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# Birth Orbit of K2-33b Revealed by Kinematic Model of Tidally Interacting Binaries

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**Abstract**

*K2-33b, discovered recently, is one of the youngest planets discovered till date. This infant planet and their likes hold the key to unraveling the origin and formation processes of planets . K2-33b is just within co-rotation orbit and the silicate dust sublimation orbit. The Age of the system is only 5 to 10My. Under the circumstances in-situ formation is the only permitted scenario. The disk migration within proto-planetary disc of M3 dwarf stars is categorically ruled out. The kinematic model of tidally interacting binaries, when applied to Star-Planet system, very convincingly explains the detection of K2-33b as transiting ice giant at 0.04AU. Kinematic model gives the following scenario: Within 1My K2-33b completes the planet formation through agglomeration-sticking into pebbles, pebbles colliding and lumping to km-sized planetesimal, planetesimals growing to planetary embryo of 10M+ by collision and coalescing and planetary embryo in turn enveloping itself with gas by runaway gravitational accretion to 70 M+ terminated only after the gas-dust is cleared from the inner region leaving a gaping void at the first co-rotation orbit. K2-33b tumbles into sub-synchronous orbit because of inherent instability at the inner co-rotation orbit and gets trapped in gravitational runaway death spiral. Along this collapsing spiral trajectory K2-33b travels from first co-rotation orbit of $aG1 = 6.76818\times10^{9}$ m to the present orbit $a = 6.11856\times10^{9}$m in 5.567My to 11.34 My. While it is spiraling inward it is making K2-33 spin faster and K2-33b is collapsing inward. This in-spiral collapse rate at the present time is -2244m/y to -1121m/y which can be confirmed in near future. In next 8.9 yrs to 17.8 yrs it is destined to lose 20km as semi-major axis decay. Eventually when it crosses the Roche's Limit at $1.65\times10^{9}$m either K2-33b tidally vaporizes into a gaseous ring around K2-33 or K2-33b is engulfed by K2-33. This tidal pulverization will occur in future about 347,000 y to 487,000 y from now. This is the scenario predicted by Kinematic Model and observed by discoverers of K2-33 system.*

**Keywords:** Kinematic Model, Seismic Model, Sub-Synchronous Orbit, Super-Synchronous Orbit, Triple Synchrony Orbit, Clarke's Orbit

## 1. Introduction

A Neptune-sized transiting planet closely orbiting a 5-10 million years old star" has been discovered by Trevor J. David et.al. [1]. The discovery of the Neptune sized planet K2-33b, which is one of the youngest planets discovered till date, and the current orbit of this infant planet, infancy in astronomical terms, is the validation of the kinematic model of tidally interacting binary pairs first proposed and later expanded in [2-5]. Here it is assumed that disk migration, migration via tidal circularization of an eccentric planet, through, for example the Kozai-Lidov Mechanism, planet-planet scattering or secular chaos are not applicable over a short time scale of 5-10My [6-8]. K2-33b has been born as Gas-Ice Giant by runaway gravitational enveloping of 10 Earth mass Planetary Embryo by gas. For the formation of Planetary Embryo snowline need not be evoked as the nuclear fusion furnace of K2-33 is only 90% turned on within first 1My and inner part is at 100K temperature which is well below silicate sublimation temperature and allows amorphous ice coated dust to stick and agglomerate into km-sized planetesimals. Some of these collide and coalesce into planetary embryo which by gravitational run-away gas enveloping grow into 70 earth mass planet. The gravitational accretion of planetary embryo was terminated due to the paucity of gas, once it had achieved Neptune Mass, and the planet formation process had cleared the inner region of gas and dust. The stellar dust disk is limited to 2AU extending outward. K2-33b has been born in-situ at inner co-rotation orbit. It has spiraled inward from aG1 = 6.76818× $10^9$m to a = 6.11856× $10^9$m in about 5.567My to 11.34 My. The inward collapse rate at the present time is -2244m/y to -1121m/y. In another 8.9 yrs to 17.8 yrs, K2-33b would have spiraled in by 20km. Eventually when it crosses the Roche's Limit either K2-33b tidally vaporizes into a gaseous ring around K2-33 or K2-33b is engulfed by K2-33. Hence inward collapsing spiral is also referred to as death spiral. This tidal pulverization will occur in future about 347,000 y to 487,000 y from now. One of the youngest planets, K2-33b, discovered by David et.al. exactly fits the scenario developed by the kinematic model of tidally interacting binaries [1]. In this paper K2-33 planet hosting star (PHS) and K2-33b planet constitute the tidally interacting binary.

