

## A Study Note on Mathematical Modelling of Ship Stability

Ambika and Shreedevi Kalyan\*

Department of Mathematics, Sharnbasva University,  
Kalaburagi

\*Corresponding Author

Shreedevi Kalyan, Department of Mathematics, Sharnbasva University, Kalaburagi.

Submitted: 2024, Nov 11; Accepted: 2024, Dec 03; Published: 2024, Dec 13

**Citation:** Ambika. Kalyan, S. (2024). A Study Note on Mathematical Modelling of Ship Stability. *Petro Chem Indus Intern*, 7(4), 01-06.

### Abstract

This paper work describes the modelling of a stability of ship. Floating bodies subject to the disturbances such as external force, gravity force and etc. The stability depends on the location of the body, weight, ship structure. The concept of ship stability can be found in terms of applications of buoyancy force, centre of gravity, moments. An approach using the concept of finding equilibrium steady states corresponding to the radii value is presented. Some numerical examples are given to illustrate the effectiveness of the presented work.

### 1. Introduction on Stability

The concept of ship stability is one of the most significant areas of centre of attention in ship design and operation, not only to guarantee the safety of the ship, cargo, crew and passengers, but also to enable proper conditions for completion of all the processes on a ship. It is worth mention that the work of literatures [1-6], who used the concepts of Buoyancy. Ship stability is certainly a subject of paramount significance in the field of Naval Architecture, the design and operation of ships and floating units having a fundamental role. Moreover, “stability” is a notion which has a very broad significance in Naval Architecture, especially embracing ship stability fundamentals with ship dynamics and eventually safety of the ship[see ref.[7-11]. In this admiration, research in the field has arrived considerable attention within the whole maritime community, ensuing in the modern-day evolution of the subject to the integrated idea of “ship stability, dynamics and safety” as it is being presently acceptable.

#### 1.1 Basic Definitions

##### 1.1.1 Buoyancy Force

The capacity of an object to float is simply defined as a term “Buoyancy”. When any object is placed in any fluid, the fluid exerts an upward force this upward force is known as Buoyancy Force. This is because of the pressure exerted on the object by the fluid. It is the force on the body which allows it to float or sink when it is placed in fluid,.

##### 1.1.2 Gravitational Force

It is a force that attracts any two objects with mass. We call the gravitational force attractive because it always tries to pull masses together, it never pushes them apart.

##### 1.1.3 Archimedes’ Principle

A body in liquid experience a Buoyant upward force equal to the liquid displaced by that body.

##### 1.1.4 Ship Stability

The surface ship’s stability can be divided into two parts,

- Intact Stability
- Damaged Stability

##### • Intact Stability

The stability of a surface ship when the intactness of its hull is maintained, and no compartment or watertight tank is damaged or freely flooded by seawater.

##### • Damaged Stability

The stability of a surface ship includes the identification of compartments or tanks that are subjected to damage and flooded by sea water, followed by a prediction of resulting trim and draft conditions.

##### 1.1.5 Equilibrium

It can be defined as the state of an object in which two or more counter-influences, whether internal, external, or a combination of both act on the body voiding each other to keep the object in the same state as it was.

### 2. A Study on Ship Stability

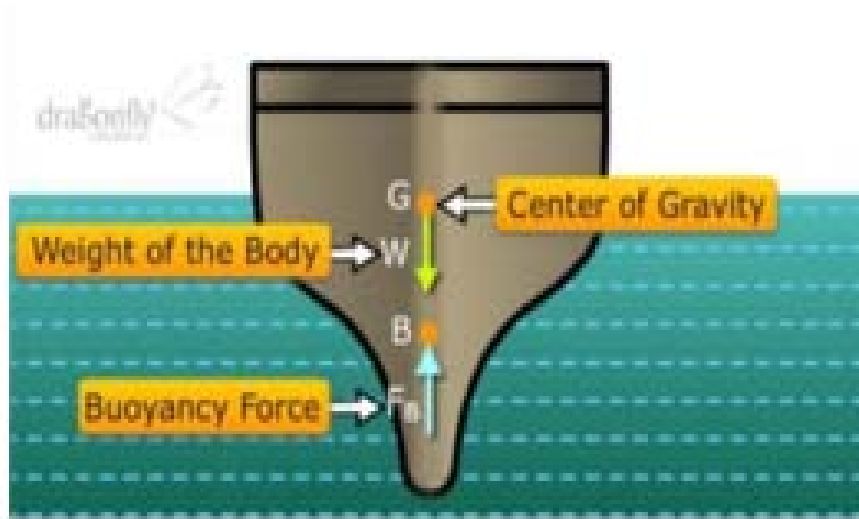
#### 2.1 Introduction

This technical paper concludes with a method that helps to find the meta centric height in order to analyse the stability of the ship. In this way, a very few prevailing principles are followed by the naval engineer, who is able to determine which of the existing criteria fits

