includes all the points in oute of the studied base, that is, the confidence index in this region is 100%. The approximation lines (in red) have thei In the last graph, we can see a strong convergence between the values $P \times I$, showing a relationship between them. cycles for a given \mathcal{O} to reach the internal lines (in red) and internal lines (in red) are obtained as the internal lines (in red) and internal lines (in red) and internal lines (in red) as the internal lines (in red The fourth graph shows all the points (*P xI*) that are part of the studied database²⁷, the first three detail excerpts from the fourth graph. It can be seen that the points (in blue) represent the total number of *P* even cycles and the respective number of *I* odd cycles for a $\sum_{n=1}^{\infty}$ of Each ONE, while the internal lines (in Eq. at botanica as the best (linear) approximation to the cited points, the two external lines (in green) limit the values within a specific region that includes al even points (blue) *P*x*I* meet (referring to the study base).

7. Limit Region

 \mathbf{p}) approximation to the cited points; the two external lines (in green) lines (in green) lines (in green) limit the two external lines (in green) The two straight line segments in green represent the limit region where all the even points (blue) $P \times I$ meet (referring to the study has base). The straight lines have the following components: *R* is a performent the limit region where all the even points (blue) P x I meet (referring to the steps)

$$
R_s(\text{reta superior}) \longrightarrow 0.500000 \ast P + 189.304 \tag{15}
$$

$$
R_i(\text{reta inferior}) \longrightarrow 0.500000 \ast P - 210.500 \tag{16}
$$

means that for a certain number of cycles P we will obtain within the range presented the value of $\sim I \pm 200$ points. It can be seen that they are in fact parallel (same angular coefficients) and the width (spacing) between them is ≃ 400 points, which

ONE indicating adherence of the Linear model to the presented distribution: From the presented adjustment equation (last of the four graphs) we have a correlation coefficient of $R^2 = 0.999607$, very close to

$$
Ciclos_{Impares} = 0.500000 \cdot Ciclos_{Pares} + 0.194725 \tag{17}
$$

Normal distribution of the numbers $\in \mathbb{N}^*$ as will be seen later. It should be noted that although the precision is not absolute, as the equations work with Real numbers subject to rounding, the cycles P, I are still *P* with with the range presented the value of *D* with a value of *I* ± 200 points. *C* and *L* + 200 points. *C* a equations work with Real numbers subject to rounding, the cycles P , I are still positive natural numbers, and the Collatz sequence is deterministic, despite the lack of knowledge about the evolution of the same sequence in relation to all possible numbers $\in N^*$,
 T Math Techniques Comput Math. 2024 It can be seen that the correlation between the number of P even cycles and *I* odd cycles ($I \approx 0.5 \times P$) is in accordance with the these approximations and the use of statistical methods are used. The equation (9) does not assertively indicate that ∀ number xi will end in um after the Total Cycles (=P+I) however the terms A,B,C and D shown in the code in Figure 5 or the equivalent terms (A), (B), (C) and (D) present in the equation (4) provide resources for an approximation based on the initial number xi and possible points $I \times P$, $(C = I + P)$.

It is also noted that the limit lines can be better defined by adopting smaller segments for the values x_i .

In the following graphs (Figure 9) it is possible to see for the studied database the Histograms [8] relating to term A of the equation (4), the relationship between odd and even cycles (even cycles include all cycles in which there is a simple division by two) and it is also observed that the number of odd cycles is smaller than the number of even cycles (total), and in the last graph that any cycle (in the studied database) has more than 60% of even cycles. It is important to note that the value *Amax* is obtained for *xⁱ* = 87381*, I* = 1*, P* = 18, base) has inote than 00% or even eyeres.

It is important to note that the value A_{max} is obtained for $x_i = 87381, I = 1, P = 18$, this combination also generates the smallest relation I/P present in the studied base, on the other hand it represents the largest relation P/C as can be seen in the last graph of Figure 9.

It is also observed that the value of A which depends on x_1 accounts for more than 79% of the value of x_n . bserved that the value of *x* It is also observed that the value of **A** which depends on *x*¹ accounts for more than 79%

Série de Collatz: A, ciclos I/P, e ciclos P/C

8. Maximum value for *xi*

will be less than this maximum x_m , it can be concluded that the maximum value for *x*^{*i*}, $d(7)$ produces the following output: $\left[\frac{11}{7}, \frac{11}{7}, \frac{12}{7}, \frac{12}{7}, \frac{13}{7}, \frac{10}{7}, \frac{10}{7}, \frac{11}{7}, \frac{10}{7}, \frac{10}{7}, \frac{11}{7}, \frac{10}{7}, \frac{10}{7}, \$ $\textit{collatz}_d(7) = [7, 22, 11, 34, 17, 52, 13, 40, 5, 16, 1]$ It is assumed that in the Collatz Sequence there is a maximum value where $x_i = x_m$ from which any subsequent (or previous) value will be less than this maximum x_m , it can be concluded that the maximum value is even, and it will be divided by a number 2^{*y*}.

The number 52 corresponds to the maximum (in this sequence), which will later be divided by $2^y = 22 = 4$ becoming the odd number 13. Once a maximum is reached, the subsequent numbers will necessarily be smaller, evidencing a decrease in the series. If the same series were only of increases, it would not be limited in a final cycle, that is, it would tend to infinity beyond what is observed that than ONE, thus converging in successive iterations to the final cycle $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$. according to Lagarias [6] heuristic predictions made using tested probabilistic models indicate that this factor is on average lower

of natural number $\mathbb{N} = \{0, 1, 2, 3\}$ and the set of non-negative even numbers $\mathbb{D} = \{0, 2, 4, 6, \ldots\}$ it contains to the maximum of the m seen that both have the same Car- dinality 28 | N $=$ \mathbb{P} $=$ \aleph_0 [9], the same occurs with the set of numbers that are powers of $2\mathbb{E}_2$ = $\{0, 2, 4, 8, 16, ...\}$, the set of odd numbers I also has the same cardinality of N. In short, when dealing with infinite (countable) sets that have the same cardinality ($|N| = |P| = |\mathbb{E}_2| = |\mathbb{I}| = \aleph_0$), it follows that the probability distribution for \forall and the number xi present Considering the set of natural numbers $N = \{0, 1, 2, 3, ...\}$ and the set of non-negative even numbers $\mathbb{P} = \{0, 2, 4, 6, ...\}$, it can be
seen that both have the same Car, dinality $28 \mid N \mid \text{= } \mathbb{P} = \mathbf{x}$. [0] the same o infinite countable sets). In the previous item, it was seen that every Collatz series has a

in these sets are equivalent, since the same sets are equipotent, making any prediction difficult to make or that it presents objective In these sets are equivalent, since the same sets are equipotent, making any prediction difficult to make or that it presents objective
trends (taking into account infinite countable sets). In the previous item, it was se $4 \rightarrow 2 \rightarrow 1$ and consequently a maximum establishing for each series a particular set of finite numbers, having the same amount of odd and even numbers [6] (before reaching the number 1). final cycle 1 \sim 1 n these sets are equivalent, since the same sets are equipotent, making any prediction difficult to make or that it presents ob ne countable sets). In the previous hem, it was seen that every contain series has a m

