

Analysis of Magnetohydrodynamic Fluid Flow

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Submitted: 2024, Nov 11; **Accepted:** 2024, Dec 03; **Published:** 2024, Dec 13

Citation: Veeresh, J., Kalyan, S. (2024). Analysis of Magnetohydrodynamic Fluid Flow. *Petro Chem IndusIntern*, 7(4), 01-09.

Abstract

The current problem MHD flow and magnetic field over a secant curved annular circular plates lubricating with couple stress fluid. In this study Incompressible fluid flow model is considered. The governing equations are evaluated analytically. Numerical computation has been carried out by using Mathematica software and results are illustrated graphically. It is established that the impact of couple stress, lubrication, boundary condition fluid improves the bearing performance.

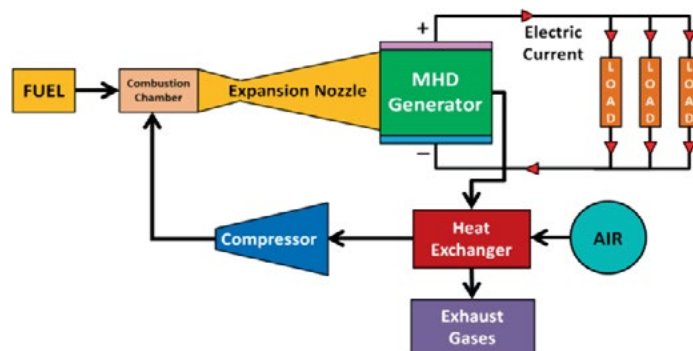
Keywords: Magnetic Field, Couple Stress, Lubrication, Boundary Condition

1. Introduction

It is the study of the behavior of the electrically conducting fluids that are plasmas, liquid metals and the saltwater it is possible in the presence of magnetic and Electric fields. Magnetohydrodynamic can also be called as magneto fluid dynamics or hydromagnetic. This magnetohydrodynamic can be defined from word magneto (magnetic field) and hydro(water) and movement(dynamics). A Magnetic Field is the vector field surrounded by Magnetic material and Electric current. It can be represented by the symbol was “B”. Magnetic field describes the magnetic influence on moving electric charges and electric current and magnetic material. This Magnetic field is discovered by “Hans Christian Oersted” he found that the “ELECTRIC CURRENT CREATES A MAGNETIC FIELD AROUND IT”. All bearing surfaces are rough to some extent, hence more interest is shown to study roughness using hydrodynamic lubrication theory recently. In the literature to discuss the effect of surface roughness on bearing system, many authors have proposed a tremendous work. First paper which discuss the effect of rough surface was by Ting [1]. Later, Burton presented rough surface by using concept of Fourier series type approximation [2]. The stochastic models for hydrodynamic lubrication of rough surface was developed by Christensen and Elord and their study derive an important results which is applicable to rough bearings was the Reynold’s equation in general form [3,4]. Based on this result, Prakash and Tiwari and Gururaj an and Prakash have done survey on different types of porous bearings with Newtonian fluid [5,6]. The combined study of roughness and lubricant additives oil for distinct porous bearing systems is studied by Naduvinamani and

from the result due to presence of lubricant additives there is an considerable influence on the characteristics of rough porous bearings [7].

MHD was given by the scientist called “Hannes Alfven”, for this he got a Nobel prize in 1970 in physics . MHD was developed in a 1942 published paper in Nature that called “Existence of Electromagnetic Hydrodynamic Waves” done by Syed Tasneem Fathima [8] shows that the combined study of rough surface in presence of MHD for circular plates results, increase in surface asperity and a large amount of load is released in the bearing and increases the response time of squeeze-film motion as compared to the smooth case [8]. The combined study on MHD and surface roughness is carried out by many authors and from their result it’s found that the surface roughness in the presence of magnetic field is significant on squeeze film characteristics of the bearings [9-11]. In the Tribology literature, so far the influence of transverse magnetic field between rough curved plates with couple stress squeeze film lubrication has not been discussed. Hence, the present paper aims to analysis the mixed discussion of rough surface and MHD over the couple stress squeeze film characteristics between curved plates and obtained numerical findings are compared with classical case studied by Hanumagowda [14]. Kalyanhas analyzed the fluid flow behavior of immiscible incompressible fluid[15-18]. MHD can be defined by the principles Induction, Lorentz force, Magnetic Reynolds Number, Navier-Stokes Equation.