### 1.1 Plain English Languge Summary

K2-33b, discovered recently, is one of the youngest planets discovered till date. K2-33b is just within co-rotation orbit and the silicate dust sublimation orbit. The Age of the system is only 5 to 10My. Under the circumstances in-situ formation is the only permitted scenario. The kinematic model of tidally interacting binaries, when applied to Star-Planet system, very convincingly explains the detection of K2-33b as transiting ice giant at 0.04AU. Kinematic model gives the following scenario: Within 1My K2-33b completes the planet formation to km-sized planetesimal, planetesimals growing to planetary embryo of 10M+ and planetary embryo in turn enveloping itself with gas by runaway gravitational accretion to 70 M+ terminated only after the gas-dust is cleared from the inner region leaving a gaping void at the first co-rotation orbit. K2-33b tumbles into sub-synchronous orbit because of inherent instability at the inner co-rotation orbit and gets trapped in gravitational runaway death spiral. Along this collapsing spiral trajectory K2-33b travels from first co-rotation orbit of aG1 = 6.76818×10^9 m to the present orbit a = 6.11856×10^9m in 5.567My to 11.34 My. While it is spiraling inward it is making K2-33 spin faster. The in-spiral collapse rate at the present time is -2244 m/y to -1121 m/y which can be confirmed in near future. In next 8.9 yrs to 17.8 yrs it is destined to loose 20 km. Eventually when it crosses the Roche's Limit at 1.65×10^9m either K2-33b tidally vaporizes into a gaseous ring around K2-33 or K2-33b is engulfed by K2-33. This tidal pulverization will occur in future about 347,000 y to 487,000 y from now.

## 2. Scenario Predicted by Kinematic Model of Tidally Interacting Binaries

According to the Kinematic Model, every tidally interacting binary has two co-rotation periods which Author calls Clarke's Orbits $a_{G1}$ and $a_{G2}$. Planet is born at $a_{G1}$. This orbit is a gravitational energy maxima hence unstable. The planet tumbles long of $a_{G1}$ or short of $a_{G1}$. This tumbling takes place by perturbation due to solar winds or cosmic showers.

If the planet falls long of $a_{G1}$, it is launched on a super-synchronous expanding spiral path by gravitational sling shot mechanism [4]. It spirals out from $a_{G1}$ to $a_{G2}$. At $a_{G2}$, which is energy minima, radial velocity becomes zero and spiraling stops. The planet remains stable at total energy minima. It may either continue orbiting at a radius of $a_{G2}$ or it may be deflected on a collapsing spiral orbit. This is the fate which Earth-Moon system will face in the future. Moon is launched on an expanding spiral orbit and destined to get locked-in with Earth at outer Clarke's Orbit $a_{G2}$ [3]. But earlier than this lock-in, Earth-Moon system will be swallowed by the Red Giant Stage of our Sun.

If the planet falls short of $a_{G1}$, as Phobos has done in the case of Mars-Phobos binary, former gets trapped in a collapsing spiral orbit and eventually gets pulverized at Roche's Limit or if it is hard enough it makes headlong collision with Mars. According to Author's calculations Phobos is losing 20cm/y and in 10My it is destined to be tidally pulverized into a gaseous ring around Mars or to be totally engulfed by Mars [9,10].

A third scenario in tidally interacting binary is Pluto-Charon well studied by New Horizon space probe in July 2015. Here Charon is in a tidally locked-in position in triple synchrony state namely:

$$P_{orb} = P_{spin\_pluto} = P_{spin\_charon} = 6.4d \qquad 1$$

## 3. Kinematic Analysis

### 3.1 Deduction of Primary Spin Velocity/Orbital vel = ω/Ω Equation

In Kinematic Model, any binary system has two triple synchrony orbits which Author refers to as inner and outer Clarke's Orbits and in Earth-Moon system they are referred to as inner geo-synchronous orbit ($a_{G1}$) and outer geo-synchronous orbit ($a_{G2}$). Moon tidally evolves out of Inner Clarke's Orbit ($a_{G1}$). If it tumbles short of $a_{G1}$, secondary rapidly spirals-in by a gravitational runaway process to its certain destruction as Phobos is trapped in a death-spiral around Mars and if it tumbles long of $a_{G1}$ then through Gravitational Sling Shot-an impulsive torque- secondary is launched on an outward spiral path as Moon is with respect to Earth [10,11]. But as the differential between orbital velocity and spin velocity of primary grows, tidal stretching and squeezing sets in the primary body which leads to tidal dissipation which causes a rapid exponential decay of the impulsive torque. In super synchronous orbit primary's tidal bulge leads the radius vector joining primary and secondary. This 'lead angle' causes secular deceleration of the primary and angular momentum transfer from primary to secondary for angular momentum conservation. From then onward the Moon coasts on its own until it locks into the outer Outer Clarke's Orbit ($a_{G2}$). But through out this tidal evolutionary history the Total Angular Momentum is conserved neglecting Sun's tidal drag on E-M system hence we have the following Conservation of Momentum equation:

$$J_T = C\omega + (m^* a_{present}^2 + I)\Omega = [C + (m^* a_{G1}^2 + I)]\Omega_{aG1} = [C + (m^* a_{G2}^2 + I)]\Omega_{aG2} \qquad 2$$