Consider also the set $\mathbb{D}_7 = [7, 22, 11, 34, 17, 52, 13, 40, 5, 16]29$ whose cardinality is: \mathbb{D}_{η} = 10, it is clear that the amount of even numbers is identical to the amount of odd numbers present in \mathbb{D}_{η} , both subsets (even \mathbb{D}_{γ_P} and odd \mathbb{D}_{γ}) have the same finite cardinality, in item 2 previously seen and adapted here we have: *xⁱ* ∈ D⁷*^P* = [22*,* 34*,* 52*,* 40*,* 16] = [*x*2*, x*4*, x*6*, x*8*, x*10] | *i* = {2*,* 4*,* 6*,* 8*,* 10}

$$
x_i \in \mathbb{D}_{7I} = [7, 11, 17, 13, 5] = [x_1, x_3, x_5, x_7, x_9] \mid i = \{1, 3, 5, 7, 9\}
$$

$$
x_i \in \mathbb{D}_{7P} = [22, 34, 52, 40, 16] = [x_2, x_4, x_6, x_8, x_{10}] \mid i = \{2, 4, 6, 8, 10\}
$$

A fact that allows us to assume that any $x_i \in \mathbb{D}$, has the same probability of occurrence, that is, there is the same number of increases and decreases in the series, however, it is observed that in the decreases or decrements made by divisions by even numbers there are compound divisions, dividing more than once by two, which is in accordance with the observations seen in the same item 2, that is: A fact that allows us to assume that any $x_i \in \mathbb{D}_7$ has the same probability of A fact that allows us to assume that any $x_i \in \mathbb{D}$, has the same probability of occurrence, that is, there is the same number of increases

$$
\rho_i = \frac{x_{(2i+1)}}{x_{(2i)}},
$$
 being: $\rho_i = \left[\frac{1}{2^1}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}\right] = \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right].$

R x i n f in terms of $[C, I]$ and *P i x* and *P i x* where we can see that: $C = I + P$ *and* $P > I$ *. x* and *p*^{*i*} $\frac{1}{2}$ *coulatz*(*x*) (Figure 4) aready presented *n* $\frac{1}{2}$ *,* where we can see that: $C = I + P$, and $P > I$. Therefore, due to the various cumulative divisions, an even number present in the Collatz sequence will be divided more than r_{RIS} and r_{a} (*reg*) where we can see that, $C = r + 1$, and $r + 1$, μ and σ σ_y are named two. The function $\frac{1}{2}$ contr $\frac{1}{2}$ (Figure $\frac{1}{2}$) already presented that is, C_{iclos} , $I_{mparses}$ and P_{ares} , where we can see that: $C = I + P$, and *P*. by the number two. The function $P_{collitz}(x)$ (Figure +) and any press The result of the *result of the function is a result of the terms is a result of the terms and in the terms and in the terms in the terms* $\frac{1}{2}$ part is, and the various cumulative divisions, an even number present in the Conatz and P where we can see that $C = I + P$ and $P > I$ p_{max} correspond to the subsets of the subsets of the series p_{max} is the series of the series p_{max} is the series of the ser re due to the verious cumulative divisions, on even number present in the Colletz sequence will be divided more than once σ y are number two. The function $\frac{1}{2}$ collatz(x) (Figure τ) aready presented the composition that is, $C_{i\n\textrm{clos}}$, $I_{m \text{parse}}$ and P_{ares} , where we can see that: $C = I + P$, and $P > I$. by the number two. The function $r_{collatz}(x)$ (Figure 4) already presented the composition of the sequence in terms of [C, I, P]30, Therefore, due to the various cumulative divisions, an even number present in the Collatz sequence will be divided more than once

 T_{total} T_{total} T_{total} T_{total} T_{total} and T_{area} and T_{area} correspond exactly to the cardinality of the Collatz seq (*i*), that is $|\psi_7| + |\psi_7| + |\psi_8| = 10$ where $|\psi_7| + 5$ and $|\psi_7| + 11$ for the respective sets. function *r* collatz(7) is [16,5,11], the terms $C_{i_{\text{other}}}$, I_{numer} and P_{inter} correspond exactly to the cardinality of The respective sets:
The respective sets:
 $T_2(7) = 76, 12, 10, 20, 10, 5, 16, 8, 4, 21$ The result of the function *r_collatz(7)* is [16,5,11], the terms $C = I$ *m* **Property of the cardinal contract of the corresponding to the subsets of the series** *collatz_seq(7)***, that is** $|C| = |T| + |D| = 16$ **where** $|T| = 5$ **.** of the series *condue* seq(*t*), that is $|\mathcal{C}_7| = |1_7| + |1_7| = 10$ where $\mathcal{C}_7 = [7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 21]$ *P*_p result of the function r collatz(7) is [16.5, 11], the terms \overline{C} I and $\sum_{i \in \{0\}}$ of the series $collatz$ $seq(7)$, that is $\|C\| = \|C\| + \|P\| = 16$ where $\|$ is of the function r collatz(7) is [16.5, 11], the terms $C = I = \text{and } P = \text{corres}$ $\int_{\mathcal{A}} f(z) \cdot \frac{z}{z} \cdot e^{-\alpha(z)} \cdot \int_{z}^{z} f(z) \cdot \frac{1}{z} \cdot \int_{z}^{z} f(z) \cdot \frac{z}{z} \cdot \int_{z}^{z} f(z) \cdot$ The function *r_collatz*(7) is [16,5,11], the terms C *icloses and P_corres* **P**
P_iclos² correspond to the cardinal integration of $\begin{bmatrix} P & P & P & P \\ P & P & P \end{bmatrix}$, the series *collatz_seq(7)*, that is $|C| = \mathbb{I} \cup \mathbb{I} \cup \mathbb{I} \cup \mathbb{I} \cup \mathbb{I} \cup \mathbb{I}$ and $|D| = 11$ is the settes condiz seq (7), that is $|\psi_7| = |\psi_7| = |\psi_7| = 10$ where $|\psi_7| = 5$ and $|\psi_7| = 10$ where $|\psi_7| = 7$ and $|\psi_7| = 10$ *Pares* correspond exactly to the cardinality of the subsets of the series *collatz_seq(7)*, that is the calendary of the subsets and in the respective sets: T_{max} Therefore, T_{max} are the various cumulative divisions, and T_{max} and T_{max} in the respective sets: $t_1 - t_1$ if $\frac{1}{2}$ and $\frac{1}{2}$ if $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ if $\frac{1}{2}$ and $\frac{1}{2}$ and The result of the function *r_collatz*(7) is [16,5,11], the terms *Ciclos*, *I mpares* and *Pares* correspond exactly to the cardinality of the subsets of the series *collatz_seq*(7), that is $|C_7| = |\mathbb{I}_7| + |P_7| = 16$ where $|\mathbb{I}_7| = 5$ and $|\mathbb{P}_7| = 11$ for the respective sets: T_{max} The function $r_collatz(7)$ is [16,5,11], the terms C_{iclos} , I_{mpares} and P_{ares} correspond exactly to the cardinality of the su *collatz_seq*(*f*), that is $|\mathbb{C}_7| = |\mathbb{I}_7| + |\mathbb{F}_7| = 16$ where $|\mathbb{I}_7| = 5$ and $|\mathbb{F}_7| = 11$ for the respective sets:
11. 34. 17. 52. 26. 13. 40. 20. 10. 5. 16. 8. 4. 21.

$$
C_7 = [7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2],
$$

particular set of finite numbers, having the same amount of odd and even numbers [6]

(before reaching the number **1**).