2. Lubrication

Lubrication is defined as the process of applying a lubricant to reduce friction between surfaces in relative motion, heat generation and energy loss. Here Lubricants are liquids, Greases. Lubrication can take part in several mechanical system and Industrial application lowering friction between moving parts. For example, if we apply a Grease or oil on the surface lubricant creates a thin layer that separates from the surface. This layer not only lowers the friction and also generates the heat. This lubricant was used also in capsicum soaps from the thousands of years. The growth of lubrication was accelerated for the industrial revolutionary. The scientist called “Osborne Reynolds” he worked the fundamental study of the lubrication, and “Sir Issacs Newton” he gave that the Newton’s study of viscosity was give the ground work for the understanding fluid lubrication.

2.1 Application of Magnetic Field

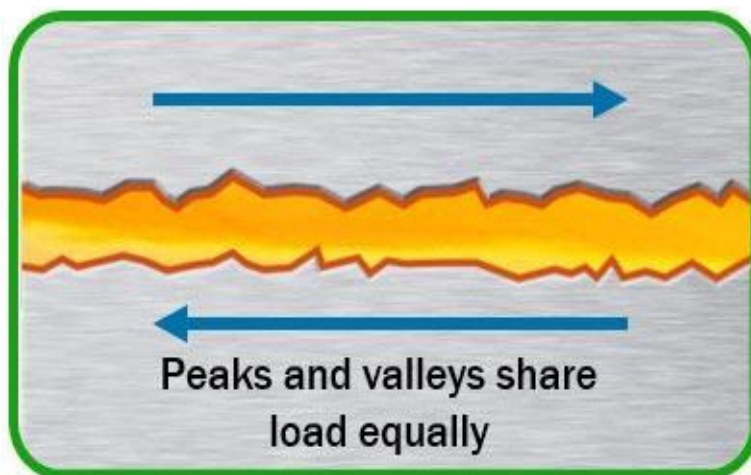
- Electromagnetic devices like motors, generators, transformer and inductors.
- It can also be used in magnetic storage like hard drives and magnetic tape.
- And also in Medical imaging like MRI (Magnetic Resonance Imaging) scanner will use
- strong magnetic field to produce images of the inside of the

body.

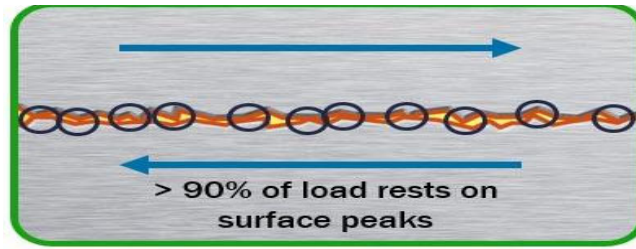
- In power generation it can be used to convert mechanical energy to electrical energy and also electrical energy to mechanical energy.
- In electrical motor conversion of electrical energy to mechanical motion was used by magnetic field.
- For the direction we use compass, magnetometers in this magnetic field are used.
- Magnetohydrodynamics can be used in Astrophysics and space science for the solar physics and the stellar interior.
- It can be used in Geophysics and Earth sciences for the Geomagnetism and for the planetary magnetospheres.
- It can be used in Engineering and Technology for the MHD power generation and MHD pumps and propulsion.
- It can be used in Fusion Energy Research for the Tokamaks and magnetic confinement fusion.
- It can be used in Environmental and Biological science for the environmental flows and biological fluid dynamics.

2.2 Types of Lubrication

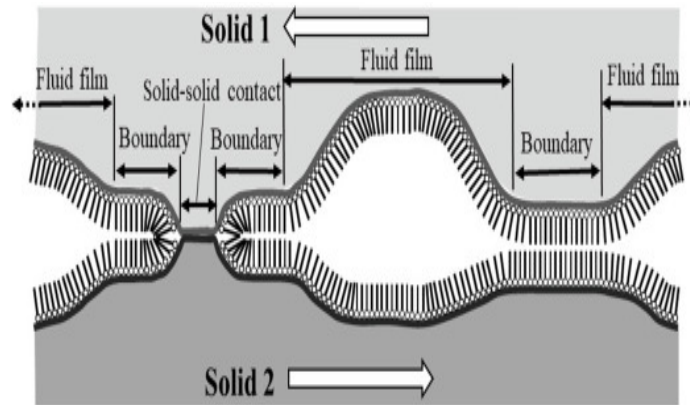
1. Fluid Film Lubrication: This type is used to making a thin layer of lubricant between the moving surfaces to reduce friction and wear.



