

operator :*V*End*S*, *V*, and *T* is a linear

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A Technical Lemma on Unitary (\mathfrak{g} **,** K **)-Modules** operator :*V*End*S*, *V*, and *T* is a linear **Current Research in Statistics & Mathematics (ISSN: 2994-81 A Technical Lemma on Unitary (** $\text{I}\,\text{J}$ **,** *K***)-Modules** operator :*V*End*S*, *V*, and *T* is a linear

Francisco Bulnes* Current Research in Statistics Burnet Volume 3, Issue3: page No: 01……

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Chalco, Mexico. *Tecnológico de Estudios Superiores Chalco (TESCHA) Chalco, Mexico maximally superiores Chalco (1ESCITA) In a pre-hilbertian structure of an unitary (g, K) -* $Teenológico de Estudios Superiores C$ **Volume 3, Issue3: page No: 01……** on *U*- , where *U*⁺ and *U*-

Corresponding Author Corresponding Author Francisco Bulnes, Tecnológico de Estudios Superiores Chalco (TESCHA)
Chalco, Mexico. operator :*V*End*S*, *V*, and *T* is a linear r**esponding Author**
isco Bulnes, Tecnológico de Estudios Superiores Chalco (TESCHA)

, where *U*⁺ and *U*-

spinor space of *V*.

Submitted: 2024, Oct 01; Accepted: 2024, Nov 04; Published: 2024, Nov 20 element . In the technical lemma, we want Submitted: 2024, Oct 01; Accepted: 2024, Nov 04; Published: 2024, Nov 20

o_p. Mexico.

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Abstract
In a pre-hilbertian structure of an unitary (**§**, K) -module underlie spinor subspaces that are spin invariant modules under right In a pre-nuberian structure of an anitary (\mathcal{G} , K)-module underlie spinor subspaces that are spin invariant modules under right
and left actions of G and that are images of endomorphisms restricted on t belonging to In a pre-hilbertian structure of an unitary $(\mathbf{\mathfrak{C}}, \mathbf{\mathit{K}})$ -module un tian structure of an unitary (\mathfrak{g} , K) -module underlie spinor subspaces that are spin invariant modules under right \mathbf{r} is the compact image of all the endomorphisms of all the endom subspaces of *SV* , the set of elements . In the technical lemma, we

on *U*-

Keywords: Pre-Hilbertian Structures, Spinor Space, Spinors, Unitary (\mathbf{g} **, K) Modules** Keywords: Pre-Hilbertian Structures, Spinor Space, Spinors, Unitary ($\int \int f(x) dx$) Modules Keywords: Pre-H ce. Spinors, Unitary $($ \mathfrak{m} , K) Modules

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1. Introduction

establish that $\mathfrak{so}(V)$ is the compact image of all the $2²$ endomorphisms of the Lie algebra $S(V)$ restricted to the $\langle \gamma(v)v, w \rangle = -\langle u, \gamma(v)v \rangle$, auvant In a pre-hilbert in an unitary g, *K* μ of an unitary g, *K* μ -module unita spinor subspaces that are spin invariant modules under \mathcal{L} . Mean Lemma decomposition $g = t + p$. However, the nature of their
dimension on R w subgroup of G , has spin modules. These can be related
Let G be a real reductive group. The by tensor product and are finally subspaces spinor space of V characterized by spinors. Likewise, for example
 $S(U) = \{y(x) \in \text{End}(S) | y(x)^2 = 0, y(x) \}$ $S(V) = {\gamma(v) \in \text{End}(S) \mid \gamma(v)^2 = -(v, v)}$ (v) $\otimes I$) = γ (v) T, where γ is the Dirac
Definition 2.2. A (g, K) -module is unitary if there is a pre-
 $V \rightarrow \text{End}(S)$), $v \in V$ and T is a linear isomorphism. Here I^+ is the identity mapping on U^+ , or $w \in V$, $\langle \cdot \rangle$ is the top-ideal lomporary we went $\langle \cdot \rangle$ such that subalgebra t [1]. This has advantages to establish linear $2²$ Mathematics Substitute Substitute Substitute Substitute $2²$ One condition inherent of those unitary (g, *K*)-modules whose endomorphisms are in g is that these must be endomorphisms in the corresponding compact maximal
 $\frac{1}{2}$ torus T, which is isomorphic to the standard torus T and \mathcal{F} between $(\cdot, \langle , \rangle)$ is determined for the p on *U*, where *U*⁺ and *U* are subspaces of *S*(*V*), the set of **Lemma 2.1.** In *S*(*V*) exis $\forall y \in V$ and $y, w \in S(V)$ with $\forall v \in V \text{ and } u, w \in S(V) \text{ with } \gamma(v) \in \text{End}(S),$
ct image of all the $\frac{1}{2}$ $\frac{1}{2}$ restricted to the algebra $t \subset \mathfrak{g}$, considering the Cartan torus T, which is isomorphic to the standard torus T
and whose Lie group is an compact Abelian Lie $\gamma(v)^2 = -(v, v)I$, $\forall v \in V$. actions of *G* and that are images of endomorphisms restricted on t $T(\gamma(v))\otimes I + \gamma(v) \otimes I) = \gamma(v)T$, where γ is the Dirac
Definition 2.2 \land (x E) module is unitary if there element γ (v). In the technical lemma, we want γ *In such that* Dirac operator :*V* End*S*, *V*, and *T* is a linear isomorphism. Here operator $(\gamma : V \to \text{End}(S))$, $v \in V$, and T is a linear hill on U, where U⁺ and U are subspaces of $S(V)$, the set of Let *module underlie spinor subspaces that are spin* efficient γ (*O j*. In the technical left and γ we wand *module underlie spinor subspaces that are spin that are images of endomorphisms restricted on t belonging to the Lie algebra g.* $\frac{\Delta V}{\Delta}$