 \mathbb{I}_{7} = [7, 11, 17, 13, 5] and \mathbb{P}_{7} = [22, 34, 52, 26, 40, 20, 10, 16, 8, 4, 2]. P]³⁰, that is, *Ciclos, Impares* and *Pares*, where we can see that: *C* = *I* + *P*, and *P >I*. $\sum_{i=1}^{n}$ is $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ if $\sum_{i=1}^{n}$ is $\sum_{i=1}^{n}$ if \sum P]³⁰, that is, *Ciclos, Impares* and *Pares*, where we can see that: *C* = *I* + *P*, and *P >I*. $\mathbb{I}_7 = [7, 11, 17, 13, 5]$ and $\mathbb{P}_7 = [22, 34, 52, 26, 40, 20, 10, 16, 8, 4, 2].$

rable 2 shows the distribution of numbers that are powers of 2 (ress than 5000) as a function of the exponent γ , remembering that any even number within the series (result of $3 \times x_i + 1$) will necessarily be divided by the non-zero probability that it (result of $3 \times x_i + 1$) will be divided more than once by 2. Table 2 shows the distribution of numbers that are powers of 2 (less than 5000) as a function of the exponent γ , remembering that ance 2 shows the distribution of numbers that are powers of 2 (less than 5000) as tribution of every home that are now can able than 5000, as a function of the cynonenty expression that

The function *nPares(x)* (Figure 10) provides the values from table 2 for $x = 5000$, in the console of the Python: *divisores* = $np.array(nPares(5000))[: 2]$ which will give the following answer:

 $np.array(nPares(5000))$: 2] which will give the following answer:
divisores[0] = array([2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096]) $\langle \cdot, \cdot \cdot \rangle$ ($\langle \cdot \rangle$, $\langle \cdot \$ $(1, 2, 3, 4, 5, 6, 1, 1)$ is identical to the amount of even numbers is identical to the amount of $(1, 2, 3, 1)$

and

Table 2: Even divisors where: $2^{\gamma} \le 5000$

Whose sum (sum(divisors[1])) is 2500, that is, there are 2500 available divisors, these being powers of 2, the function nPaires(numero) whose sum (sum(urvisors $\lceil r \rceil$)) is 2500, that is, there are 2500 available divisors, these being powers of 2, the function in alrea (has $\lceil 2r \rceil$) indicates the original number and, if necessary, the immediately hig P]³⁰, that is, *Ciclos, Impares* and *Pares*, where we can see that: *C* = *I* + *P*, and *P >I*. t - t sequence will be divided more than α and β is the number of t and α t and β is t [2:] indicates the original number and, if necessary, the immediately higher pair (to be used if the number informed be odd). P]³⁰, that is, *Ciclos, Impares* and *Pares*, where we can see that: *C* = *I* + *P*, and *P >I*.

particular set of finite numbers, having the same amount of odd and even numbers [6]

```
\frac{1}{2} def nPares (num) :
          import numpy as np
3 \mid \text{consta} = \text{int}(\text{num})4 #valores =[[" n " ,"C" ," I " ,"P" ," nucleo "]]
 5 valores = []
 6 while (\text{const} > 1):
 7 c_num = str (conta)
8 \mid \text{resp} = \text{divByPot2}(\text{c\_num})\begin{array}{c|c} 9 & \text{values.append} (\text{resp } [1]) \ 10 & \text{contains} \end{array}\text{consta} = 211 if conta \leq 1:
\begin{array}{c|c}\n 12 & \text{break} \\
 13 & \text{matrix}V = np.\n\end{array}matrix V = np.array (values)14 expoente, ocorrencia = np. unique (matrizV, return_counts=True)
15 return expoente, ocorrencia
```
Figure 10: Code: *nPares*()

Previously in table 1 we presented some results of the relationship between I and P, that is: between the number of Odd cycles in \mathbf{e} have: reviously in the ^r we presence some results
relation to the number of Even cycles [31]. where 'apparently' we have:

$$
\lim_{x_1 \to \infty} r_collatz(x_1) \longrightarrow \frac{I}{P} \approx 0.50
$$

Table 3 shows the distribution for number NN8, where:

 $[C, I, P], E\gamma = r_{collatz}1(NN8) \longrightarrow [74671, 25016, 49655], E\gamma.$

e number NN8) is composed of 25016 elements, which correspond to 2^{γ} , γ in the first column of the tal correspond to 2*^γ*, *γ* in the first column of the table. 3. The matrix *Eγ* (for the number NN8) is composed of 25016 elements, which correspond to 2^{*γ*}, *γ* in the first column of the table. 3.
 $\frac{122.231}{222.231}$ [32,33].

 $\mathcal{L} = \frac{1}{2} \left(\frac{1}{2} \mathbf{N} \mathbf{S} \right)$ Table 3: Even divisors: r_collatz(NN8)

 $T = \frac{1}{2}$ – Even divisors: r_collatz the operations occur on even numbers, however the distribution of division operations follows an exponential distribution, column
converges (%) and Figure 11. The last column of Table 3 refers to the houristic argument pre *γ* is a dividible 11. The fast column of table 5 feters to the neuristic argument presented by Eagarias[0], in which
a factor (MF) between two consecutive odd integers should be \approx 3/4 < 1, it can be seen in Table 3 t tic argument suggests that, on average, iterations in a trajectory tend to decrease in size, so that there should i divergent trajectories "(translated by the author). occurrence (%) and Figure 11. The last column of Table 3 refers to the heuristic argument presented by Lagarias[6], in which the multiplicative factor (MF) between two consecutive odd integers should be $\sim 3/4 < 1$, it can be seen in Table 3 that MF ≈ 0.7579 .
"this having group out away that our guarantee is a trainer in a trainer and to decreas "this heuristic argument suggests that, on average, iterations in a trajectory tend to decrease in size, so that there should be no
divergent trajectories "(translated by the author) operations follows an exponential distribution, column occurrence ($\frac{1}{\sqrt{2}}$ operations, which is in agreement with the function collatz $d(x_1)$, that is, 50% of the operations occur on odd numbers and 50% of
the operations again an aver numbers however the distribution of division operations foll A number x_i (even) will be divided by 2^{γ} , it can be seen that the number of divisions by 2γ is exactly equal to the number of odd

divergent trajectories"(translated by the author)..