2. Boundary Lubrication: This type is used to protect the layer by preventing direct metal-to-metal contact.



3. Solid Lubrication: It can be used when the solid materials like graphite, molybdenum disulfides are involve in lubricants. These are used for reducing the friction and wear between sliding surfaces by forming a solid film that separates the moving parts.



2.3 Application

- It can be used in Automotive industry like engines, transmissions to minimize the friction and heat generation.
- It can be used in industrial machinery as manufacturing equipment, conveyor systems and heavy machinery to proceed smooth operation.
- It can be used in Aerospace as aircraft engines, landing gear.

- It can be used in Geomechanics to analyze the stress and deformation of the geological materials.
- And also in Nanotechnology it is used at the Nanocycle for the mechanical behavior of materials.
- It is also used in mechanical engineering for the design and analysis of materials and structures.

3. Couple Stress

Couple stress can be defined as it is a conception in a continuum mechanics that enlarges classical stress theory with the effect of rotational forces and moment at the microscopic level. It is an advanced topic that effect from the microstructural rotational forces and moment. It is especially used for the material to describe with significant internal structures such as biological tissues, foams and some engineered material. couple stress can also introduces the additional tensor, the couple stress tensor. Here stress tensor can be represented as that distributes within a material and couple stress tensor will represents a distribution of the internal moments within a material. On couple stress more scientist has worked mainly are "Ronald S.Rivin" in 20th century he gave that contribution to the theoretical foundation of the couple stress theory, then "Eric Reissner" he extended the classical elasticity to include couple stress and also developed the theory of couple stress and also problems in mechanics.

3.2 Boundary Condition

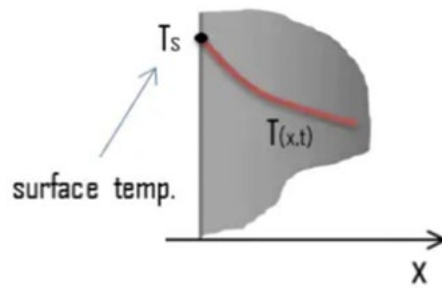
Boundary condition was the essential tools in the mathematical modeling mainly for the study of differential equations. These defines how the physical system interacts with it surroundings at the boundaries. Boundary condition are needed to analyses the behavior of the systems at boundaries for the domain in heat conduction, fluid flow and structural analysis. Boundary conditions developed in mathematics, physics and engineering with the contribution of more scientist and given the more thought of history in Boundary Condition. In those "Joseph Fourier" (1780-1830) he given the Fourier series and Fourier transform which are essential for solving the heat conduction problems with the boundary conditions, then "Carl Friedrich Gauss (1774-1855) he made the significant contribution for the theory of potential and boundary value problems in electrostatics and the gravitation, then "George Green (1793-1843) he developed the 'Greens functions' this method used solving differential equation with boundary conditions.

3.1 Application

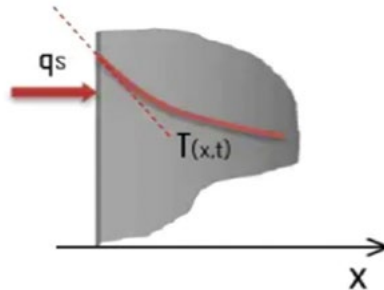
- Couple stress can be used in biochemist for the behavior of the biological tissues.
- It can be used in Micromechanics for understanding the behavior of material with Microstructure.

3.3 Types of Boundary Condition

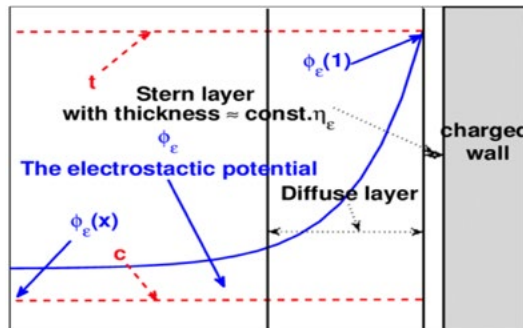
- 1. Dirichlet Boundary Condition:** It defines the specific value of the function at the boundary.
Ex: Heat conduction in a rod.