whose ordomorphisms are in \star is that these must be $\overline{}$
2. Mean Le $\frac{C}{C}$ $\frac{C}{C}$ $\frac{1}{\sqrt{1}}$ **C** \sum_{ν} Liet group is an compact Abelian Liet \sum_{ν} (*v*)² – guheroun of *G* has spin modules. γ (*b*). In the technical lem k is the \overline{f} the Lie clocking $S(1)$ is estimated to the Spinors, Unitary (g, *K*) Modules whose en Tesuntico C C C C C D $\text{D$ Unitary (g, *K*)-Module. *Curr Res Stat Math, 3*(3), ϵ ndomorp *I*, *in the corresponding compact may imal S*(*I*) see *S*(*I*) $\frac{1}{2}$ expected by $\frac{1}{2}$ *V*}, $\langle \cdot \rangle$ *such the structure such that* , t [1]. This has advantages to establish linear Unitary (g, *K*)-Module. *Curr Res Stat Math, 3*(3), characte₁ Spinors, Unitary (g, *K*) Modules A pre-hilbertian structure of an unitary g, *K* μ -module is the Hermitian structure is the Hermitian structure is the Hermitian structure is the Hermitian structure in the Hermitian structure is the Hermitian structure g is that these must be ear ablish linear and $\frac{1}{2}$ and $\frac{1}{2}$. $t \subset \mathfrak{g},$ $\mathfrak{g} = \mathfrak{t} + \mathfrak{p}.$ so(*V*)

isomorphisms and define a restriction of $\mathfrak{so}(V)$ sobre Tecnológico de Estudios Superiores Chalco (TESCHA) $\wedge VX$. $\Delta V \Delta$. isomorphisms and define a restriction of $\mathfrak{so}(V)$ sobre **2. EXECUTE: 2. EXECUTE Definition** 2.1. **Let** *Let**Let**Let**b**Let V**b**Let**d**Let**d**let**d**let**d**let**d**let**d**let**d**let**d**let**d**let**d**let**d**let**d**let**d**let**d**let**d**let**d**let* les is extended a restriction of $\mathcal{U}_{\mathcal{U}}$ and $\mathcal{U}_{\mathcal{U}}$ so $\mathcal{U}_{\mathcal{U}}$

space to V, , *is determined for the pair*

such that

,

the nature of their
Definition 2.1. Let V be a vector space of finite space to $(V, (,))$ is determined for the pair μ . However, the nature of their
dimension on R with inner product (,). Then a spinor space to $(V, (,))$ is determined for the pair *I, V* **Definition 2.1.** Let V be a vector space of finite uim **Definition 2.1.** *Let V* be a vector space of finite *aimension on* ℝ *with inner product* (, *). Then a spinor* act maximal **Definition 2.1. Let V** *Let V* **a vector space of** *Let* **V and** *W* **and** *W* \int *index T inner space to* $(V, (,))$ is determined for the pair

$$
\gamma(v)^2 = -(v, v)I, \quad \forall v \in V.
$$
 (1)

In be related
Let G be a real reductive group. Then, $S(V)$ is the
whenever nor space of V. *spinor space* of *V*. $\frac{1}{2}$ $\frac{1}{2}$ α α $S(V)$ is the space of *S* is the *n* is the $\frac{1}{2}$ spinor space of *V*.

$$
S(V) = \{ \gamma(v) \in \text{End}(S) \mid \gamma(v)^2 = -(v, v)I, \forall v \in V \},
$$

$$
\text{Dirac}
$$

 $V \to \text{End}(S)$, $v \in V$, and T is a linear hilbertian Structure (,) on V such that $\forall X \in \mathfrak{g}, k \in K$ and u, **T**, which is isomorphic to the standard torus *T* and whose Lie group is an $W \in V$, **Definition 2.2.** A g, *K*-module is *unitary* if there is a pre- $W \in V$, $W \in V$, $w \in V$, *w w W w W* , *W W* , *W*

Example the subspaces of $S(V)$ the set of Lemma 2.1. In $S(V)$ exists a pre-hilbertian structure by tensor product and are finally subspaces characterized by spinors. Likewise, for example *T I I T*, where is the \langle , \rangle such that λ , *Such that* λ $\forall v \in V$ and *u* b, the set of Lemma 2.1. In $S(V)$ exists a pre-hilbertian structure $\forall v \in V \text{ and } u, w \in S(V) \text{ with } \gamma(v) \in \text{End}(S),$ \langle , \rangle *such that* element $\gamma(n)$ in the technical lemma we want