Figure 11: Occurrence of Pairs (r_collatz1(NN8))

approximated equation. It can be seen from Table 3 that the probability of an even number subsequent to the operation $3 \times x_i + 1$
being divided by TWO is 100%³⁴ and in this process being divided again by a power of TWO t approximated equation. It can be seen from Table 3 that the probability of an even number subsequent to the operation $3 \times x_i + 1$
being divided by TWO is 100%³⁴, and in this process being divided again by a power of TWO, $\frac{3}{4}$ that the probability. The graph in (Figure 11) illustrates the exponential distribution of even numbers according to occurrence (%) in Table 3 (NN8) and being divided by 1 wO is 100%, and in this process being divided again by a power of 1 wO, that is, being divided where $γ^{35} ∈ ℕ^*$ between 22 and 2∞ is given by: nential distribution of even numbers according to occurrence $(\%)$ in Table 3 (NN8) and \Box ^P(1*.*⁵¹ [≦] *^γ* [≦] 14) = ¹⁴ $\frac{1}{2}$ that the probability of an even number subsequent to the operation $\frac{1}{2}$ $\frac{$ The graph in (Figure 11) illustrates the exponential distribution of even numbers according to occ

$$
\mathbb{P}(1.51 \le \gamma \le 14) = \int_{1.51}^{14} e^{(-0.6909 \times \gamma)} \simeq 50\%
$$

Table 3 also shows that $\Sigma(occurrences)$ when $\gamma > 1$ is $\Sigma \approx 50\%$, which also implies that for 25016 increases there will be 49655 2, which on average corresponds to: $2\left(\frac{49655}{25016}\right)$ ≈ 3.9584, and this average value in turn approaches the heuristic argued proposed by Lagarias, that is: $\frac{3}{2}$ ≈ $\frac{3}{2502}$ ≈ 0.75 [36]. d proposed by Lagarias, that is: $\frac{3}{4} \simeq \frac{3}{3.9584} \simeq 0.75$ [36]. divisions by 2, which on average corresponds to: $2(\frac{49000}{25016}) \approx 3.9584$, and this average value in turn approaches the heuristic argument presented and proposed by Lagarias, that is: $\frac{3}{4} \simeq \frac{3}{2.0584} \simeq 0.75$ [36]. s to: $2^{\left(\frac{49655}{25016}\right)} \approx 3.9584$, and this average value in turn approaches the heuristic argument $\frac{3}{4} \simeq \frac{3}{3.9584} \simeq 0.75$ [36]. divisions by 2, which on average corresponds to: $2\left(\frac{49655}{25016}\right) \approx 3.9584$, and this average value in turn presented and proposed by Lagarias, that is: $\frac{3}{4} \simeq \frac{3}{3.9584} \simeq 0.75$ [3.4258.1]

orting that the occurrence of divisions by two, that is, divisions by 2^{*z*}, is in accordance with the exponential distributions to $\frac{1}{2}$. of numbers ∈ N[∗], as also seen in Table 2. It is worth noting that the occurrence of divisions by two, that is, divisions by 2^{*z*}, is in accordance with the exponential distribution of numbers $\in N^*$, as also seen in Table 2. is in accordance with the exponential distribution of numbers ∈ N∗, as also seen in Table

The following graph in Figure... shows the average of the exponential factor $e^{(-0.6909 \times \gamma)}$ for $2^{68} + 1 \le x_i \le 2^{68} + 10001$ (only the odd numbers) numbers). av T following graph in Figure... shows the average of the exponential factor $\frac{1}{2}$

9. Conclusion

The Big Question

istion
al numbers when subjected to the Collatz sequence always end in the cycle 4→2→12 a1
... T following graph in Figure... shows the average of the exponential factor of the exponen Do all natural numbers when subjected to the Collatz sequence always end in the cycle $4 \rightarrow 2 \rightarrow 1$?
Statistical evidence and some auxiliary programs point in this direction, but they are not emphati

This is expected since the operation is on a set of infinite numbers $\in \mathbb{N}^*$, and the tools used are limited in relation to the internal representation of the numbers supported by the machines used. However, with the help of binary operations on numbers expressed
in test fame (thing), have numbers and as NN9 man model as The spection 2 × 0 + 1 and area divi Statistical evidence and some auxiliary programs point in this direction, but they are not emphatic in admitting such a conclusion. in text form (string), huge numbers such as NN8 were worked on. The operation $3 \times x_i + 1$ and even division by TWO are relatively simple to perform with binary operations and are a great help in overcoming the representation barrier. numerical intrinnsect to current computers. Such results operating on 'large' numbers of the order of 103000 were in accordance with the expected and proclaimed results.

It is worth highlighting based on what was previously presented in item 4.1.2 and appendix A:

In the Collatz Conjecture there is only one limit cycle formed by the stable points: $1\rightarrow4\rightarrow2\rightarrow1$.

The non-existence of other internal limit cycles in the Collatz Conjecture, and also in accordance with the equation (12) transcribed here: $\Psi + \Omega = 1$, remembering that $0 < \Psi < 1$ and $0 < \Omega < 1$ result in:

 $\frac{1}{4}$ In fact, any odd number when multiplied by THREE and added to ONE will result in an even number, always in Any and all numbers $x_i \in \mathbb{N}^*$ (even $xi \to \infty$) when subjected to the Collatz Conjecture will invariably end in the limit cycle formed by the stable points: 1→4→2→1. equation [∈] ^R as per Fig. 11 (occurrence (%)= *^e*−0*.*6909*^γ*), *^γⁱ* = 1*.*⁵¹ and *^γ^f* = 14. ³⁶ The difference between the value ³

27

The Collatz Conjecture is definitely not only a challenge, it is also a fertile field for using the tools available in mathematics. At each stage, new tools and/or observations (sometimes previously neglected) are present. In one of these surprises, we can say: Any number $x_i = 2^{\sigma} - 1$, $\sigma \in \{0, 2, 4, 6, 8, ...\}$ will always be divisible by three.

The function *divide_3*(*expI, expF*) helps in the verification, but the demonstration of such a statement is beyond the scope of this work, and will be left for a possible future work!