best with the requirements of the ships function. And theoretical concepts have been discussed. A study has been done on the ship stability by using the prevailing concepts. Finally, some numerical examples were given to illustrate the theory.

## 2.2 Stability of Ship

Ship stability deals with how a ship behaves at sea, both in still water and in waves. Stability calculations focus on the centre of gravity, centre of buoyancy, and metacentre of the ship.



When a body undergoes an angular displacement towards right side, then the volume displaced is larger in right side and lesser in left side. Thus, the centre of Buoyancy shifts to the right from B to B' and a couple is formed which tries to rotate the body. Here the point of intersection of the vertical line through

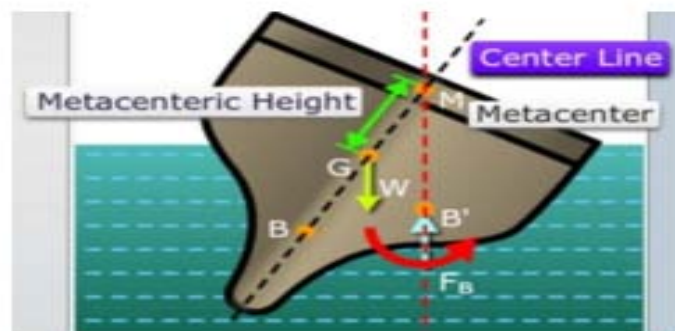
Stability of equilibrium of a Floating body depends, upon the location of the metacentre. If a ship floats, mass density of the ship is less than that of the water. Weight of the ship is same as the water displaced by the ship.

To be in equilibrium,

- Buoyant force must be equal to the weight of the ship displaced by the water.
- Gravitational force and buoyant force should lie on the same vertical line.
- Volume displaced is equal on both sides

the new centre of buoyancy B' and the centre line G is turned as Metacentre. Metacentric height (GM) = BM- Band the distance by which the metacentre lies above the centre is gravity is known as Metacentric height

$$\text{Metacentric height (GM)} = \text{BM} - \text{BG}$$



## 2.3 Types of Equilibrium

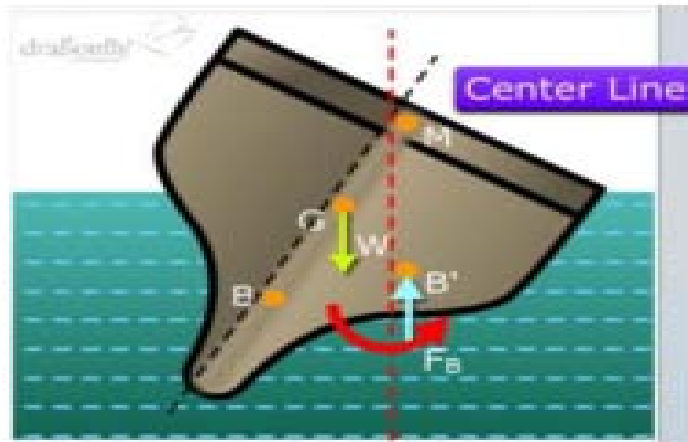
The concept behind the understanding of intact stability of a floating body is that of Equilibrium. There are three types of equilibrium conditions that can occur, for a floating ship, depending on the relation between the positions of centre of gravity and centre of buoyancy.