Lemma 2.1. *In S*(*V*) *exists a pre-hilbertian structure*

 $S = \{ x \text{ and } u, w \in \mathbb{R} \}$, $S = \{ y \text{ and } u, w \}$, $S = \{ x \text{ and } u, w \}$, $S = \{ x \text{ and } u, w \}$, $S = \{ x \text{ and } w \}$, $S = \{ x \text{ and } w \}$, $S = \{ x \text{ and } w \}$, $S = \{ x \text{ and } w \}$, $S = \{ x \text{ and } w \}$, $S = \{ x \text{ and } w \}$, $S = \{ x \text{ and } w \}$, $S = \{ x \text{ and } w \}$,

$$
S(V) \text{ restricted to the } \langle \gamma(\nu)\nu, w \rangle = -\langle u, \gamma(\nu)\nu \rangle,
$$
 (2)

, *w u*, , (2)

For proof, see and $[2-4]$. $\sum_{i=1}^{n}$ or form structure given by a product or form ($\sum_{i=1}^{n}$). \overline{p} to \overline{p} $\frac{150 \text{ A}}{100 \text{ A}}$ ror prooi, Γ or is the truck the Hermitian structure given by a product of the Hermitian structure given by a product of the s decomposition gas F

restricted to the algebra t g, considering the Cartan

A pre-hilbertiana structure of an unitary (g, *K*)-module

A pre-hilbertiana structure of an unitary (g, *K*)-module

is the Hermitian structure given by a product or form $(,)$. Hermit A pre-hilbertiana structure of an unitary (g, *K*)-module). A pretorus **T**, which is isomorphic to the standard torus *T* A pre-hilbertiana structure of an unitary (\mathfrak{g}, K) -n $A₁$ torus **T**, which is isomorphic to the standard torus *T* \overline{A} torus **T**, which is isomorphic to the standard torus *T*

A pre-hilbertiana structure of an unitary (g, *K*)-module

is the Hermitian structure given by a product or form (,) and (,)

is the Hermitian structure given by a product or form (,) and (,)

Lemma 2.2 (F. Bulnes). In a pre-hilbertian structure). the Lie algebra g restricted to the algebra t. subspace whose endomorphisms are endomorphisms of $\wedge V$ X, res
the Lie algebra $\mathfrak g$ restricted to the algebra t. corresp of an unitary (\mathfrak{g}, K) -module can be constructed a spinor subspace \mathfrak{g} is subgroup of *GR* α ⁿ modules. and whose Lie group is an compact Abelian Lie subgroup of *G*, has spin modules. These can be related \mathbf{v} the **Lemma 2.2** (F. Bulnes)**.** *In a pre-hilbertian structure* and whose Lie group is an compact Abelian Lie by tensor product and are finally subspaces \ddot{h} $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ **g** restricted to the algebra **i**. **Example 1** corresponding *Property mode entrolling phisms are entromer phisms by*
the Lie algebra **g** restricted to the algebra **t**. Cartan involution

(g, *K*)-*module can be constructed a spinor subspace whose endomorphisms are endomorphisms of the Lie algebra* g *restricted to the algebra* t. *Proof.* Let **q** be a semi simple Lie algebra on **p** with *whose endomorphisms are endomorphisms of the Lie algebra* g *restricted to the algebra* t. \overline{P} \overline{C} \overline{D} *Proof.* Let $\mathfrak g$ be a semi simple Lie algebra on p with constructed a spinor s
(S((w) - $\mathfrak{so}(V)$ with S() Cartan involution θ and corresponding Cartan decomp $(S((W) = \mathfrak{so}(V))$ with $S(W)$ osition $g = t + \mathfrak{p}$. Consider a vector space V of finite 3. Applications of Lemma dimension with inner product (,). Let $S(V)$ be the **Example 3. 1.** Little is spinor space to $(V, ())$. dimensional representations a dimensional representations [6, 7]. $\sum_{k=1}^{\infty}$ consider a vector space V of finition $\mathfrak{g} = \mathfrak{t} + \mathfrak{p}$. Consider a vector space V of finition dim
. *Proof.* Let α be a semi simple Lie algebra on p with $C \leftarrow \begin{matrix} 1 & \cdots & 1 & \cdots & 0 & 1 \end{matrix}$ of $\begin{matrix} 1 & \cdots & 0 & \cdots & 1 \end{matrix}$ *Proof.* Let \bf{g} be a semi simple Lie algebra on \bf{p} with Cartan involution θ and corresponding Cartan decomposition $\bf{g} = t + \bf{p}$. Consider a vector space V of finite dimension with inner product (,). Let

If $V=\mathfrak{p}$, the lemma follows trivially, since the spinor \langle , \rangle on $S(n)$ is such that it is a subspace with a t - application of t invariant inner product and the endomorphisms of transform appl End(S(p)) are images of End(S(p)) restricted to t. $\qquad \qquad$ of SU (p, q) an module of $S(p)$ whose pre-hilbertian structure given for expressed by module of *S*(p) whose pre-hilbertian structure given for \langle , \rangle on *S*(\upmu) is such that it is a subspace with a t invariant inner product and the endomorphisms of If $V = \mathfrak{p}$, the lemma follows trivially, since the spinor module of *S*(p) whose pre-hilbertian structure given for and corresponding Cartan decomposition g t p. Consider a vector on *S*(p) is such that it is a subspace with a t invariant inner product and the endomorphisms of End($S(\mathfrak{p})$) are images of End($S(\mathfrak{p})$) restricted to t. space *V* of finite dimension with inner product , . Let *SV* be the spinor whose pre-hilbertian structure given for , on *S*p is such that it is a invariant inner product and the endomorphisms p

of the endomorphism of the endorphisms of the endorphisms of the end of the end of the end of the end of t. p. end *S*(p) restricted to t. a. **N** $\frac{1}{2}$ However, the said X-bilineal form conforms to a pre- \mathbb{R}^7 *s*, \mathbb{R}^8 If *V* =g, then we extend (,) to a X-bilineal form on *V*. \mathbf{r} are independent to the set of \mathbf{r} If *V* =g, then we extend (,) to a X-bilineal form on *V*. If $V = \mathfrak{g}$, then we extend (,) to a X-bilineal form on V. If *V* =g, then we extend (,) to a X-bilineal form on *V*. hilbertian structure on $S(V)$ If $V = \mathfrak{g}$, then we extend (,) to a X-bilineal form or Lowever, the said *V* bilineal ferm conforms to a v \mathbf{b} ilhertien structure on $\mathcal{S}(V)$