References

- 1. Baker, G. L., & Gollub, J. P. (1996). *Chaotic dynamics: an introduction*. Cambridge university press.
- 2. [Barina, D. \(2021\). Convergence verification of the Collatz problem.](https://doi.org/10.1007/s11227-020-03368-x) *The Journal of Supercomputing, 77*(3), 2681-2688.
- 3. Grauer, J. A. (2021). Analogy Between the Collatz Conjecture and Sliding Mode Control.
- 4. Hartnett, K. (2019). Mathematician Proves Huge Result on Dangerous Problem. *Quanta magazine.*
- 5. Hayes, B. (1984). On the ups and downs of hailstone numbers. *Scientific American, 250*(1), 10-13.
- 6. [Lagarias, J. C. \(1985\). The 3 x+ 1 problem and its generalizations.](https://doi.org/10.1080/00029890.1985.11971528) *The American Mathematical Monthly, 92*(1), 3-23.
- 7. Monteiro, L. H. A. (2002). *Sistemas dinâmicos*. Editora Livraria da Física.
- 8. Helene, O. A., & Vanin, V. R. (1991). *Tratamento estatístico de dados em física experimental*. Editora Blucher.

Appendix A

Appendix A
Comparison Between Ω Calculated and Approximated

The text in this appendix was highlighted and treated as complementary, serving to The text in this appendix was highlighted and treated as complementary, serving to The text in this appendix was inginighted and treated as complementary, serving to
support the observations seen in the item 4.1.2, in which when $x_k \to \infty$ it is observed that $\Psi = \frac{3^I}{2^P} \rightarrow 0$ and due to this fact the value 1 present in the formula $3 \times x_i + 1$ was disregarded, since this is much smaller than the product $3 \times x_1$, $(1 \ll \ll (3 \times x_1))$. Therefore, the value of Ω was calculated using both methods (I) and (II), namely: that $\Psi = \frac{3^I}{2^I} \to 0$ and due to this fact the value 1 present in the formula $3 \times x_i + 1$ that $\Psi = \frac{1}{2^p} \to 0$ and due to this fact the value **1** present in the formula $3 \times x_i + 1$ was disregarded, since this is much smaller than the product $3 \times x_1$, $(1 \ll \ll (3 \times x_1))$. T was calculated using both methods (I), namely: T and (I), namely: namely: namely: namely: namely: namely: T was uistegarued, since this is much smaller than the product $3 \times x_1$, $(1 \le \le \theta \times x_1)$
Therefore the value of O was calculated using both methods (I) and (II) namely: $t = \frac{3^I}{2^P} \rightarrow 0$ and due to this fact the value 1 present in the formula $3 \times x_i + 1$ garded, since this is much smaller than the product $3 \times x_1$, $(1 \ll \ll (3 \times x_1))$. T this appendix was highlighted and treated as complementary, serving to support the observations seen in the item 4.1.2, in which when $x_k \to \infty$ it is observed
 $x_k = \frac{3^I}{2^I} \to 0$ and due to this fact the value 1 present in the formula $3 \times x_i + 1$ $\frac{d}{dt} = \frac{1}{2^p} \rightarrow 0$ and due to this fact the value **1** present in the formula $3 \times x_i + 1$
(arded, since this is much smaller than the product $3 \times x_1$, $(1 \ll \ll (3 \times x_1))$). $T = \frac{1}{2}$ was calculated using both methods (I) and (I), namely $\frac{1}{2}$ The text in this approximate was highlighted and treated as complementary, serving to the service of t $\frac{3^I}{4}$ = 0.25 ¹ α been in the nem 4.1.2, in which when $x_k \to \infty$ is observed
d due to this fact the value 1 present in the formula $3 \times x_i + 1$ $v_2 = 2^p$ \rightarrow 0 and due to this fact the value 1 present in the formula $3 \times x_i + 1$ and x_i a T_{total} and T_{total} is the value of Ω was calculated using both methods (I) and (II), namely:

approximated.

APPENDIX A – Comparison between Ω calculated and

$$
\underbrace{\sum_{j=(I-1)}^{0} \left(3^{j} * \left[\prod_{i=(I-j)}^{I} (\rho_{i})\right]\right)}_{(B)+(C)+(D)=\Omega} = \Omega \qquad (I)
$$
\n
$$
\lim_{(x^*,\Psi)\to(\infty,0)} \left(\frac{\Omega}{1-\Psi}\right) = \Omega \qquad \therefore \quad \left(\frac{\Omega}{1-\Psi}\right) = \Omega \qquad (II)
$$

The following graphs using the function *omega_1e2(xi, xn, salto)* show the modulus The following graphs using the function *omega_1e2(x_i, x_n, salto)* show the modulus
of the difference heterographs using the indices of the (1) and (1) , \vdots , $|O(I) - O(II)|$. of the difference between the values obteined in (I) and (II), i.e. $|\Omega(I) - \Omega(II)|$: $\sum_{i=1}^{\infty} \frac{1}{i} \sum_{i=1}^{\infty} \frac{1}{i} \sum_{i=1}^{\infty$ of the difference between the values obteined in (I) and (II), i.e. $|\Omega(I) - \Omega(I)|$:

differences, while in red the approximated exponential trend line according to values of the according to values $\sum_{i=1}^{n}$ is $\sum_{i=1}^{n}$ calculated values $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ **Figure 12:** Graphs: $abs(\Omega(I) - \Omega(II)) \times x_i$

In Fig. 12, the sequences in blue are the calculated values of the modulus of the In Fig. 12, the sequences in blue are the calculated values of the modulus of the
differences, while in red the approximated exponential trend line according to values differences, while in red the approximated exponential trend line according to values
of **a**, **b** and **c** in each graph, note that the residual value **c** when $x_i \rightarrow \text{inf}$, we have wh (*x*∗*,*Ψ)→(∞*,*0) such an approximation (when $x_i \to \inf$) is acceptable as well as the limits: $\lim_{x_i \to \infty} c \to 0$, demonstrating that equations (I) and (II) lead to similar results, and that In Fig. 12, the sequences in blue are the calculated values of the modulus of the modulus of the sequences in blue calculated values of the modulus of the sequences in blue calculated values of the modulus of the sequences α , while in red the approximated exponential trend line according to values In Fig. 12, the sequences in blue are the sequences in blue are the modulus of the mod Fig. 12, the sequences in blue are the calculated values of the modulus of the such an approximation (when $x_i \rightarrow \inf$) is acceptable as well as the limits:

$$
\lim_{(x^*,\Psi)\to(\infty,0)}\left(\frac{\Omega}{1-\Psi}\right) = \Omega, \qquad \lim_{x^*\to\infty}(x^*) \longrightarrow \Omega
$$

The following graphs (Fig. 13) present both methods for calculating Ψ , (I) and (II), note that when $x_i \to \inf$ the value of Ψ also tends to zero $(\Psi \to 0)$.

