

# Analytical Modeling of a Triple-Band Artificial Magnetic Conductor

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## Abstract

Understanding and optimizing the behavior of artificial magnetic conductors require modeling them through analytical means, which can be challenging due to their complex structures. In this paper, a triple-band artificial magnetic conductor is presented, exhibiting an in-phase reflection of incident waves at 1.21 GHz, 1.61 GHz, and 2.46 GHz. The design is based on a triple square loop structure, where each frequency is controlled by one square loop. Additionally, an analytical model is proposed to predict the operating frequencies of the artificial magnetic conductor.

**Keywords:** Artificial Magnetic Conductor, Frequency Selective Surface, Transmission Line Model

## 1. Introduction

Artificial magnetic conductor is a type of metamaterial that exhibits properties similar to a perfect magnetic conductor [1]. Due to its ability to reflect incident waves with zero reflection phase, it has been implemented in many antenna designs as a back reflector to enhance their gain. The complexity of AMC structures makes it challenging to model them through analytical methods, and in most previous publications, full-wave simulation was the main analytical approach. Nevertheless, analytical methods remain of great interest and need to be explored.

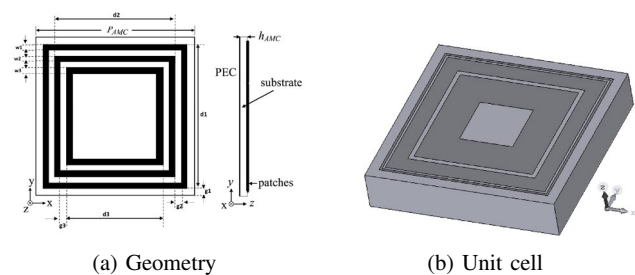
In this paper, a triple-band AMC based on a square loop structure is introduced, where each frequency can be controlled independently, facilitating its design. Moreover, an analytical model is proposed that predicts where the three null reflection phases occur. Section 2 will present the triple-band AMC design along with its analytical model. In Section 3, a comparison between simulation and modeled results will be shown and discussed. Finally, a conclusion will be drawn in Section 4.

## 2. The Triple-Band AMC

### • Design

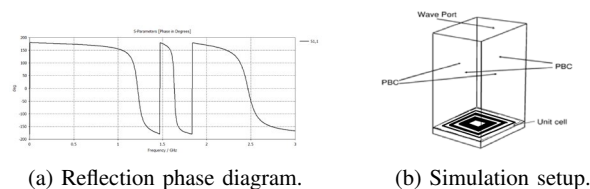
The triple-band AMC unit cell (shown in Figure 1) is based on [2], where the structure is composed of three square loops with dimensions  $d_1 = 22.29\text{mm}$ ,  $w_1 = 0.305\text{mm}$ ,  $d_2 = 21.68\text{mm}$ ,  $w_2 = 2.14\text{mm}$ ,  $d_3 = 15.64\text{mm}$ ,  $w_3 = 4.32\text{mm}$ , printed on a substrate with relative permittivity  $\epsilon_r = 10.2$  and dimensions  $p = 25\text{mm}$ ,  $h = 5\text{mm}$ . The AMC operates at three frequencies, where each frequency is controlled by one square loop respectively.

To obtain the reflection phase diagram of the AMC shown in Figure 2a, a unit cell model in CST Microwave Studio software using the frequency domain solver with periodic boundary



**Figure 1: The Proposed Triple-Band AMC**

Conditions was simulated. A perfect magnetic and electric conducting wall was imposed on  $\pm y$  and  $\pm x$  directions, respectively. The waveguide port was placed in the far-field region and was used to excite the signal along  $+z$  while perfect electric ( $E_t = 0$ ) was applied due to a full ground along  $-z$ .



**Figure 2: Simulation Overview: Setup and Results**

These boundary conditions were used to imitate the periodic nature of the structure (Figure 2b). The simulation results show resonance at 1.21 GHz, 1.61 GHz, and 2.46 GHz, at which the AMC can reflect incident waves with a zero-reflection phase, having the same characteristics as PMC.