(Lemma 2. 1), and thus of the spin module $(\mu, S(V))$ The first generalized $h \mu \in \mathcal{B}(\mathcal{V})$ (Lemma 2. 1), and thus of the spin module $(\mu, S(V))$ (Lemma 2. 1), and thus of the spin module (, *S (V))* $\forall \mu \in \mathfrak{so}(V)^*$ $\forall \mu \in \mathfrak{so}(V)^*$ If *V=*p, the lemma follows trivially, since the spinor

considering $\mu \in \mathfrak{so}(V) \to \text{End}(S(V)))$, which is a (g, constructed by [K -module in the pre-hilbertian space
representations of considering $\mu \in \mathfrak{so}(V) \to \text{End}(S(V)))$, which is a (g, considering so *V* End (*S*(*V)))*, which is a (g, *K*)-module in the pre-hilbertian space $\mathcal{S}(\mathcal{S}(n))$ when $\mathcal{S}(\mathcal{S}(n))$ on *S*(p) is such that it is a subspace with a t -

$$
S(V) = H(K) \otimes 1_{V^{\pm}}.\tag{3}
$$

By the demonstration of the Lemma [2], there exists a t can be determined for the set of the land t, and then $\mathfrak{g} = \mathfrak{so}(V)$, where $\mathfrak{so}(V)$ is the compact unitary modules of image of endomorphism of the Lie algebra $S(V)$ and others. Its unit restricted to the subalgebra t. [7] using boduct $\langle \cdot \rangle$ on $S(V)$. Thus $\gamma(v) | t = \gamma(v) X$ discrete series. $\sum_{i=1}^{N}$ and $\sum_{i=1}^{N}$ is the compact of compact $\sum_{i=1}^{N}$ is the compact of compact $\sum_{i=1}^{N}$ is the compact of compact $\sum_{i=1}^{N}$ is the compact of compact of compact $\sum_{i=1}^{N}$ is the compact of compac *K*)-module in the pre-hilbertian space representations to *SU*(*p*, *q*) was constructed using *L*2 - \mathcal{C} structure on an underlying homogeneous \mathcal{C} oduct $\langle \cdot \rangle$ on $S(V)$. Thus $\gamma(\nu)|t = \gamma(\nu) X$ anscreie series. baigeora **I**. representations to *SU*(*p*, *q*) was constructed using *L*2 - $\frac{1}{\sqrt{2}}$ structure on an underlying homogeneous h broduct $\langle \cdot \rangle$ on $S(V)$. Thus $\gamma(\nu)|t = \gamma(\nu) X$ discrete series. The *K*)-module in the pre-hilbertian space representations to *SU(^p*), there exists a *t* can be accommoded tor complex structure $\log_{10}(r)$ gebra *t*. $\begin{bmatrix} 9 \end{bmatrix}$ using spin By the demonstration of the Lemma [2], there exists a t
invariant inner product \langle , \rangle on $S(V)$. Thus $\gamma(\nu)|t = \gamma(\nu) X$ t, and then $\mathfrak{g} = \mathfrak{so}(V)$, where $\mathfrak{so}(V)$ is the compact image of endomorphism of the Lie algebra $S(V)$ restricted to the subalgebra t. restricted to the subalgebra t. By the demonstration of the Lemma [2], there exists a t

image of endomorphism of the Lie algebra *S*(*V*)

restricted to the subalgebra t.

In particular, if (g, K) -module is unitary, said prehilbertian structure induced by the product $($, $)$ is a v_r that is to say, in each one of their restrictions on of $\land V$ X, respect to t, these restrictions are the $corresponding$ images of $\mathfrak{so}(V)$. Then can *constructed a spinor subspace* $W \subset S(V)$ *such that* $\mathcal{S}(G(W)) = \mathfrak{so}(V)$ with $\mathcal{S}(W)$ an unit ple Lie algebra on p with constructed a spinor subspace $W \subset S(V)$ such that End esponding Cartan decomp $(S((W) = \mathfrak{so}(V))$ with $S(W)$ an unitary (\mathfrak{g}, K) -module. **Proposition** Lie algebra on p with p with p with p with $\frac{1}{2}$ *tricted to the algebra* t. corresponding images of $\mathfrak{so}(V)$. Then can be of *SU* (*p*, *q)* and of *SU* (2, 2) to the problem no solved $(S((W) = so(V))$ with $S(W)$ an unitary (S, K) -module. ϵ , the case of ϵ is the case of ϵ in ϵ in ϵ in ϵ in ϵ of ϵ in ϵ i corresponding images of $\mathfrak{so}(V)$. Then can be Hermitian form and is a sesquilineal form in each complex of the corresponding cohomology on $\wedge V$ X,