C = Moment of Inertia of the Primary around its spin axis.
I= Moment of Inertia of the Secondary around its spin axis.
And m*= reduced mass of the secondary = m/(1+m/M) where m = the mass of the secondary and M= mass of the primary.
From Kepler's Third Law:

$$\Omega_{aG1} = \frac{B}{a_{G1}^{3/2}} \quad and \quad \Omega_{aG2} = \frac{B}{a_{G2}^{3/2}} \quad where \ B = \sqrt{G(M+m)} \qquad 3$$

From Classical Mechanics the Synchronous Orbit is the same as the Inner Clarke's Orbit calculated in Kinematic Framework for vanishingly small mass ratio m/M. In Classical Mechanics, the synchronous orbit is defined as:

$$a_{sync}^{3/2}\Omega_{orb} = a_{sync}^{3/2}\omega_{primary} = B \qquad 4$$

In Sharma 2023B (Iapetus hypothetical sub-satellite re-visited and it reveals celestial body formation process in the Primary-centric Framework. presented at 39th COSPAR Scientific Assembly, Mysore, India from 14th July to 20th July 2012,) the correspondence between Newtonian Formalism of Synchronous Orbit and Kinematic Formalism was found as shown in Figure 1.

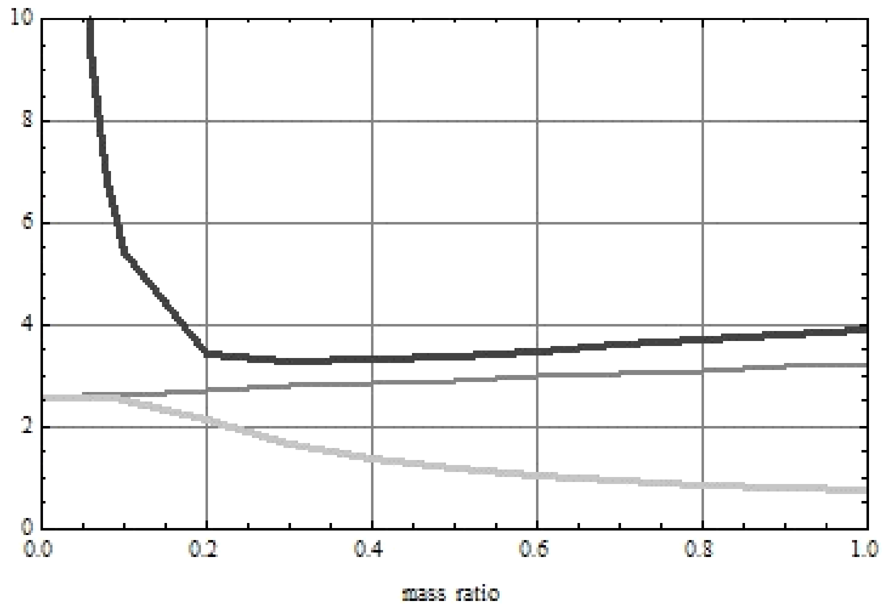


**Figure 1:** Plot of $a_{synSS}$ ($\times R_{Iap}$)[thin gray], $a_{G1}$ ($\times R_{Iap}$)[thick gray] and $a_{G2}$ ($\times R_{Iap}$) [thick black] as a function of 'q'(mass ratio)

Inspection of Figure 1 tells us that at infinitesimal values of 'q' , asynSS is the same as $a_{G1}$ and only one Clarke's Orbit is perceptible. But at larger mass ratios the two (classical and kinematic formalism for $a_{G1}$) rapidly diverge. Author's analysis till now has confirmed that $a_{G1}$ is the correct formalism for predicting the inner triple synchrony orbit in a binary system at $q < 0.2$.
At mass ratios greater than 0.2, $a_{G1}$ is physically untenable and only $a_{G2}$ is perceptible. Outer Triple Synchrony Orbit seems to converge but does not actually converge to the classical formalism but remains offsetted right till the limit of q =1. Here again only outer Clarke's Orbit is perceptible but the actual Star pairs satisfy the Kinematic formalism and not the classical formalism.

So Kinematic Formalism, though satisfies the correspondence principle at $q \sim 0$, is a theory in its own right. Till date there exists no formalism for two triple synchrony orbits in Classical Newtonian Mechanics in the mass ratio range 0.0001 to 0.2.
For mass ratio less than 0.0001, binaries remain in inner Clarke's Configuration stably which is predicted by Classical Newtonian Formalism also.

At mass ratios greater than 0.2 right up to unity, star pairs remain in outer Clarke's Configuration stably and its magnitude is more than Newtonian prediction.

For mass ratios $0.0001 < q < 0.2$, Outer Clarkes configuration is the only stable orbit and secondary is catapulted from $a_{G1}$ by Gravitational Sling Shot mechanism and it migrates out of that configuration. If it is at $a > a_{G1}$ the pair spirals out with a time constant of evolution and if $a < a_{G1}$ then the pair spirals-in on a collision course again with a characteristic time constant of evolution.