- Stable Equilibrium

- Unstable Equilibrium
- Neutral Equilibrium

### 2.3.1 Stable Equilibrium

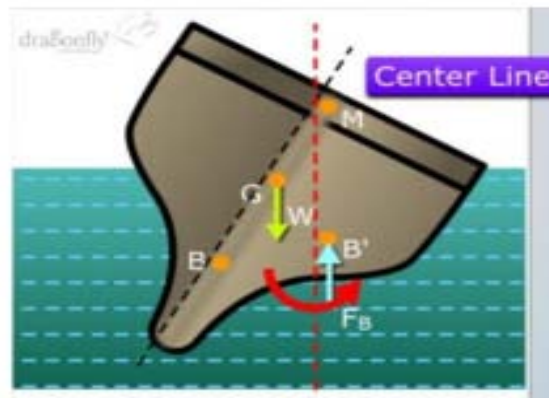
When M is above G, then the restoring couple is formed by the buoyant force and the weight of the body which tends to change the body to original position.  $GM > 0$  means the ship is stable



### 2.3.2 Unstable Equilibrium

When M is below G, then overturning couple is formed by the

Buoyancy force and the weight of the body which tends to sink the body from its original position.  $GM < 0$  means the ship is unstable.

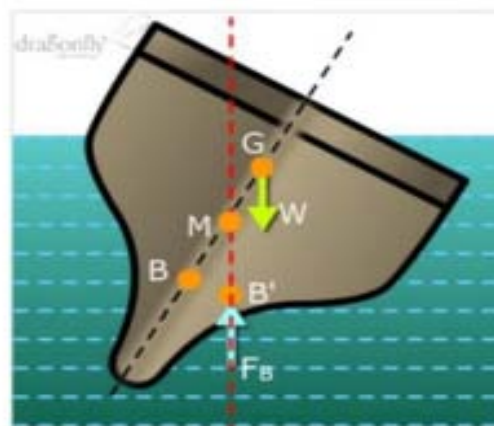


### 2.3.3 Neutral Equilibrium

When M coincides with G, then line of action of buoyant force and weight of the body are collinear and passes through same point due

to this the ship neither returns to its original position nor increases its displacement.

$GM = 0$  means the ship is neutrally stable.



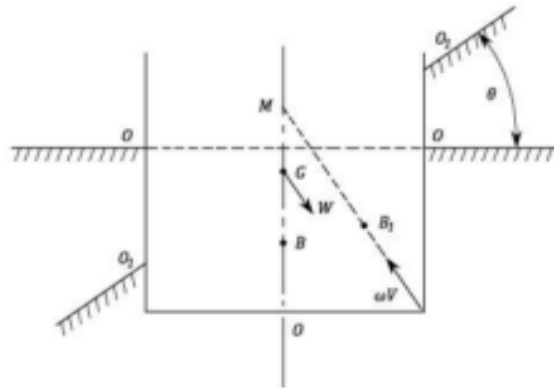
## 2.4 Theories on Ship Stability

### 2.4.1 The Stability of Partially Submerged Bodies

Let G be the centre of gravity, B be the centre of buoyancy and OO be the original water surface. O1 O1 be the new water line,  $\theta$  be the angle of tilt and GM be the metacentric height.

G remains in the same position but B moves to B1

Righting couple =  $W GM \sin\theta$  [∴ where W is the mass of the body]



### 2.4.2 The Determination of Metacentric Height

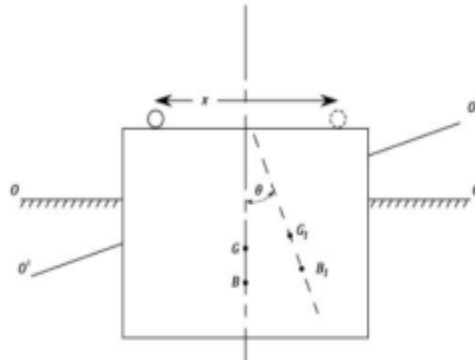
Let  $W$  be the weight of the ship plus its load, A small load  $w$  is moved to a distance of  $x$  and causes a tilt of angle  $\theta$ . The ship is now in a new position of equilibrium with  $B'$  and  $G'$  lying along the vertical through  $M$ .