 T following graphs \overline{F} present both methods for calculating Ψ and Ψ

 T following graphs (Fig. 13) present both methods for calculating Ψ (T) and Ψ

Figure 13. *Graphi* **Figure 13:** Graphics: Ψ × x_i

evident that \sharp is any value of $x^* \neq 1, x^* \neq 2, x^* \neq 4 \mid x^* \in \mathbb{N}$ that satisfies the equation (12), which leads to the conclusion that \exists is just and exclusively a limit cycle in the Collatz Conjecture Conjecture. Thus, with what has been exposed in this appendix and seen previously, it is Conjecture. (12), which leads to the conclusion that $\frac{1}{2}$ is just and exclusively a limit cycle in the Collatz in the Coll

In addition, hypothetically consider the equation (12) assuming that $\Psi \rightarrow 1$ (which is in disagreement with the graphs in Fig. 13), as seen previously $\Psi x_k + \Omega = x_k$ we will
possessibly have that $\Omega \to 0$, we obtain that: $\Psi x_k + \Omega = x_k$ or $\Psi x_k = x_k$. necessarily have that $\Omega \to 0$, we obtain that: $\Psi x_k + 0 = x_k$ or $\Psi x_k = x_k$ is in the graphs in Fig. 13^{*P*} , it follows that $\Psi x_k + 0 = x_k$ or $\Psi x_k = x_k$ $\text{that } \Omega \to 0$, we obtain

Given that $\Psi = \frac{3^I}{2^P}$, it follows that $x_k \simeq x_k \times 2^{(I * \log_2(3) - P)}$, for the expression $2^{(I*\log_2(3)-P)}=1$ to be true we have: $I*\log_2(3)=P$, remembering $\Omega=B+C+D$ where $B=\Psi$ it follows that (when $\Psi\to 1$, $\Omega\to 0$) $\Omega=\Psi+C+D$; we have that $\Omega\sim 1$ which $B = \frac{\Psi}{3}$, it follows that (when $\Psi \to 1, \Omega \to 0$) $\Omega = \frac{\Psi}{3} + C + D$. we have that $\Omega > \frac{1}{3}$ which contradicts the hypothesis previously formulated. The following graphs (Fig. 14) illustrate
that: $Q > 0$ and $\Psi \neq 1$ reinforcing that the hypothesis $\Psi \rightarrow 1$, $Q \rightarrow 0^{37}$ is not valid that: $\Omega > 0$ and $\Psi \neq 1$, reinforcing that the hypothesis $\Psi \to 1, \Omega \to 0^{37}$ is not valid. ven that $\Psi = \frac{3}{2^p}$, it follows that $x_k \simeq x_k \times 2^{(1+\log_2(j)-1)}$, for the expressi
 P _{*k*} + 0, we obtain the vector I_k or $(2) - P$ remembering $Q - R + C + D$ wh ven that $\Psi = \frac{3^2}{2^p}$, it follows that $x_k \simeq x_k \times 2^{(1*\log_2(3)-P)}$, for the expression $B^2 = 1$ to be true we have: $I * log_2(3) = P$, remembering $\Omega = B + C + D$ where

Appendix B Relationship between I and P 1 Equation (4) possibilities as a function of I, P. 1 Equation (4) possibilities as a function of I, P. 1 Equation (4) possibilities as a function of I, P. **1. Equation (4) Possibilities as a Function of I, P** p between 1 and P
(4) Possibilities as a Function of I, P

1. Equation (4) I ossibilities as a function of I, P.
The term (C) of the equation (4) presents a certain complexity in the formation of the sets that contain the values of γ_i , having a distribution of the elements γ_i in arrangements $(AR(\gamma_i) = (I-1)!)$ different³⁸, the function $r_collatz1('7')$ displays as the answer: $\frac{1}{2}$ $\$ or the sets that contain the values of $\frac{1}{l_1}$, having a distribution of the elements $\frac{1}{l_1}$ in
arrangements $(AR(\gamma_i) = (I-1)!)$ different³⁸, the function $r_collatz1('7')$ displays as the arrangements (*AR*(*γi*)=(*^I* [−] 1)!) different³⁸, the function *^r*_*collatz*1(′ answer: (C) of the equation (4) presents a certain complexity in the formation \mathbf{e} $\mathop{\mathrm{ne}}$ $\frac{1}{2\pi}$ (*ARC*(*|z*) = (*I* − 1).) different 3, the function *r*_*collatz*1(α) displays as the $\overline{\mathbf{a}}$ displays as the set of $\overline{\mathbf{a}}$ $\Gamma(\mathcal{C})$ of the equation (4) presents a certain complexity in the formation arrangements ($AR(\gamma_i) = (I - 1)!$) different³⁸, the function *r_collatz*1('7') displays as the $\overline{\mathbf{a}}$ answer:

$$
[C, I, P, E\gamma] = r_collatz1('7') \longrightarrow ([16, 5, 11], \underbrace{[1, 1, 2, 3, 4]})_{(E\gamma)}
$$

The original set $E\gamma = [1, 1, 2, 3, 4]$, which are the exponents to be applied to $2^{-E\gamma}$, resulting in the set $\rho_i = \left[\frac{1}{2^1}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}\right] = \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right]$. The set $E\gamma$ can be rewritted ways, i.e. in $(I - 1)$: arrangements, without thanging the result of $\prod_{i=1}^{k} (p_i)$. resulting in the set $\rho_i = \left[\frac{1}{2^1}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}\right] = \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right]$. The set $E\gamma$ can be rewritten in several ways, i.e. in $(I - \overline{1})!$ arrangements, without changing the result of $\prod_{i=1}^{n}(\rho_i)$. The original set E ^{*F*} $[1, 1, 2, 3, 4]$, which are the exponents to be applied to 2−⁷, σ in the set $a - 1$, $\frac{1}{2}$, $\frac{1$ resulting in the set $p_i = \lfloor 2^i, 2^i, 2^i, 2^i, 2^i, 2^i \rfloor = \lfloor 2^i, 2^i, 2^i, 2^i, 2^i \rfloor$. The set *E*_{*l*} can be rewritten in
several ways, i.e. in $(I-1)!$ arrangements, without changing the result of $\prod_{i=1}^n (\rho_i)$. The original set $E\gamma = [1, 1, 2, 3, 4]$, which are the exponents to be applied to $2^{-E\gamma}$, resulting in the set $\rho_i = \left[\frac{1}{2^1}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}\right] = \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{16}\right]$. The set $E\gamma$ can be rewritten in several ways, i.e. in $(I - 1)!$ arrangements, without changing the result of $\prod_{i=1}^n (\rho_i)$. n the set $\rho_i = \left[\frac{1}{2^1}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}\right] = \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right]$. The set $E\gamma$ can be rewritten in ys, i.e. in $(I - 1)$! arrangements, without changing the result of $\prod_{i=1}^{n} (\rho_i)$. e original set E' = [1, 1, 2, 3, 4], which are the exponents to be applied to 2 $'$,
n the set $\rho_i = \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right] = \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right]$. The set $E\gamma$ can be rewritten in n the set $\rho_i = \left[\frac{1}{2^1}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}\right] = \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right]$. The set $E\gamma$ can be rewritten in ys, i.e. in $(I-1)!$ arrangements, without changing the result of $\prod_{i=1}^n (\rho_i)$ ys, i.e. in $(I-1)!$ arrangements, without changing the result of $\prod_{i=1}^{n}(\rho_i)$. n the set $\rho_i = \left[\frac{1}{2^1}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}\right] = \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right]$. The set $E\gamma$ can be rewritten in $\mathcal{L} = \{ \mathbf{S} \mid \mathbf{S} \}$ is the term (C) of equation (C) of equation (4) as can be term (4) as can be te