Time Constant of Evolution is in inverse proportion of some power of mass ratio. For q = 0.0001, it is Gy and as q increases, time-constant decreases from Gy to My to kY to years. This is valid for mass scale encountered in Solar and Exo-Solar Systems. Between 0.2 to 1, a solar nebula falls into outer Clarke's Configuration by hydro-dynamic instability within months/years.

For q being vanishingly small, the calculation of the man-made Geo-synchronous Satellite's orbit of 36,000Km above the equator has been done by Kinematic Formalism. This calculation has been done in Author's personal communication: http://arXiv.org/abs/0805.0100

Rewriting (2) we obtain:

$$\frac{J_T}{C\Omega} = \left[\frac{\omega}{\Omega} + \left(\left(\frac{m^*}{C}\right) a_{present}^2 + \frac{I}{C}\right)\right] = \left[\frac{\omega}{\Omega} + \theta_2' a_{present}^2 + \theta_1\right] \qquad 5$$

Substituting Ω= B/a$^{3/2}$ in (5) we obtain:

$$\left(\frac{J_T}{CB}\right) a^{\frac{3}{2}} = \left[\frac{\omega}{\Omega} + \theta_2' a_{present}^2 + \theta_1\right] \qquad 6$$

Rearranging the terms in (6) we get:

$$\frac{\omega}{\Omega} = \frac{LOM}{LOD} = \left(\frac{J_T}{CB}\right) a^{\frac{3}{2}} - (\theta_2' a^2 + \theta_1) = E a^{\frac{3}{2}} - F a^2 \qquad 7$$

$$where\ E = \frac{J_T}{BC}\ and\ F = \left(\theta_2' + \frac{\theta_1}{a^2}\right)$$

The System Parameters of K2-33 are tabulated in Table 1.

| parameter | symbol | magnitude | uncertainty | comments |
|---|---|---|---|---|
| K2-33 | | | | |
| Spectral Type | M3(M dwarf) | | ±0.5 | |
| Stellar radius | $R_{☼}$(×$R_{\odot}$) | 1.1 | ±0.1 | |
| Stellar mass | $M_{☼}$(×$M_{\odot}$) | 0.31 | ±0.05 | |
| Stellar spin | $P_{spin\ star}$(d) | 6.3 | ±0.2 | |
| Luminosity | Log($L_{☼}$/$L_{\odot}$)(dex) | -0.83 | ±0.07 | |
| Effective Temperature | $T_{eff}$(K) | 3410 | ±75 | |
| K2-33b | | | | |
| Spin period | $P_{spin\ planet}$(d) | 5.42513 | $^{+0.00028}_{-0.00029}$ | Planet is tidally locked |
| Orbital period | $P_{orb}$(d) | 5.42513 | $^{+0.00028}_{-0.00029}$ | |
| Planet radius | $R_P$(×$R_+$) | 5.76 | $^{+0.62}_{-0.58}$ | |
| Planet mass | $M_P$ | 1.024×10$^{26}$ | 6$M_+$ to 70$M_+$ | 3.58416×10$^{25}$ Kg to 4.181252×10$^{26}$ Kg. |
| Semi-major axis | a(AU) | 0.0409 | $^{+0.002}_{-0.0023}$ | |
| Black Body equi-Librium Temp. | $T_{equi}$(K) | 850 | ±50 | |

**Table 1: System Parameters of K2-33 System**

### 3.2 Analysis of K2-33 and K2-33b in Kinematic Framework [4]

First (Orbital Period/ Spin Period of Star) expression is set up from system parameters in Table 1. This gives:

$$\frac{\omega}{\Omega} = \frac{Orbital\ Period}{Spin\ Period\ of\ K2-33} = E \times a^{1.5} - F \times a^2 \quad 8$$

$$where\ E = \frac{J_T}{B \times C} = 1.85426 \times 10^{-15} \frac{1}{m^{3/2}}\ and\ F = \frac{m}{1+\frac{m}{M}} \times \frac{1}{C}$$

$$= 7.088 \times 10^{-22} \frac{1}{m^2} \quad 9$$

In (9):

$$m = mass\ of\ K2-33b = 1.024 \times 10^{26} Kg\ and\ M = mass\ of\ K2-33 = 6.169 \times 10^{29} Kg$$

$$B = \sqrt{G(M+m)} = 6.4172 \times 10^9 \frac{m^{\frac{3}{2}}}{s}; \quad 10$$

$$C(moment\ of\ inertia\ of K2-33\ around\ spin\ axis) = \frac{2}{5} M \times R_{star}^2$$
$$= 1.444 \times 10^{47} Kg - m^2\ \ here\ R_{star} = 765.05 \times 10^6 m \quad 11$$

$$I(moment\ of\ inertia\ of K2-33b\ around\ spin\ axis) = \frac{2}{5} m \times R_{planet}^2$$
$$= 5.516 \times 10^{40} Kg - m^2\ \ here\ R_{planet} = 36.7 \times 10^6 m \quad 12$$

The calculation of the moment of inertia (Eqs.11 and 12) implicitly assumes that both star and planet are homogeneous spheres, thereby ignoring the observational evidence for exceptionally strong central condensation of these celestial bodies. For mathematical tractability this assumption is made. Ignoring the observational evidence will cause only quantitative errors in calculation but the general qualitative trends will be correctly predicted.