In particular, if (g, *K*) -module is unitary, said pre-

pplications of Lemma $\mathbf a$ $\mathbf a$ extension of said representations to the case of infinite case of infinite case of infinite case of infinite c
The case of infinite case dimensional representations [6, 7]. **3. Applications of Lemma 3. Applications of Lemma**

 $ext{(,)}.$ Let $S(V)$ be the **Example 3. 1.** Little representations as cuspidal forms and infinite dimensional representation e pre-hilbertian structure given for expressed by a spinor decomposition. A concrete that it is a subspace with a $t -$ application of this, we can see the works to the twistor ext and the endomorphisms of transform applied to finite dimensional representations of End($S(\mathfrak{p})$) restricted to t. of $SU(p, q)$ and of $SU(2, 2)$ to the problem no solved $\frac{\text{d} \ln \ln(\theta(\mathbf{r}))}{\text{d} \ln \theta}$ of the globalization of finite dimensional $\frac{1}{2}$ (,) to a λ -bifurcal form on ν .

representations to the study of the Universe and the α λ -bilinear form comorms to a pre-
extension of said representations to the case of infinite $\lim{ \text{reson } S(V)}$ dimensional representations [6]. nodules induced by hyperbolic G -orbits G_h) can be ion of finite dimensional ations $[8]$. ation of finite dimensional end(*S*(c)) are interested to the explobalize mining unnerstorial representations (possibly some dic grobalization of the differential and infinite dimensional representations (possibly some G-modules induced by hyperbolic G-orbits G_h) can be expressed by a spinor decomposition. A concrete application of this, we can see the works to the twistor transform applied to finite dimensional representations of $SU(p, q)$ and of $SU(2, 2)$ to the problem no solved $d = 111'$, $c = c \cdot c$, the corresponding to the corresp representations to the study of the Universe and the extension of said representations to the case of infinite dimensional representations [6]. prehilbertian structure of the spin modules. Rawnsley of the grobalization of fillic difficultional

where $\sin(\nu)$. Thus $y(\nu)\mu - y(\nu)$ and decrease server the examination of the corresponding first generalized twistor transform to group $\mathfrak{so}(V) \to \text{End}(S(V))$, which is a (g, representations with the idea of conform group was constructed by [7] of certain class $SU(p, q)$ called ladder $1_{V^{\pm}}$. (3) representations. These representations are those that be determined for analytic continuation of the rete series. The classification of the corresponding $\lim_{x \to a}$ of the Lie algebra $S(V)$ and others. Its unitarization was demonstrated firstly in [9] using spinor structure underlying in the prehilbertian structure of the spin modules. Rawnsley image of endomorphism of the Lie algebra *S*(*V*) [7] of certain class $SU(p, q)$ called ladder therefore any possible action of Weyl group is lost on nary
... retain a general harmonic theory to indefinite a general here the second theory to induce the second term in the
Independent of the contract of the second term in the second term in the second term in the second term in th $V(f(n, a))$ colled ladder representations to *Superintential* α ² the ϵ is the parameter for the parameter for the representation, ϵ the representation, ϵ therefore any possible action of Weyl group is lost on ana et *al*,develop a general harmonic theory to indefinite $\mathcal{C} \mathcal{I} \mathcal{U}$ which is not the ladder more of the ladder more of the ladder more of the ladder representations to *SU*(*p*, *q*) was constructed using *L*2 - representations. Finally, all the set of ladder space is linked to the parameter for the representation, the parameters as in the discrete series \mathcal{G} \mathbf{f} therefore any possible action of \mathbf{f} ytic et *al*,develop a general harmonic theory to indefinite $\frac{m}{\sqrt{2}}$ which is more of the ladder be determined for analytic continuation of the space is linked to the parameter for the representation, $\frac{1}{\sqrt{2}}$ $r = \frac{1}{2}$ and $r = \frac{1}{2}$ of certain class et *al*,develop a general harmonic theory to indefinite representations. Finally, all the set of ladder C *SUIT CALCOITS* to $D\cup \{p, q\}$ valid radicithe parameters as in the discrete series of the discrete series of the discrete series of the discrete series of the discrete seri andoptive boson encoloneousloss are consideration of μ X-respect to the these extititions are the constrained in the section of So proof, second (2.41) **Let g** real hypersurface in particular, if (g, *R*) -woulds is unitary, and proposition with the semi-model of the semi-model of the semi-model of the semi-model of the semi-model on product of th representations with the idea of conform group was representations of $SU(p, q)$ called ladder representations. These representations are those that can be determined for analytic continuation of the discrete series. The classification of the corresponding unitary modules of maximum weight are given in [8] and others. Its unitarization was demonstrated firstly in $\begin{bmatrix} 0 \end{bmatrix}$ using spinor structure underlying in the The first generalized twistor transform to group

cohomology where the Penrose transform plays an et al, develop a general harmonic theory to indefinite metrics which include more of the ladder metrics which include more of the ladder representations. Finally, all the set of ladder representations to $SU(p, q)$ was constructed using L^2 . important role. In the result obtained in the choice of a complex structure on an underlying homogeneous space is linked to the parameter for the representation, therefore any possible action of Weyl group is lost on the parameters as in the discrete series [10]. cohomology where the Penrose transform plays an representations. Finally, and the set of the set of ladder metrics metrics which include more of the ladder ladder