However, such arrangements will impact the term (C) of equation (4) as can be seen below for two specific sets 39 : $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ (*Pi*). seen below for two specific sets 39 :

$$
\rho_i^E = [\frac{1}{2^1}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}], \qquad \qquad \rho_i^M = [\frac{1}{2^3}, \frac{1}{2^2}, \frac{1}{2^1}, \frac{1}{2^4}]
$$

 α equation (4), where I Rremembering the item (*C*) of the equation (4), where *I* = *n* = 5: Remembering the item (C) of the equation (4), where $I = n = 5$:

$$
\underbrace{\sum_{j=(n-2)}^1 \left(3^j * \left[\prod_{i=(n-j)}^n(\rho_i)\right]\right)}_{(C)}
$$

Adopting ρ_i^E , ρ_i^M seen previously, it is possible to calculate the dyadic fractions (α) , (β) and (δ) for the two series of ρ : $\frac{1}{2}$ and (*δ*) for the two series of *ρ*: A ^{*i*} (*δ*) for the two series of *ρ*: P_i , P_i for the two series of *ρ*: and (*δ*) for the two series of *ρ*: ρ_i^E , ρ_i^M seen previously, it is possible to calculate the dyadic fractions (α) , (β)

$$
\underbrace{3^{3} * [\frac{1}{2^{1}} * \frac{1}{2^{2}} * \frac{1}{2^{3}} * \frac{1}{2^{4}}]}_{(\alpha^{E})} + \underbrace{3^{2} * [\frac{1}{2^{2}} * \frac{1}{2^{3}} * \frac{1}{2^{4}}]}_{(\beta^{E})} + \underbrace{3^{1} * [\frac{1}{2^{3}} * \frac{1}{2^{4}}]}_{(\delta^{E})}
$$
\n
$$
\underbrace{0.0263671875}_{(\alpha^{E})} + \underbrace{0.017578125}_{(\beta^{E})} + \underbrace{0.0234375}_{(\delta^{E})} = \underbrace{0.0673828125}_{(C^{E})}
$$
\n
$$
\underbrace{3^{3} * [\frac{1}{2^{2}} * \frac{1}{2^{1}} * \frac{1}{2^{1}} * \frac{1}{2^{4}}]}_{(\alpha^{M})} + \underbrace{3^{2} * [\frac{1}{2^{1}} * \frac{1}{2^{1}} * \frac{1}{2^{4}}]}_{(\beta^{M})} + \underbrace{3^{1} * [\frac{1}{2^{1}} * \frac{1}{2^{4}}]}_{(\delta^{M})}
$$
\n
$$
\underbrace{0.10546875}_{\alpha^{M})} + \underbrace{0.140625}_{\alpha^{M})} + \underbrace{0.09375}_{\alpha^{M})} = \underbrace{0.33984375}_{\alpha^{M})}
$$

$$
\underbrace{0.10546875}_{(\alpha^M)} + \underbrace{0.140625}_{(\beta^M)} + \underbrace{0.09375}_{(\delta^M)} = \underbrace{0.33984375}_{(C^M)}
$$

It becomes clear from previous considerations that a large part of the complexity in α is becomes than 1.0 in provisity considerations that a large part of the complexity in ving the equation (4) consists in solving the term (C) of the same, since the It becomes clear from previous considerations that a large part of the complexity in α equation (*λ*) consists in solving the term (*C*) of the sample starts with the coefficients γ_i are of the order of $(I-1)!$. solving the equation (4) consists in solving the term (C) of the same, since the possible
expansion with the coefficients ϵ_k are of the order of $(L-1)$ arrangements with the coefficients γ_i are of the order of $(I-1)!$. ³⁸ *"From the viewpoint of this heuristic argument, the central difficulty of the problem lies in understanding in* It becomes clear from previous considerations that a large part of the complexity in

2. The Conjecture Considering Real Numbers.
2 The conjecture considering Real Numbers.

Certainly considering even and odd numbers in the case of $x_i \ge 0 | x_i \in \mathbb{R}^*$ makes
ense since these numbers, as they can be fractional or irrational, do not present
The conjecture must be adapted, and in this case only little sense since these numbers, as they can be fractional or irrational, do not present parity. The conjecture must be adapted, and in this case only:

$$
collatz_reais(x_i)^{40} \Longrightarrow x_n = \frac{3 \times x_i + 1}{4}
$$

 $\frac{3}{4}$ is in agreement with Lagarias' heuristic argument[6], in which the tive factor (MF) between two subsequent numbers must be $\sim \frac{3}{4} < 1$, in the graph in Fig. 14 the cycle is observed limit defined by the circle with radius UM to which all sequences converge⁴¹. Remembering that according to **Monterio**^[7] "to know which an sequences converge. Remembering that according to **MOILERO** $\lbrack \rbrack$ to know the characteristics of an orbit it is necessary to know its eigenvalue λ , which corresponds to the product of each eigenvalue relative to the fixed points x_i^* in the orbit", as follows: reement with Lagarias' heuristic argument $[6]$, in which the tive factor (MF) between two subsequent numbers must be $\sim \frac{3}{4} < 1$, in the The factor $\frac{3}{4}$ is in agreement with Lagarias' heuristic argument [6], in which the multiplicative factor (MF) between two subsequent numbers must be $\sim \frac{3}{4} < 1$, in the The factor $\frac{3}{4}$ is in agreement with Lagarias' heuristic argument [6], in which the multiplicative factor (MF) between two subsequent numbers must be $\sim \frac{3}{4} < 1$, in the $\frac{3}{4}$ is in agreement with Lagarias' heuristic argument[6], in which the ative factor (MF) between two subsequent numbers must be $\sim \frac{3}{4} < 1$, in the Fig. 14 the cycle is observed limit defined by the circle with radius IIM to $df(x) = 3$

$$
\lambda = \tfrac{df(x)}{dx}|_{x_i^*} = \tfrac{3}{4}
$$

 $\lambda = \frac{\omega_0 \omega}{dx} |_{x_i^*} = \frac{2}{4}$
the orbit is stable, which allows us to state that such a cycle repeats indefinitely once the value **UM** is reached. As $\lambda < 1$ the orbit is stable, which allows us to state that such a cycle repeats indefinitely once the value **UM** is reached. − *4*
ich allows us to

Figure 15: Polar Graph: $\frac{(3 \times x_i+1)}{4}$

3 A Practical Example (Brute Force) num= **x**^{*i*}(or none for 0 λ ⁱ λ), steps= 50 (number of trajectory), julian the trajectory (number of trajectories), julian the trajectory), julian the trajectory (number of trajectory), julian the trajectory (numb ⁴⁰ The function *collatz_reais*() without any parameter generates the graph in Fig. 14, the parameters are