Anelastic effects of tidal interaction have been considered in the analysis because those are central to tidal dissipation and to the orientation and the mis-alignment of the tidal bulges of the star and planet. It is this mis-alignment which gives rise to the torque.

$$J_T = total\ angular\ momentum\ of\ K2-33system$$

$$= C \times \frac{2\pi}{P_{spin\_star}} + \left(I + \frac{m}{1+\frac{m}{M}} \times a^2\right) \times \frac{2\pi}{P_{orbital}} = 1.72 \times 10^{42} \frac{Kg - m^2}{s} \quad 13$$

In (13)

$$P_{spin\_star} = 544320s\ and\ P_{orb} = P_{spin\_planet} = 468731.23s \quad 14$$

In (14) Author is assuming that K2-33b is in synchronous orbit hence:

$$P_{orb} = P_{spin\_planet}$$

The semi-major axis of the elliptical orbit of K2-33b is:

$$a = 0.0409AU = 6.11856 \times 10^9 m \quad 15$$

$$mean - stellar\ density = \rho_{star} = \frac{0.34g}{cc}\ and$$

$$mean\ planet\ density = \rho_{planet} = \frac{m}{\frac{4}{3} \times \pi \times R_{planet}^3} = \frac{0.495gm}{cc}$$

Therefore Roche's Limit (Ida et.al. 1997):

$$a_{Roche} = 2.45R_{star} \times (\frac{\rho_{star}}{\rho_{planet}})^{1/3} = 1.65 \times 10^9 m \quad 16$$

From (2) theoretical value of Orbit Period/Spin Period = 0.8609 at 'a' = 6.11856× $10^9$ m and present observed value of Orbit Period/Spin Period = 0.861 as calculated from Table 1. So our main equation is correctly set up.

### 3.2 Calculation of Co-Rotation Orbits of K2-33b

By setting (8) equal to Unity we obtain the co-rotation orbits of K2-33b:

$$inner\ co-rotation\ orbit\ or\ inner\ Clarke'sOrbit = a_{G1} = 6.76818 \times 10^9 m \quad 17$$

$$outer\ co-rotation\ orbit\ or\ outer\ Clarke'sOrbit = a_{G2} = 6.84177 \times 10^{12} m \quad 18$$

$$2:1\ Mean\ Motion\ Resonance\ Orbit = a_2 = 1.08056 \times 10^{10} m \quad 19$$

$$present\ semi-major\ axis\ of\ K2-33b's\ Orbit = a = 6.1185582 \times 10^9 m \quad 20$$

Comparing (17) and (20) it is obvious that K2-33b is in sub-synchronous orbit.

K2-33(star)-K2-33b(planet) Radius vector leads the tidal bulge in K2-33 , planet K2-33b spins-up Planet Hosting Star(PHS) K2-33 and Planet K2-33b approaches PHS K2-33 because of transfer of angular momentum and orbital energy from Planet K2-33b to PHS K2-33.

### 3.3 The Radial Velocity of Recession/Approach [4]

#### 3.3.1 The Tidal Torque Formulation

For the calculation of the spiral trajectory we need the radial velocity of recession in case of super-synchronous configuration and velocity of approach in case of sub-synchronous configuration. The time integration of the reciprocal of radial velocity gives the non-Keplerian Transit time from its inception to the present orbit. This transit time should be equal to the age of the secondary body. For arriving at the velocity of recession, we need the tidal torque.

The Tidal Torque of Planet on the star(Planet Hosting Star) and of star(PHS) on the Planet = Rate of change of angular momentum hence

$$Tidal\ Torque = T = \frac{dJ_{orb}}{dt} \quad 21$$

But Orbital Angular Momentum:

$$J_{orb} = m^* a^2 \times \frac{B}{a^{3/2}} = m^* B\sqrt{a} \quad 22$$

Time Derivative of (22) is:

$$T = \frac{dJ_{orb}}{dt} = \frac{m^* B}{2\sqrt{a}} \times \frac{da}{dt} \quad 23$$

In super-synchronous orbit, the radius vector joining the planet and the center of the PHS is lagging PHS tidal bulge hence the Planet is retarding the PHS spin and the tidal torque is BRAKING TORQUE.