The moment due to Centre of gravity is  
 $w x = W * GM \theta$

### 2.5 Theoretically

The ship tilts from its original water line  $OO'$  to new water line  $O'O'$  and it moves to an angle  $\theta$ . Due to the movement of the wedge of water from  $A_1AC$  and  $D_1DC$  the center of Buoyancy moves from  $B$  to  $B'$

$$GM = \frac{wx}{w\theta}$$



The change in the momentum of the buoyancy force

$$= wV \times BB'$$

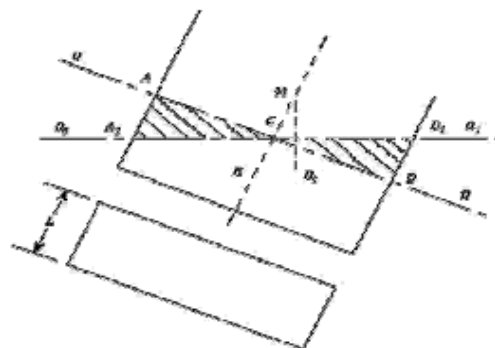
$$= wV \times BM\theta \quad [\text{where } \theta \text{ is small}]$$

The volume of the wedge is  $ACA'$

$$= \frac{1}{2} \times \frac{b}{2} \times \frac{b}{2\theta} \times L = \frac{b^2 L \theta}{8}$$

The Moment of the Couple due to the movement of the wedge

$$= \frac{wb^2 L \theta}{8} \times \frac{2b}{3}$$

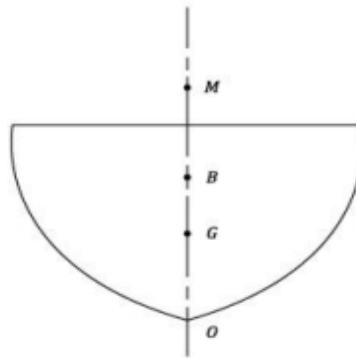


$$wV \times BM\theta = wb^3L\theta/12$$

$$BM = b^3 L / 12V$$

$$= I/L$$

Where I is the Second Moment of Area of the Water Plane Section



V is the volume of water displaced

If the values of G & B are known GM can be determined.

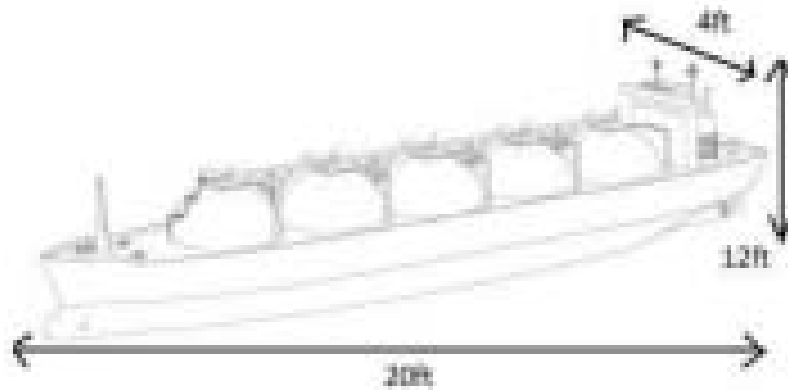
$$GM = BM + BG$$

$$= BM + BO - OG$$

## 2.6 Application Problems on Ship Stability

### Problem 1

A ship measuring 20ft. by 12ft. and 4ft deep, weighs 12 tons. It carries a load of 8 tons. The ship is in sea water with a density of 64 lb/cu. ft. Find its meta centric height and establish the angular tilt which will result if the load is moved by one ft. sideways.



### Solution

$$\text{Moments above the base} = (12+8) \times OG$$

$$= 12 \times OGP + OGL$$

$$OG = 12 \times 2 + 8 \times \frac{5}{(12+8)}$$

$$= 3.2 \text{ ft.}$$

$$\text{The volume of water displaced } V = (12+8) \times \frac{2240}{64} = 700 \text{ft}^3$$

$$\text{Depth of immersion } h = \frac{700}{20} \times 12 = 2.197 \text{ ft.}$$

$$\text{The height } OB = \frac{h}{2} = 1.458 \text{ft}$$

$$BM = \frac{Lb^3}{12V} = \frac{(20 \times 12^3)}{(20 \times 700)}$$

$$\text{Metacentric height} = BM + OB - OG$$

$$= 4.143 + 1.458 - 3.2$$

$$= 2.401 \text{ft}$$

The moment due to the Movement of the Load = 8ft tones

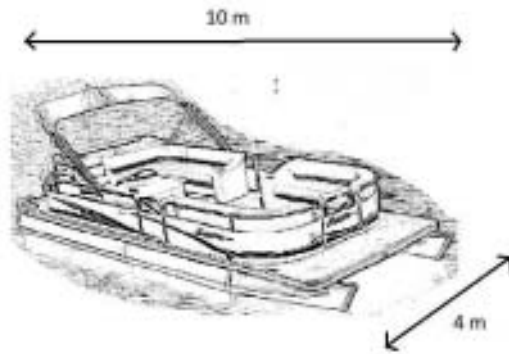
The moment due to the movement of the centre of gravity  $W\theta$

$$\theta = \frac{8}{48} = \frac{1}{6} \text{radian} \quad W\theta = 20 \times 2.40$$