et *al*,develop a general harmonic theory to indefinite

manifold M , whose complex structure of T is given by the Hermitian form Φ with signature (p, q) such that $p+q = N + 1$, with dim T= N+1. We consider the Lie group underlying in the complex manifold defined for $G = SU(p, q)$, which is a subgroup of $SL(N + 1, \mathbb{C})$ and which preserves Φ . The projective space $\mathbb{P} = \mathbb{P}^N(\mathbb{C})$ divide to *G* in three open *G*-orbits: \mathbb{P}^+ , $\mathbb{P}^{\text{-}}$ and \mathbb{P}^0 , where divide to *G* in three open *G*-orbits: ℙ⁺ \mathbb{R}^n We consider the vector complex space **T** of a complex $\frac{1}{\sqrt{2}}$ $\frac{a}{2}$ G in three open G-orbits: \mathbb{P}^+ , $\mathbb{P}^{\text{-}}$ and \mathbb{P}^0 , where *K-finite* A Technical Lemma on Unitary g, *K* -modules 5

 $\mathbb{P}^+=\{\text{lines } \subset \mathbf{T}|\Phi\big|_{\text{lines}} \geq 0 \Leftrightarrow \Phi\geq 0, \Phi=0, \text{if } \Phi\big|_{\text{lines}} = 0\},\$ This stat \mathbb{R}^n and \mathbb{R}^n ^ℙ⁺ ⁼lines 0 0, 0, if 0, lines lines **T** (4) , where $\mathbf I$ $\mathbf{H} = \mathbf{H} \cdot \mathbf{H} \cdot \mathbf{H}$ (4) , where \mathbf{I} And (4) And

$$
\mathbb{P} = \{ \text{lines } \subset \mathbf{T} \mid \Phi \big|_{\text{lines}} \le 0 \Leftrightarrow \Phi \le 0, \Phi = 0, \text{ if } \Phi \big|_{\text{lines}} = 0 \}. \quad \text{structure}
$$
\n
$$
\text{(5)} \quad \text{The last}
$$

Then \mathbb{P}^0 , is a real hypersurface in \mathbb{P} . \Box endomo Then \mathbb{P}^0 , is a real hypersurface in \mathbb{P} , are the open *G*-orbits that determine the construction of the

 $\sum_{i=1}^{\infty}$ construction of the sesquifilear app subspace
construction of the sesquilineal appearing more simple \mathbb{P}^+ , and \mathbb{P}^- , are the open *G*-orbits that determine the \langle , \rangle in SU (p, q). Said sesquilineal appe prehilbertian structure whose restriction to germs of the I sheaf o $(-n-p)$ corresponds to the $-n-p$ power of the consider *Hp-1* tautological bundle of lines on \mathbb{P}) of the complex operator is the module which is the spin space spin*N*, 1. Then sheaf of the *n*-*n*-*n*-*p* corresponds to the tautor of the tautor $\langle \cdot \rangle$ in *SU* (*p*, *q*). Said sesquilineal appearing induces a **T** $\frac{1}{2}$ which determines an inner product \mathbf{r} , \mathbf{r} and \mathbf{r} tautological bundle of lines on \mathbb{P}) of the complex holomorphic bundle sesquilineal appearing more simple , in *SU p*, *q*. Said sesquilineal construction of the sesquifficul uppearing more simple sheaf o $(-n-p)$ corresponds to the $-n-p$ power of the \mathcal{L} construction of the sesquilineal appearing more simple appearing induces a prehilbertian structure whose restriction to germs of the \langle , \rangle in *SU* (*p*, *q*). Said sesquilineal appearing induces a $\mathcal{L}_{\mathcal{F}}$

underlies a spinor subspace in the unitary g, *K* -module

$$
\mathbf{T} \to SU(p, q) \cong SL(N+1, \mathbb{C})/SO(N+1, \mathbb{C}),\tag{6}
$$

which determines an inner product (α) on the product (α) on the product (α

which determines an inner product (,) on the $\frac{1}{2}$ r
cohomological space cohomological space $\frac{1}{2}$ $\left(\begin{array}{c} 0 & 0 \\ 0 & 0 \end{array} \right)$ $\frac{1}{2}$ t_{obs} which determines an inner product $(,)$ on the cohomological space