Consider the number 753257 whose graphs were seen in Fig. 7 item 4.2, from the equation (5) doing: $\prod_{i=1}^{n}(\rho_i) = \prod_{i=1}^{P}(\frac{1}{2}) = \frac{1}{2^P}$ $\frac{d}{d}$ $rac{1}{2}$ $rac{1}{2}$, $rac{1}{2}$ votion (5) doing: Π^n (a) = Π^p (1) = 1 equation (5) doing: $\prod_{i=1}^{n}(\rho_i) = \prod_{i=1}^{P}(\frac{1}{2}) = \frac{1}{2^P}$ Consider the number 753257 whose graphs were seen in Fig. 7 item 4.2, from the of each trajectory occurs on the circle with radius UM

we will have: x_i ⁱl here α \dot{e} will have.

of each trajectory (see angle $(A) \implies h(753257) = 753257 * \frac{3^I}{2^P}$ (= 0.80) $\Rightarrow h(753257) = 753257 * \frac{3^P}{2^P}$ (= 0*.8098463884774229*)

look for the values of *I* and *P* that meet the above, also observing the equation (11), the Remembering that this value "*generally*" is \geq 0.78 (data in table 1), we must function *acha* $ABD(n,m,c)$ available in the file Collatz Files constructed in accordance with the limit range (two lines in green) seeks to find the values that satisfy the condition above, it is important to note that up to 400 possible answers of $I \times P$ can be presented, since a region was delimited where the confidence index for the study base is 100%. 42 with the limit range (two limits in green) seeks to find the values that satisfy the values that satisfy the condition

Item	А	Pares	Impares	$A + B + D$	$I/P(\%)$
1	0.886717	72	33	0.949217	0.458333
$\overline{2}$	0.841688	80	38	0.904189	0.475000
3	0.798946	88	43	0.861447	0.488636
$\overline{4}$	0.898814	91	45	0.961315	0.494505
5	0.853172	99	50	0.915672	0.505051
6	0.809846	107	55	0.872347	0.514019
7	0.911077	110	57	0.973578	0.518182
8	0.864812	118	62	0.927312	0.525424
9	0.820895	126	67	0.883396	0.531746
10	0.923507	129	69	0.986008	0.534884
11	0.876610	137	74	0.939111	0.540146
12	0.832095	145	79	0.894595	0.544828
13	0.936107	148	81	0.998607	0.547297
14	0.789840	153	84	0.852341	0.549020
.					

Table 4: acha_ABD(753257, 0.5, 0.194725)

rect one corresponds to item 6, that is $P = 107, I = 55$ (r_collatz(str(753257)) \implies erves to show the most appropriate one among these results without having to calculate $\frac{1}{2}$ for the most appropriate one among these results writted having to calculate the complete Collatz Conjecture as presented in the equation (2) . Among the possibilities that the function *acha_ABD(n,m,c)* presented, the cor-Among the possibilities that the function *acha_ABD(n,m,c)* presented, the cor- $[162, 55, 107]$). This method (via brute force) presents many candidate values, and only

\mathcal{L}_{c} Collatz Collatz Collatz Conjecture as presented in the equation (2). **Appendix C** Setting up the Python / PyCharm environment **Creating the project**

 $3.11.6$ in a Linux environment $6.5.12\text{-}200. \text{fc}38. \text{x}86_64$ and the IDE interface <code>PyCharm</code> 2022.1.3 (Community Edition), let's create the initial project (those who have already done so can skip this step). Start the PyCharm program and create a new project the so-called Python_Collatz as shown in the following figure: \overline{Q} and confidence index index index in this case study is 100% since all points of the Study Base are included within \overline{Q} Once the Python environment is set up, the version currently used is: Python α note that the confidence index in this case study is 100% since all points of the Study Base are included with α

Figure 16: Creating the Project Fig. 16 – Creating the Project

displayed (Fig.16). Note that the print_hi(name) function is automatically created and triggered internally in the main code. When creating the project, the initial screen with the main.py file (initial) is

Figure 17: Main.py file (initial)

 $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, we get in eq. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ of $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ of $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ By making the changes to the code (example), our main.py file (initial) will look like this:

Figure 18: Code: main.py

Lines 13 and 14 shown in the code in Figure 17 were included only so that it is possible to test the environment beforehand.

1.1 Including Files

Lines 13 and 14 shown in the code in the code in Fig. 17 were included other files:
Before running the main.py program we must include other files: test the environment beforehand.

 $__$ init $__$.py⁴⁴, BaseDados.py, funcoes.py, Collatz_Files.py $\frac{1}{44}$ BaseDados.py, $\frac{1}{\sqrt{2}}$ α is put β BaseDados.py, f Conaug_r register

ers to the __init__.py file and adjusts the environment to import local functions developed in files separate main.py within the working directory. T following list refers to the following list refers to the environment and adjusts the environment of environment and adjusts the environment of the environment and adjusts the environment of the environment of the env to instruct to the __mit__.py the and adjusts the environment to import focal functions developed in mes separate
hip the working directory to import local functions developed in files separate from main.py within the working The following list refers to the __init__.py file and adjusts the environment to import local functions developed in files separate from

```
1 # todo comentário em Python inicia com o caracter #2 \# este arquivo tem o objetivo de indicar para o programa
             3 \# main . py a localiza ção
4 \mid \# das funções auxiliares
             5 \vert5
             7 from funcoes import funcoes
             8 from Collatz_Files import Collatz_Files
             \mathcal{L}_\text{max} from Collatz import Collatz import Collatz import Collatz import Collatz in the Collatz 
            5
6 from BaseDados import *3 + 1 \# todo comer
\overline{5} from functions in port functions in possible functions \overline{5} from \overline{5}
```
 $\frac{1}{2}$

Figure 19: *Code: ___init____.py*

by, Collatz_{_Files.py, BaseDado} T_{max} functions with the final contained in electronic form when requested by to admost always publication they will be available in the uncetory indicated field. Fig. 19 – Code: *__init__.py* very high, due to this the files will be made available in electronic form when requested by very mgn, que to this the mes will be made available in electronic form when requested by
email to the author, after publication they will be available in the directory indicated here. The files cited: main.py, funcoes.py, Collatz_Files.py, BaseDados.py and conjecturas.py present almost two thousand lines of code, the probability of incorrect typing is

) high, due to this the files will be made available in electronic form when \mathcal{L} ematication to the author, after publication they will be available in the directory indicated here. The directory indicated here. The directory indicated here. The directory indicate α email to the author, after publication they will be available in the directory indicated here. **Appendix D Conflict of Interest**

does not generate conflicts of interest in general! developments and information made public are acknowledged in this article, applying due This article is based on research and various articles of a public nature, the reference to the same authors cited. This is an analysis of a public domain topic, which

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