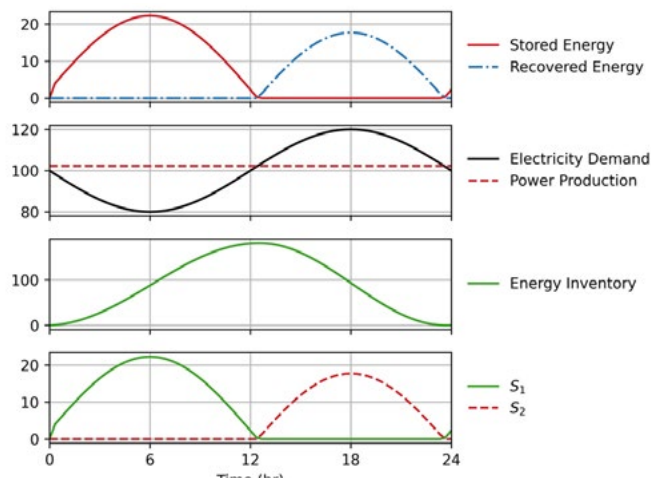
2. Neumann Boundary Condition: It defines the specific value of the derivative of the function at the boundary.
 Ex: Heat conduction in a rod with insulated end.



3. Robin Boundary condition or Mixed Boundary condition: It is defined as combination of Dirichlet and Neumann boundary condition that involves both functional value and its derivative.
 Ex: Heat conduction in a rod with convective heat transfer.



4. Periodic Boundary Condition: It was used when the problems of domain is periodic, that means the function values repeat after a certain interval.
 Ex: Heat conduction in a symmetric plate.



3.4 Application of Boundary Condition

- In physics it can be used as heat conduction, wave propagation, electromagnetism.
- In Engineering it can be used as structural analysis, Fluid dynamics, Thermal analysis.
- In mathematics it can be used as solving partial differential equations, modeling dynamics system.

3.5 Applications of Advanced Magnetohydrodynamics

Advanced MHD is used in the Astrophysics and the space physics for understanding the behavior of the plasmas in the solar corona, solar wind and planetary magnetospheres. It is used in the nuclear fusion for the magnetic confinement fusion devices like tokamaks and the stellarators. It is used in Geophysics for earth's molten

outer core.

3.6 Mathematical Formulation

Figure 1 exemplifies the geometry of squeeze film between the secant curved annular circular plates. The thickness of the film h for the squeeze film can be given by the secant function and h_m be the minimum film thickness, B_0 is applied along the direction of z -direction. It is supposed that the fluid film is thin, the body couples and body forces are ignored. Under these assumptions of hydrodynamic lubrication theory valid to thin film and Stokes theory for couple stresses, the continuity equation and the Magnetohydrodynamic (MHD) momentum equation in polar form becomes.

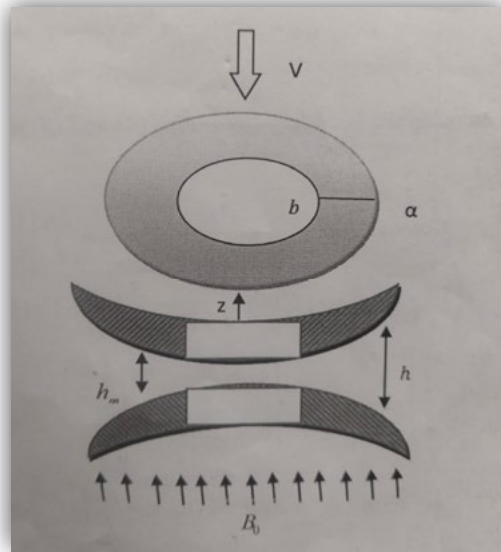


Figure 1: A Incompressible Fluid is Field in the Channel in Presence of Magnetic Field; Considering All Transport Properties are Constant the Governing Equation Becomes

$$\mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \sigma B_0^2 u = \frac{\partial p}{\partial r}$$

$$\frac{\partial p}{\partial z} = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0$$

The relevant boundary conditions are

- At the upper surface $z = h$ $u = 0$;
- At the lower surface $z = 0$.