In sub-synchronous orbit, the radius vector joining the planet and the center of the PHS is leading PHS tidal bulge hence the Planet is spinning up the PHS and the tidal torque is ACCELERATING TORQUE. This is a gravitational runaway process and Planet is trapped in a death spiral.

Author has assumed the empirical form of the Tidal Torque as follows:

$$T = \frac{K}{a^Q}\left[\frac{\omega}{\Omega} - 1\right] \quad 24$$

(24) implies that at Inner Clarke's Orbit and at Outer Clarke's Orbit, tidal torque is zero and (23) implies that radial velocity is zero and there is no spiral-in or spiral-out.

At Triple Synchrony, Planet-PHS Radius Vector is aligned with PHS tidal bulge and the system is in equilibrium and no radial migration. There are two roots of ω/Ω=1: Inner Clarke's Orbit and Outer Clarke's Orbit. As shown in Total Energy Profile (Sharma 2015, Sharma 2024), Inner Clarke's Orbit $a_{G1}$ is total energy maxima and hence is unstable equilibrium state and Outer Clarke's Orbit $a_{G2}$ is total energy minima and hence stable equilibrium state. In any Binary System, secondary is born at $a_{G1}$. This is the CONJECTURE assumed in Kinematic Model. From this point of inception, Secondary may either tumble short of $a_{G1}$ or tumble long of $a_{G1}$. If it tumbles short, satellite gets trapped in Death Spiral and it is doomed to its destruction. If it tumbles long, satellite gets launched on an expanding spiral orbit due to gravitational sling shot impulsive torque which quickly decays due to the growing differential of ω and Ω and the resulting tidal heating. After the impulsive torque has decayed, the satellite coasts on it own toward final lock-in at $a_{G2}$.

Equating the magnitudes of the torque in (23) and (24) we get:

$$\frac{m^*B}{2\sqrt{a}} \times \frac{da}{dt} = \frac{K}{a^Q}\left[\frac{\omega}{\Omega} - 1\right] \quad 25$$

Rearranging the terms in (25) we get:

$$V(a) = Velocity\ of\ recession = \frac{2K}{m^*B} \times \frac{1}{a^Q}\left[Ea^2 - Fa^{2.5} - \sqrt{a}\right]m/s \quad 26$$

The Velocity in (26) is given in m/s but we want to work in m/y therefore (26) R.H.S is multiplied by $31.5569088\times10^6$s/(solar year).

$$V(a) = \frac{2K}{m^*B} \times \frac{1}{a^Q}\left[Ea^2 - Fa^{2.5} - \sqrt{a}\right] \times 31.5569088 \times \frac{10^6 m}{y} \quad 27$$

In (27) there are two unknowns: exponent 'Q' and structure constant 'K'. Therefore two unequivocal boundary conditions are required for the complete determination of the Velocity of Recession/approach.

First boundary condition is at a = a2 which is a Gravitational Resonance Point where ω/Ω = 2 [Rubicam 1975] and V(a) is at its maxima and radial acceleration is zero (Sharma 2015).

i.e. $(Ea^{3/2} – Fa^2) = 2$ has a root at $a_2$.

Therefore at $a = a_2$, $(\delta V(a)/\delta a)(\delta a/\delta t)|_{a2} = 0$.
On carrying out the partial derivative of V(a) with respect to 'a' we get the following:

$$At\ a_2,\quad (2-Q)E \times a^{1.5} - (2.5-Q)F \times a^2 - (0.5-Q) = 0 \quad 28$$

$$At\ a_2,\quad (2-Q)E \times a^{1.5} - (2.5-Q)F \times a^2 - (0.5-Q) = 0 \quad 29.$$

Solving (27) we obtain: Q =3.4586.
Now structure constant (K) has to be determined. This will be done by trial error so as to get the right age of K2-33 exo-solar system i.e. 5-10My. Rewriting (27) and substituting the best fit values of the exponent and constants E and F we obtain the structure constant 'K'.

$$V(a) = \frac{2K}{m^*B} \times \frac{1}{a^Q}\left[E \times a^2 - F \times a^{2.5} - \sqrt{a}\right] \times 31.5569088 \times \frac{10^6 m}{y} \quad 27$$

$$Transit\ Time = \int_{a_{G1}}^{a} \frac{1}{V(a)}da \quad 30$$

(30) tells us that the planet is born at $a_{G1}$ and it tumbles into sub-synchronous orbit and hence it is launched in a collapsing spiral orbit. Since its age is 5 to 10 My hence its transit time should be taken over a range of 5 to 10My and unknown 'K' will be calculated.