**Result:** The metacentric height is 2.40ft and the angle is  $\theta$  is 90 31'

### Problem 2

A boat 10m by 4m in plan, weighs 280 kN and floats in sea water of density 1025 kgm<sup>-3</sup>. A steel tube weighing 34 kN is placed longitudinally on the deck. When the tube is in a central position, the center of gravity for the combined mass is on the vertical axis of symmetry 0.25m above the water surface. Find the metacentric height?



### Solution

Weight of boat + load = 280 + 34  
= 314 kN

Weight of seawater displaced =  $1025 \times 9.8 \times 10 \times 4 \times \text{Draught}$

$$\text{Draught} = \frac{(314 \times 1000)}{(1025 \times 9.8 \times 10 \times 4)}$$

= 0.781m

$$\text{BM} = \frac{I}{V} = \frac{1/12 \times 10 \times 4^3}{4 \times 10 \times 0.781}$$

= 1.707m

The center of gravity is  $BG = 0.25 + \frac{0.781}{2} = 0.640\text{m}$

$GM = BM - BG$

= 1.707 - 0.640

= 1.067m

**Result:** The metacentric height is 1.067m

### 3. Conclusion

From our analysis the stability of the ship depends on the metacentre and the centre of buoyancy plays an important role in Metacentre. When metacentre is equal to zero the ship remains in neutral equilibrium and when it is less than or greater than the ship remains unstable and stable respectively.

### References

- Pérez, R., & Riola, J. M. (2011, January). Case study of damage stability criteria of Merchant vessels and Warships. *In Damaged Ship International Conference* (pp. 26-27).
- Pérez, F. R., & Rodríguez, J. M. R. (2011). DAMAGE STABILITY CRITERIA IN AIRCRAFT CARRIERS. *Journal of Marine Technology & Environment*, 1.
- Priyadharsini, S., Basheer, S. M., Kaushalya, R., & Sneka, M. (2018). A study on Motion of a free falling body in Kinematic Equation. *International Journal for Research in Applied Science & Engineering Technology*, 6, 118-124.
- Riola, J. M., & Pérez, R. (2009). Warship damage stability criteria case study. *Journal of Maritime Research*, 6(3), 75-100.
- Sarchin, T. H. (1962). Stability and buoyancy criteria for US Naval surface ships. Bureau of Ships, Department of the Navy, Washington DC, Published by: The Society of Naval Architects and Marine Engineers, SNAME Transactions 1962, Paper 5, paper: T1962-1 Transactions.
- Surko, S. W. (1994). An assessment of current warship damaged stability criteria. *Naval Engineers Journal*, 106(3), 120-131.
- International journal of multidisciplinary research and studies Vol.02, ISSUE 01, A STUDY ON MATHEMATICAL MODELLING OF SHIP STABILITY, accessed Nov 27 2024.
- Malarvannan, J., Sivasubramanian, C., Sivasankar, R., Jeganathan, M., & Balakumari, M. (2016). Shading of building as a preventive measure for passive cooling and energy conservation—A case study. *Indo-Asian Journal of Multidisciplinary Research (IAJMR)*: ISSN, 2454-1370.
- Malarvannan, J., Sivasubramanian, C., Sivasankar, R., Jeganathan, M., & Balakumari, M. (2015). Life Science Archives (LSA).
- Sivasankar, R., Sivasubramanian, C., Malarvannan, J., Jeganathan, M., & Balakumari, M. (2015). Life Science Archives (LSA).
- Ashok, J., Kumar, S. S., Kumar, P. S., & Jeganathan, M. (2015). Life Science Archives (LSA)

**Copyright:** ©2024 Shreedevi Kalyan, et al. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.