lines on **P**) of the complex holomorphic bundle

 \overline{a}

 L^2 -
 H^{p-1}($\vert \mathbb{P}^+ \vert$, \mathfrak{o} (-n - p)), and $\forall \phi, \psi \in H^{p-1}(\vert \mathbb{P}^+ \vert, \mathfrak{o}$ (-n - p)), $\sum_{i=1}^{n}$ and $\sum_{i=1}^{n}$ -modules $\sum_{i=$, of p), $\frac{1}{2}$. T_{max} tion, underlies a spinor subspace $\delta(\phi \cup T\psi)$ in the unitary $\lim_{\epsilon \to 0}$ (α , *K*) -module $H^{p-1}(\mathbb{P}^+|, \mathfrak{o}(-n-p)).$ the image of the Dirac operator is the module $\delta(\phi \cup T\psi)$ which is the spin space spin $(N, 1)$. Then t_{inter} is the module operator is the module of the module t_{inter} the image of the Dirac operator is the module underlies a spinor subspace in the unitary $O(\psi \cup \psi)$ which is the spin spax |, o (-n - p)). (μ, Λ) -module Λ^* (μ |, ν (-n - p)). $\left(\begin{array}{ccc} 1 & 1 \\ 1 & 1 \end{array}\right)$, $\left(\begin{array}{ccc} 1 & 1 \\ 1 & 1 \end{array}\right)$, $\left(\begin{array}{ccc} 1 & 1 \\ 1 & 1 \end{array}\right)$, $\left(\begin{array}{ccc} 1 & 1 \\ 1 & 1 \end{array}\right)$ \mathcal{L} subspace \mathcal{L} $H^{p-l}(\mathbb{D}^+)$ o $(-n - n)$ and $\forall \phi$ $\psi \in H^{p-l}(\mathbb{D}^+)$ o $(-n - n)$. $\delta(\phi \mid \text{Tuv})$ which is the spin space (α, V) modulo $H^{p-l}(\mathbb{P}^+| \alpha(n,n))$ $H^{p-1}(\mathbb{P}^+|, \mathfrak{o}(-n - p)),$ and $\forall \phi, \psi \in H^{p-1}(\mathbb{P}^+|, \mathfrak{o}(-n - p)),$ the linage of the Dirac operate (\mathfrak{g}, K) -module $H^{p-l}(\mathbb{P}^+|, \mathfrak{g}(-n-p)).$

Then a concrete application of the pre-hilbertian plex structure where underlie spinor subspaces (whose $\frac{1}{10}$ by linear endomorphisms are the representations of SU (p, that q) is the following result: linear endomorphisms are the representations of *SU* (*p*, Then a concrete application of the pre-hilbertian structure where underli-

 $T_{\rm eff}$ statement is a version of the Eastwood theorem in $\mathcal{T}_{\rm eff}$ LIE **I neorem 3.1.** Let \cdot be the inner product on $H^{\perp}(\mathbb{P}),$ for $\mathfrak{o}(-n-p)$) positive defined. Then the subspace $H^{p-l}(|\mathbb{P}^+|,$ **Theorem 3.1.** *Let* $\langle \cdot, \cdot \rangle$ *be the inner product on* $H^{p-1}(\mathbb{P}^+)$, **The inner** *s S*
 o(-n - p)) *contains classes of cohomology that are* dense spinor subspaces in a Hilbert space H. Then are **dense** spinor subspaces in a Hilbert space H. Then are K-finite vectors to the representation of SU (p, q) on H. $\frac{1}{2}$ $\frac{1}{2}$ o (-n - p)) *contains classes of cohomology that are dense spinor subspaces in a Hilbert space H*. *Then are K*-*finite vectors to the representation of SU* (*p*, *q*) *on H*. **Theorem 3.1.** Let $\langle \cdot \rangle$ be the inner product on $H^{p-1}(\mathbb{P}^{\cdot}),$ $\mathfrak{g}(-n - p)$) *positive defined. Then the subspace* π (\mathfrak{g}), *g*(-*n* - *p*)) contains classes of conomology that are *k*-*finite vectors to the representation of <i>a finite directors to finite a representation on H. <i>i nen are* **Theorem 3.1.** Let $\langle \cdot, \cdot \rangle$ be the inner product on $H^{p-1}(\mathbb{P}^+),$ **o**($-n - p$)) *positive defined. Then the subspace* $H^{p-1}(\mathbb{P}^+),$ $p(-n - p)$ contains classes of cohomology that are *K*-*finite vectors to the representation of SU* (*p*, *q*) *on H*.

sessuit more simple substitute $m = 0$.
 i = $\sin(N + 1, \mathbb{C})$, are endomorphisms of the spinor (7) where V , is admissible and unitary (g, V)-module with and unitary (g, K)-module with α *K*-*f K*-*finite vectors to the restwood incordination of* μ *. <i>Supportion in [7]* spinor subspaces underlying in all pre-imbertial structure of the unitary module $H = (\mathbb{F}^n)$, \mathbf{v} (-n - p)). $\sum_{i=1}^{\infty}$ in all probability in all preendomorphisms of the afgeora $\mathfrak{g} = \mathfrak{su}(p, q)$ restricted to $\frac{1}{2}$ (iv $\frac{1}{2}$, $\frac{1}{2}$), are encomorphisms of the spinor $\sum_{i=1}^{\infty}$ of the algebra g $\sum_{i=1}^{\infty}$ $\sum_{i=1}^{\infty}$ $\sum_{i=1}^{\infty}$ respectively to the subspace of proved by twistor transform in \mathfrak{g}_1 . *K*-*finite vectors to the representation of SU* (*p*, *q*) *on H*. spinor subspaces underlying in all pre-hilbertian structure of the unitary module $H^{p-l}(\mathbb{P}^+)$, $\mathfrak{o}(-n - p)$). The last line of the Theorem 3.1 says that the endomorphisms of the algebra $\mathfrak{g} = \mathfrak{su}(p, q)$ restricted to $t = \sin(N + 1, C)$, are endomorphisms of the spinor subspaces of $H^{p-l}(\mathbb{P}^+|, \mathfrak{o}(-n - p))$. Theorem 3.1 is proved by twistor transform in [6]. subspaces of *Hp-1* which affirms the same that this in the language of the This statement is a version of the Eastwood theorem in [7], This statement is a version of the Eastwood theorem m_{Γ} . spinor subspaces underlying in all pre-hilbertian structure of the unitary module *Hp-1* \mathbf{I} $\frac{1}{\sqrt{2}}$ says the last line of the Theorem 3.1 says that the endomorphisms of the algebra $\mathfrak{g} = \mathfrak{su}(p, q)$ restricted to $t = \sin(N + 1, \mathbb{C})$, are endomorphisms of the spinor subspaces of $H^{p-1}(\mathbb{P}^+), \mathfrak{o}(-n - p)$. Theorem 3.1 is $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$ e Eastwood theorem in [7],
is in the language of the
in all pre-hilbertian
le $H^{p-1}(\mathbb{P}^+|, \mathfrak{o}(-n - p))$.
em 3.1 says that the
 $\mathfrak{g} = \mathfrak{su}(p, q)$ restricted to
norphisms of the spinor
- p)). Theorem 3.1 is
[6]. o **ℙ**