Assuming a tentative value for $V_{max}$ and inserting it in (27) at a = $a_2$ we deduce the value of 'K'. Using this 'K' in (27) and inserting this trial expression in (30) we carry out the time integral to get the transit time from aG1 to present 'a' which should be the age of K2-33b. Several iterations are carried by adjusting Vmax. By this iteration method we obtain the best fit structure constant as:

$$If\ V_{max} = +\frac{1500m}{y}\ at\ a_2\ we\ obtain\quad K = 7.57256 \times 10^{60} \qquad 31$$

(31) gives a transit time of 11.341My from aG1 to present 'a' and a transit time of 11.8281My from aG1 to $a_{Roche}$.
Therefore in (11.8281-11.341) My= 487,000y from now, K2-33b is destined to be pulverized into a gaseous ring or be engulfed by K2-33.

$$If\ V_{max} = +\frac{3000m}{y}\ at\ a_2\ we\ obtain\quad K = 1.51451 \times 10^{61} \qquad 32$$

(32) gives a transit time of 5.567My from aG1 to present 'a' and a transit time of 5.914My from aG1 to $a_{Roche}$.
Therefore in (5.914-5.567) My= 347,000y from now K2-33b is destined to be pulverized into a gaseous ring or be engulfed by K2-33.

## 4. Results and Discussions

K2-33b has been born as Gas-Ice Giant by runaway gravitational enveloping of 10 Earth mass Planetary Embryo by gas. For the formation of Planetary Embryo snowline (Sasselov and Lecor 2000) need not be evoked as the nuclear fusion furnace of K2-33 is only 90% turned on within first 1My and inner part is at 100K temperature which is well below silicate sublimation temperature and allows amorphous ice coated dust to stick and agglomerate into km-sized planetesimals. Some of these collide and coalesce into planetary embryo which by gravitational run-away gas enveloping grow into 70 earth mass planet. The gravitational accretion of planetary embryo was terminated due to the paucity of gas, once it had achieved Neptune Mass and the planet formation process had cleared the inner region of gas and dust. The stellar dust disk is limited to 2AU extending outward.

After its birth since it has tumbled short of $a_{G1} = 6.8541\times 10^9$m, the planet is caught in gravitationally runaway sub-synchronous collapsing spiral path. Along this spiral path it has spiraled inward from aG1 = $6.76818\times 10^9$m to a = $6.11856\times 10^9$m in about 5.567My to 11.34 My. While it is spiraling inward it is making K2-33 spin faster and K2-33b is collapsing inward. ***This inward collapse rate at the present time is -2244m/y to -1121m/y. This can be verified by David's Team. In another 8.9 yrs to 17.8 yrs, K2-33b would have spiraled in by 20km.*** Eventually when it crosses the Roche's Limit either K2-33b tidally vaporizes into a gaseous ring around K2-33 or K2-33b is engulfed by K2-33. Hence inward collapsing spiral is also referred to as death spiral. This tidal pulverization will occur in future about 347,000 y to 487,000 y from now.

Phobos around Mars is caught in such a death spiral losing its altitude by 20cm/y and destined to be tidally pulverized in future in 10My from now according to Kinematic Model (Sharma 2023A, Black & Mittal 2015). Moon around Earth is launched on an expanding spiral path radially receding at 3.8cm/y (Sharma et.al 2009). Phobos scenario is exactly the scenario observed by David's team in case of K2-33 exo-solar system.

## 5. Conclusions

Kinematic model of tidally interacting binaries recreates the observations made by David's team. David's team has studied this system through Kepler Space Telescope during campaign 2 of K2 mission. Pre-main sequence population of Upper Scorpius was observed by K2 mission. Planetary signals of K2-33b was verified. K2-33 is a member of Upper Scorpius OB association which is the nearest site to Earth where recently massive star formation has taken place at distance d=145±20pc. By IR studies there are clear indications that 20% of Low Mass Stars of Upper Scorpius host protoplanetary disks. 80% , which donot have protoplanetary disks, have completed the formation of exosolar systems and remaining 20% are in the advanced stage of planetary formation.

K2-33 exo-solar system has a age of 5 to 10My is confirmed by kinematics, by Hertzsprung-Russell Diagram and by eclipsing binary analysis. Infancy of K2-33 is confirmed by spectroscopic indicators of enhanced hydrogen emission and lithium absoption as confirmed by Keck Spectra.

Cool protoplanetary disk surrounding K2-33 beyond 2AU is clearly indicated by IR bump in Spitz Space Telescope spectroscopic studies of the exo-solar system. There is 50% excess in 24μm emission of the expected stellar photosphere but there is a lack of IR excess at less than 16μm [12]. This indicates absence of warm dust close to the star [13]. The spectral energy is best fitted by including dust component at 122K having an inner edge at 2AU. This indicates that the previously present protoplanetary disk has been cleared in the process of K2-33b ice-giant formation in the inner region of the exo-solar system. The inner region is clear of gas and dust is supported by chromospheres emission. It indicates that K2-33 PHS is not accreting gas.

Detection of short-period planet in the transitional disk establishes that the gravitational accretion by K2-33b has cleared the inner region of the protoplanetary disk. IR upper limit at 880μm (as measured by Atacama Large Millimeter Array and as measured by Spitzer Fluxes) put a constraint on the residual proto-planetary disk at 0.2M+. CO emission, a tracer of molecular hydrogen, was not detected. This indicates that the primordial gas disk has largely or entirely dissipated.