Solving the above governing equation using the boundary conditions, using the mathematical we get

$$\begin{aligned}
\mathbf{u} = & -e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} y - \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) - e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) + \\
& 2e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) - e \sqrt{2} \left(\sqrt{\frac{1-4AM^2}{A}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) + \\
& e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) - 2e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) + \\
& e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \sqrt{2} \sqrt{\frac{1+4AM^2}{A}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) - 2e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) + \\
& 2e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) + 2e \sqrt{2} \left(\sqrt{\frac{1-4AM^2}{A}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) - \\
& 2e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \sqrt{2} \sqrt{\frac{1+4AM^2}{A}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) + \\
& e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \sqrt{2} \sqrt{\frac{1-4AM^2}{A}} y + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) - \\
& 2e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \sqrt{2} \sqrt{\frac{1-4AM^2}{A}} y + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) + \\
& e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \sqrt{2} \sqrt{\frac{1+4AM^2}{A}} + \sqrt{2} \sqrt{\frac{1-4AM^2}{A}} y + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y \right) - \\
& e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y + \sqrt{2} \sqrt{\frac{1+4AM^2}{A}} y \right) + \\
& 2e \left(\frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y + \sqrt{2} \sqrt{\frac{1+4AM^2}{A}} y \right) - \\
& e \sqrt{2} \left(\sqrt{\frac{1-4AM^2}{A}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} + \frac{\sqrt{1-4AM^2}}{\sqrt{2}} y + \sqrt{2} \sqrt{\frac{1+4AM^2}{A}} y \right) y \Big/ \\
& \left(2 \begin{pmatrix} \tilde{\mathbf{a}} & -\tilde{\mathbf{a}} \\ \frac{\sqrt{1-4AM^2}}{A} & \frac{\sqrt{1-4AM^2}}{A} \end{pmatrix} \begin{pmatrix} -1 + \tilde{\mathbf{a}} & \\ \frac{\sqrt{1-4AM^2}}{A} & \frac{\sqrt{1-4AM^2}}{A} \end{pmatrix} \mathbf{M}^2 \right);
\end{aligned}$$

4. Result and Discussion

The Coupled nonlinear governing equations are solved by using Mathematica software and results are discussed below

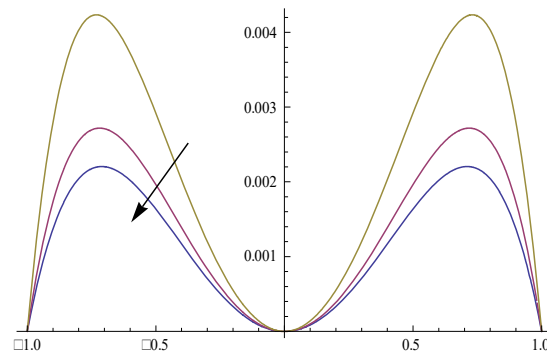


Figure 1.1: velocity profile for different values of HartmannNumber

In figure 1.1. it is observed that For $M = \{10,20,30\}$, Increase in the magnetic field decreases the flow due to Lorenz force, by keeping other parameters $p = 1, A = 5$.

Figure 1.2 velocity profile for different values of presure

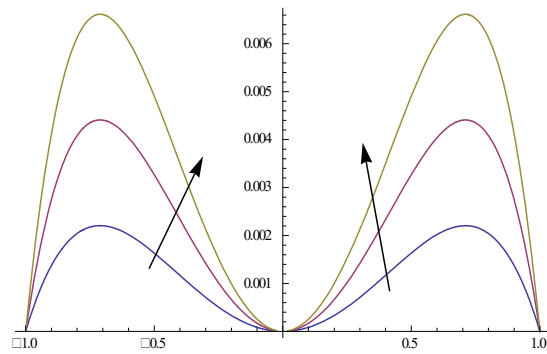


Figure 1.2: Shows the velocity profile for $p=\{1,2,3\}$, as we increase the pressure, flow enhances, this is due to Lorenz force, by keeping $A = 5, M = 10$.

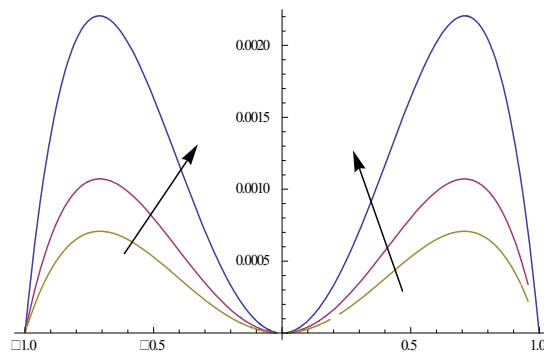
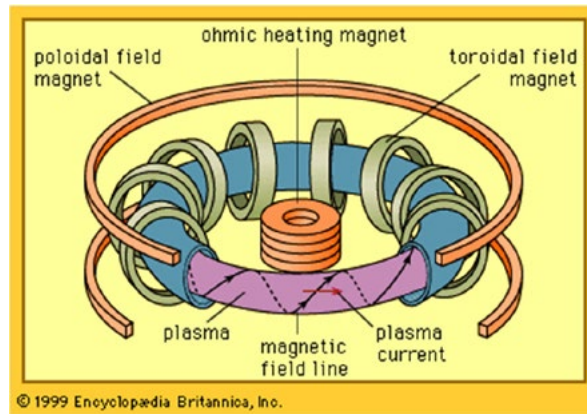


Figure 1.3: Velocity profile For $A=\{5,10,15\}$, Flow Enhances in both the Region.