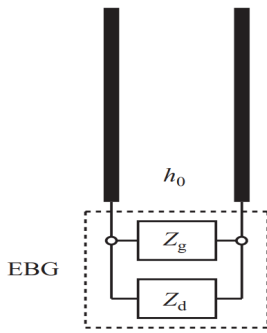
## 2.1 Analytical Model

To model the proposed triple-band AMC analytically, a transmission line model is established Figure 3. The AMC is divided into a frequency selective surface (FSS) and a slab treated as a spacing medium between the FSS and the ground plane [3]. The surface impedance  $Z_s$  is then calculated from the parallel connection of the FSS grid impedance  $Z_g$  and the slab impedance  $Z_d$ :

$$Z_s = \frac{Z_g Z_d}{Z_g + Z_d} \quad (1)$$

The reflection coefficient is then computed as:

$$\Gamma^{TE} = \frac{Z_s \cos \theta - \eta_0}{Z_s \cos \theta + \eta_0}, \quad \Gamma^{TM} = \frac{Z_s - \eta_0 \cos \theta}{Z_s + \eta_0 \cos \theta} \quad (2)$$



**Figure 3: Equivalent Transmission Line Model for Plane Wave Incidences [3].**

Where  $\eta_0$  is the free space wave impedance and  $\theta$  is the incident angle. The resonant frequency for the zero-degree reflection phase must satisfy:

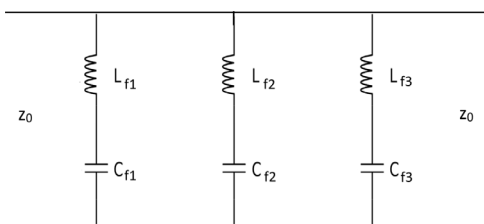
$$X_g(\omega_0) + X_d(\omega_0) = 0 \quad (3)$$

$$X_g(\omega_0) = \text{Im}(Z_g), \quad X_d(\omega_0) = \text{Im}(Z_d)$$

$Z_d$  is simply the input impedance of a TEM line section of length  $h$ , and in most interesting cases  $kdh \ll 1$ , both  $Z_{dTE}$  and  $Z_{dTM}$  are practically equal [4,5].

$$Z_d^{TE, TM} \approx j\omega\mu_0 h \quad (4)$$

The grid impedance  $Z_g$  of the FSS depends on the specific geometry used in the design. In our case, the triple-square loop is modeled using the equivalent circuit approach (shown in Figure 4). For transverse electrical (TE) wave incidence, the vertical strips act as an  $L$  impedance, and the horizontal gratings as a  $C$  impedance [6,7].



**Figure 4: Equivalent Circuit Model of the Triple Square Loop FSS**

The basic equations for calculating the values of inductance and capacitance are found in Marcuvitz [8] and are given in general form by:

$$\text{Inductance: } \frac{X_L}{Z_0} = \omega L = \frac{d}{p} \cos \theta F(p, s, \lambda) \quad (5)$$

$$\text{Capacitance: } \frac{B_c}{Y_0} = \omega C = \frac{4d}{p} \sec \theta F(p, s, \lambda) \epsilon_{\text{eff}} \quad (6)$$

$$\text{where: } F(p, s, \lambda) = \frac{p}{\lambda} \left( \ln \csc \frac{\pi s}{2p} + G(p, s, \lambda) \right) \quad (7)$$

And  $G$  is the correction term:

$$G(p, s, \lambda) = \frac{1}{2} \frac{(1 - \beta^2)^2 \left[ \left(1 - \frac{\beta^2}{4}\right) (A_+ + A_-) + 4\beta^2 A_+ A_- \right]}{\left(1 - \frac{\beta^2}{4}\right) + \beta^2 \left(1 + \frac{\beta^2}{2} - \frac{\beta^4}{8}\right) (A_+ + A_-) + 2\beta^6 A_+ A_-} \quad (8)$$