K2-33 system provides direct evidence that large planets can be found in inner region shortly after the dispersal of the nebular gas well within the snowline [14]. Meaning by gas giants and subsequently terrestrial planets can be formed at inner co-rotation orbit. In fact in the new Perspective, FGK(stars with F-type, G-Type, K-type which are consudered to be "middler" spectral classes) Star System have the Initial Mass Function (IMF) to support the formation of Jovian Planets and after the gas has been exhausted support the formation of terrestrial planets but M Dwarf star systems IMF will support only terrestrial planets hence K2-33b is likely to evolve into near-Earth like planet in due course of time.

Tidal circularization of highly eccentric planet need not be invoked. The time scale on which tidal circularization takes place is much greater than the time scale on which the disk is dispersed by planet formation process or by Poynting-Robertson drag or by photo-radiation. Even formation at large separations beyond snow-line followed by migration within the gas disk is ruled out on the ground of tenuous disk associated with Dwarf M3 stars [14]. Hence in-situ planet formation at inner co-rotation orbit is the only permissible scenario. Snowline argument for sticking-agglomeration does not come into picture.

### Author's Contributions

NASA Press Releasae 20th July 1994 on the Silver Jubilee Anniversary of Man's Landing on Moon gave the data that Moon had receded by 1m in last quarter of a century from 20th July 1969 to 20th July 1994. This exact recession velocity by LUNAR LASER RANGING(LLR) experiments comes out to be 3.8cm per year. The author had set up the equation of motion of Earth-Moon with a wee-bit imbalance in centripetal and centrifugal force to allow the recession of Moon from Earth but this was a second order ordinary linear differential equation which required two boundary conditions for a complete solution. George Howard Darwin had given one boundary condition that of outer co-rotation orbit being of orbital period of 47 days. The year 1994 NASA Press Release gave the second boundary condition. Using these two boundary conditions equation of motion of E-M was completely solved in Sharma 1995 and in Author's personal communication: http://arXiv.org/abs/0805.0100.

Author was able to explain the lengthening of day curve of our Earth by applying the planetary-satellite dynamics in the above personal communication. In 2002 an approximate theoretical fit was obtained to the observational curve and reported in The World Space Congress held in Houston Texas with the title "Lengthening of Day curve could be experiencing chaotic fluctuations with implications for Earth-Quake Prediction". The observed lengthening of day curve was obtained by John West Wells and Kaula and Harris by studying the coral fossils and marine creatures and by Charles P. Sonnett et al through the study of tidalies in ancient canals and estuaries. This work was redone with a much better fit between theory and observed curve by the title "Kinematic Model of binaries – the theoretical framework of the evolving Earth-Moon System and the correspondence between observed and the predicted day length". Proceedings of International Conference on Celestial Mechanics and Dynamical Astronomy held from 15th Dec to 17th Dec.2015 at Maulana Azad National Urdu University, Hyderabad,500032. under publication. In 2004 planetary-satellite dynamics was extended to Planets and Sun and reported as "The New Perspective of birth and evolution of our Solar System" in the 35th Cospar Scientific Assembly inParis, France. By 2009 planetary-satellite dynamics was extended to a large number of exosolar systems and "The Architectural Design Rules of Solar Systems" was presented at 5th CELMEC meet in Viterbo, Italy. In 2012 Paper No. B0.3-0011-12 Iapetus hypothetical sub-satellite re-visited and it reveals celestial body formation process in the Primary-centric Framework. presented at 39th COSPAR Scientific Assembly, Mysore, India from 14th July to 20th July 2012, the correspondence between Newtonian Formalism of Synchronous Orbit and Kinematic Formalism was found and graphically illustrated for vanishingly small mass ratios [15]. In 2016, Matija Cuk gave the "Fits and Bound Model" of Earth-Moon System. Based on this new model exact match is obtained between Observed Length of Day and Theoretical Length of Day (Sharma 2023C). Now the Author has demonstrated in the present paper that the observations of David et.al of K2-33 Exo-solar System is exactly predicted by the Kinematic Model of tidally interacting binaries.

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### Endnotes

Right now there are two competing models of tidally interacting binaries namely Seismic Model and Kinematic Model [16]. In Seismic Model, Love Number and Quality Factor of the two components of the binary system are the basic parameters and these two parameters depend upon density, rigidity, viscosity and rate of periodic forcing. These parameters are known with large uncertainties for different Planets and their PHS and hence their Tidal Evolutionary History will be arrived at with equal uncertainty in Seismic Model based analysis. In contrast Kinematic Model requires the globe-orbit parameters and the age of the system which are fairly accurately known with high level of confidence, Hence any tidally interacting system is amenable to Kinematic Model Analysis with much better accuracy and most of the time these are tractable.

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