4.1 PlasmaConfinement in a Tokamak

Here tokamakis the device used for the confine hot plasma using the magnetic field to the sustain nuclear fusion reaction. This is the one

of most researched forming for the magnetic confinement fusion and the promising results in the achievement of the conditions necessary for the fusion.



Example: A tokamak is the device used to confine the plasma using magnetic fields for the purpose of nuclear fusion. Suppose the magnetic field strength B in a tokamak is 7 Tesla, the temperature T is 10^{10} kelvin. Calculate the Larmor radius (gyro radius) for the deuterium ion (mass $m_i = 3.34 \times 10^{-27}$ kg and charge $e = 1.6 \times 10^{-19}$ C).

Solution:- Given, Magnetic field (B) = 7 Tesla
 Plasma Density (n) = 2×10^{20} particles/m³
 Temperature (T) = 10^{10}
 Larmor radius (r_L) = ?

We know that, the Larmor radius r_L given by, $r_L = \frac{v_{\perp}}{w_c}$

To calculate v_{\perp} we have, $v_{\perp} = \sqrt{\frac{2k_B T}{m_i}}$

Where k_B is the Boltzmann's constant = 1.38×10^{-23} J/K

$$v_{\perp} = \sqrt{\frac{2 \times 1.38 \times 10^{-23} \times 10^{10}}{3.34 \times 10^{-27}}}$$

$$v_{\perp} = \sqrt{\frac{2.76 \times 10^{-13}}{3.34 \times 10^{-27}}}$$

$$v_{\perp} = \sqrt{0.8263473053 \times 10^{14}}$$

$$v_{\perp} = 9.0903 \times 10^5 \text{ m/s}$$

Then, next we have to calculate the cyclotron frequency w_c

$$w_c = \frac{eB}{m_i}$$

$$w_c = \frac{1.6 \times 10^{-19} \times 7}{3.34 \times 10^{-27}}$$

$$w_c = 3.34 \times 10^8 \text{ rad/s}$$

Now we calculate the Larmor radius

$$r_L = \frac{v_{\perp}}{w_c}$$

$$r_L = \frac{9.0903 \times 10^5}{3.34 \times 10^8}$$

$$r_L = 2.72 \times 10^{-3} \text{ m}$$

Therefore the Larmor radius for the deuterium ion in this tokamak is approximately 2.72mm

4.2 Magnetic Pressure in a Plasma

Here Magnetic pressure is the crucial concept in the plasma physics and the magnetohydrodynamic. This represents the pressure exerted by the magnetic field on the plasma, this pressure

plays a significant role for the stability and confinement of the plasmas in the various applications those are tokamak, astrophysical phenomena, and the industrial processes.

Example: A plasma has a magnetic field strength B of 1 Tesla. Calculate the magnetic pressure p_m exerted by this magnetic field.

Solution:- Given, Magnetic field (B) = 1 Tesla

Magnetic pressure (p_m) = ?

We know the magnetic pressure (p_m) = $\frac{B^2}{2\mu_0}$

Where, μ_0 is the permeability of the free space ($4\pi \times 10^{-7}$ H/m)

$$p_m = \frac{B^2}{2\mu_0}$$

$$p_m = \frac{1^2}{2 \times 4\pi \times 10^{-7}}$$

$$p_m = \frac{1}{8\pi \times 10^{-7}}$$

$$p_m = 3.91 \times 10^5 \text{ pa}$$

Therefore the magnetic pressure in the plasma is approximately 39.1×10^2 kPa.

5. Conclusion

The study of MHD is unpredictable in future research area we work more on that. Some conclusion points are based on radius, pressure, Lorentz force

- When the increase in magnetic field decreases in the flow of fluid due to the Lorentz force.
- When there is increase in the pressure flow increases due to the Lorentz force.
- When Larmor radius decreases when the increase in cyclotron frequency and velocity perpendicular increases.
- Increase in the Hartmann number decreases the flow.
- Impact of couple stress, lubrication, boundary condition fluid improves the bearing performance.

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