> Example 3. 2. Another example of application is consider unitary representations such that consider unitary representations such that **Example 3. 2.** Another example of application is \mathcal{L} and the presentations such that \mathcal{L} consider unitary representations such that representations such that

gGu [11].

consider unitary representations such that

$$
H^{i}(\mathfrak{g}, K; V \otimes F^*) = \text{Hom}_{K}(\wedge^{\bullet} \mathfrak{p}, V \otimes F^*), \qquad (7)
$$

where V, is admissible and unitary (g, K) -module with infinitesimal character $\chi_{\Lambda+\rho}$, \mathfrak{p} , is a compact component References of $\mathfrak g$ ($\mathfrak g$ = t $\oplus \mathfrak p$) and F, is an unitary module as ($\mathfrak g_u$, G_u)module with $u \in (\wedge \mathfrak{p})^*$ and with a Hermitian form $\leq, >,$ such that $\forall u, v \in F$, is satisfied $g \le u$, $v > -\le gu$, $gv >$ press. $\forall g \in G_u[11].$ of \mathfrak{g} ($\mathfrak{g} = \mathfrak{t} \oplus \mathfrak{p}$) and F, is an unitary module as (\mathfrak{g}_u , G_u)- α , β , α , α , β , α ,

where \mathcal{V} is admissible and unitary (g, \mathcal{V})-module with \mathcal{V}

 $\frac{1}{\sqrt{2}}$, or $\frac{1}{\sqrt{2}}$, or $\frac{1}{\sqrt{2}}$

the spinor subspaces of *Hp-1*

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algebra g su*p*, *q* restricted to t = so(*N* + 1, ℂ), are endomorphisms of

infinitesimal character $\mathcal{L}_{\mathcal{A}}$ is a compact component component compact component c

product to the restriction B (bilinear form) to μ . The 5. Vogan, I restriction of B to \mathfrak{p} , is an image complex Bi($\wedge \mathfrak{p}$, V Lie groups (No. $\otimes F^*$) = dC_{i-1}(\wedge **p**, $V \otimes F^*$), considering the complex form. Twistors is sequence Likewise, on $\land \cdot \mathfrak{p}$, we introduce a corresponding inner 4. Vogan, D.A. sequence < , >, such that *u*, *F*, is satisfied *g*<*u*, > = <*gu*, *g*> *gGu*. Likewise, on R and R (bilinear form) to p. The restriction of B to p, is an image of B to p, is an im

$$
\dots \to \mathcal{C}^{\mathcal{C}^{\mathcal{C}}}(\mathfrak{g}, K; V \otimes F^*) \to \mathcal{C}^{\mathcal{C}}(\mathfrak{g}, K; V \otimes F^*) \to \mathcal{C}^{\mathcal{C}^{\mathcal{C}^{\mathcal{C}}}}(\mathfrak{g}, K; V \otimes F^*) \to \dots (8)
$$

Then is given the cohomology $H^i(g, K; V \otimes F^*)$ [2, 3, 5]. complex $C^i(g, K; V \otimes F^*)$ determined by the functorial diagram where appears the restriction of the corresponding endomorphisms of Lie algebra \mathfrak{g} , to the Lie algebra t, Likewise, has been used strongly the structure of the complex $C^1(g, K; V \otimes F^*)$ determined by the functorial diagram where appears the restriction of the corresponding endomorphisms of Lie algebra g , to the \mathbf{T}^{1} the strongly the structure of the complex City \mathbf{U}^{1} FINCT IS given the conomology $H(g, K, V \otimes F)$ [2, 3, 3]. Then use sentors and represents Id Hom*K* (9) trongly the structure of the F _{*[Functional Analysis, 24](https://doi.org/10.1016/0022-1236(77)90005-2)*(1), 52-106.} diagram where appears the restriction of the $of Function$ corresponding endomorphisms of Lie algebra g Then is given the cohomology $H^i(\sigma, K: V \otimes F^*)$ [2, 3, 5]. Γ (Issued strongly the strongly the structure of the Γ $\frac{1}{2}$ diagram where appears the restriction of the corresponding of the corresponding $\frac{1}{2}$ i (g/t) *V F**

Id Hom*K* (9)

$$
\wedge^i(\mathfrak{g}/\mathfrak{t}) \to V \otimes F^*
$$
\n
$$
\text{Id} \to \mathcal{P} \text{Hom}_K
$$
\n
$$
\wedge^i \mathfrak{p},
$$
\n(9)

p,

Then the lemma 2. 2, is satisfied.

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