The six circuit elements given in Figure 4:  $L_{f1}$ ,  $C_{f1}$ ,  $L_{f2}$ ,  $C_{f2}$ ,  $L_{f3}$ ,  $C_{f3}$ , are calculated as follows:

$$L_{f1} = \frac{2 \cdot (L_1 \parallel L_2) \cdot d_1}{p}, \quad C_{f1} = \frac{0.75 \cdot C_1 \cdot d_1 \cdot \epsilon_{\text{eff}}}{p} \quad (10)$$

$$L_{f2} = \frac{L_3 \cdot d_2}{p}, \quad C_{f2} = \frac{(C_1 \text{ in series with } C_2) \cdot d_2 \cdot \epsilon_{\text{eff}}}{p} \quad (11)$$

$$L_{f3} = \frac{1.455 \cdot L_4 \cdot d_3}{p}$$

$$C_{f3} = \frac{(C_1 \text{ in series with } C_2 \text{ in series with } C_3) \cdot d_3 \cdot \epsilon_{\text{eff}}}{p} \quad (12)$$

Where:

$$\begin{aligned} L_1 &= F(p, w_1, \lambda), & C_1 &= 4F(p, 2g_1, \lambda), \\ L_2 &= F(p, w_2, \lambda), & C_2 &= 4F(p, g_2, \lambda), \\ L_3 &= F(p, 2w_2, \lambda), & C_3 &= 4F(p, g_3, \lambda), \\ L_4 &= F(p, 2w_3, \lambda) \end{aligned} \quad (13)$$

The factor  $\epsilon_{\text{eff}}$  present in Equation (6) was introduced by Munk [9] with the value of  $\epsilon_{\text{eff}} = 0.5(\epsilon_r + 1)$ .

It is important to note that the equations presented here have certain conditions that need to be respected to obtain correct results, such as  $s/p \ll 1$ ,  $p/\lambda \ll 1$ , and  $p(1 + \sin \theta)/\lambda < 1$ .

## 3. Results and Discussion

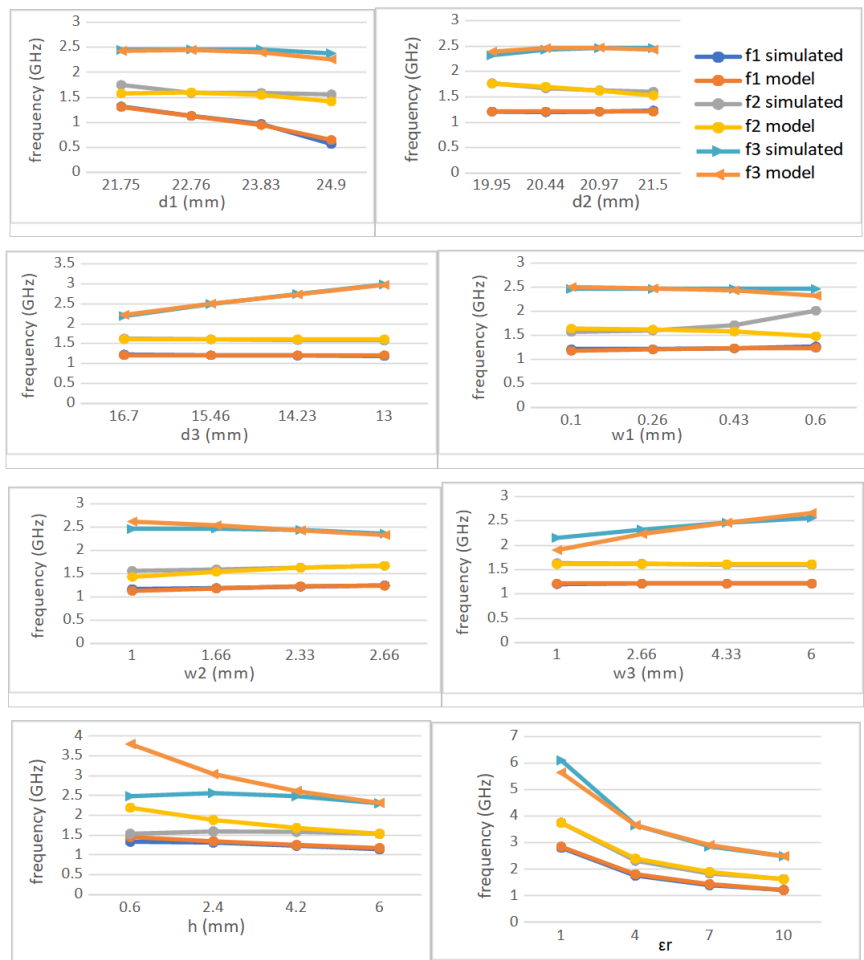
By solving Equation (3) for the proposed dimensions, we obtain the three frequencies predicted by the analytical model: 1.21 GHz, 1.61 GHz, 2.46 GHz, which are identical to the electromagnetic (EM) simulation.

To further investigate the analytical model and demonstrate how different parameters affect the resonance frequencies, a parametric study is conducted and shown in Figure 5.

The parametric study reveals that the three resonant frequencies,  $f_1, f_2, f_3$ , of the artificial magnetic conductor (AMC) are highly dependent on the square sides  $d_1, d_2, d_3$  respectively, and on the substrate characteristics such as thickness  $h$  and permittivity  $\epsilon_r$ . The results also demonstrate that varying one frequency does not significantly affect the other two frequencies, and sometimes

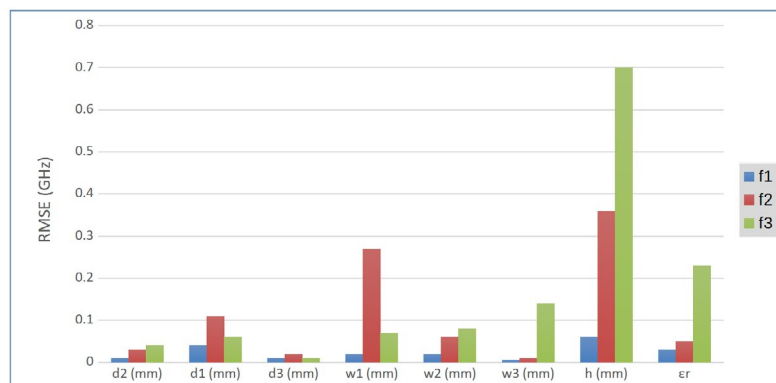
they remain unchanged. This independence allows tuning one frequency by altering the dimensions of the corresponding square loop without affecting the remaining frequencies, greatly facilitating the design process.

Lastly, the analytical model successfully predicted the three resonant frequencies for most cases. The root-mean-square error (RMSE) has been calculated for the complete parametric



**Figure 5: Results of the Parametric Study on the Triple-Band AMC**

Study, where the frequencies predicted by the model were evaluated against the electromagnetic (EM) simulation, and the results are shown in Figure 6.



**Figure 6: RMSE Evaluation for Each Parameter**

The figure shows that the analytical model has the least accuracy when it comes to the thickness parameter. Moreover, the third frequency has the highest error among all three frequencies, so more work needs to be done to improve the model.

#### 4. Conclusion

In this paper, a novel triple-band Artificial Magnetic Conductor (AMC) unit cell has been proposed. It has been demonstrated that the three operating frequencies (1.21 GHz, 1.61 GHz, 2.46 GHz) at which it reflects incident waves inphase can be set independently, significantly facilitating the design process. Moreover, an analytical model of the tri-band AMC is developed. This model can predict the frequencies at which the AMC operates with an error estimation lower than 170 MHz compared to the full wave simulation.

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