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The Ewald-Oseen Extinction Theorem in the Light of Huygens' Principle

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Abstract

The Ewald Oseen extinction theorem deals with the penetration of an electromagnetic wave from a vacuum into a polarizable and/or magnetizable medium. It states that the incident electromagnetic wave penetrates into the medium without perturbation. Within the medium, there is a polarization and/or magnetization which create(s), (i), the reflected wave, (ii), a wave which extinguishes the incident wave in the medium, (iii), the observed wave(s) in the medium, and, (iv), the wave(s) leaving the medium. The question arises, what excites the medium? For the incident wave does it not because it does not interact with the medium. Thus, while being mathematically correct, that theorem is both physically and philosophically incorrect as the excitation used has no reason but is being imposed from nothing. Moreover, it contradicts Huygens' principle according to which, (i), the incident wave is absent after having excited the sources of the secondary wavelets and, (ii), each secondary source re-irradiates only one secondary wavelet (in case of double refraction, two ones). This contradiction is examined in terms of, (i), propagators (following Feynman) and, (ii), the electric Hertz vector (following Zangwill, where his calculations are simplified). Being mathematically correct, it may be useful to treat the theorem "as if" (Vaihinger) it also were physically correct.

1. Introduction

When a light wave enters a material medium, it magnetizes and/or polarizes it. The excited medium re-irradiates light. In view of the linearity of the Maxwell equations for linear media, one could think that the total electromagnetic field in the medium is the sum of the original (incident, exciting) and the re-irradiated fields. Actually, the incident beam is not observed in the medium. The Ewald Oseen extinction theorem explains this absence in that the excited medium creates three waves: (i), one, which annihilates aka extincts the incoming wave, (ii), one that propagates trough the medium according to its optical properties, see Figure 1 (p. 8, Figure 4), and, (iii), the reflected wave [1-3].

Mansuripur comments, as if "the oscillating electrons conspire to produce a field that exactly cancels out the original beam everywhere in the medium" (p. 209). Now, it is hard to believe, that "electrons conspire" that way as this phenomenon has nothing to do with collective effects like Langmuir waves [4,5]. Huygens' secondary wavelets consist of *only one* part which, however, depends on the local propagation conditions (double refraction needs additional considerations [6-9]). Of course, *mathematically*, due to the linearity of the Maxwell equations and the linear media usually considered in the literature, the electric and magnetic field quantities can be rather arbitrarily split and combined. Anyway, the theorem has been reconsidered several times and for various materials, e.g. to name a few, and it enters some textbooks, e.g. For a review of various interpretations of the extinction process, see [10,11].

Sein states, ". . . the extinction theorem is essentially an expression of Huygens' principle for the incident field inside the medium." According to Lian, if Huygens' principle is mathematically formulated such that, in a vacuum or in homogeneous isotropic media, back-scattering is absent, "it must satisfy" the extinction theorem (p. 5 II). Using simple cases, we will show that the extinction theorem and Huygens' principle are mathematically equivalent but not physically.

Thus, to clarify the physical content of the Ewald Oseen extinction theorem and, in particular, its relationship to Huygens' principle, this article proceeds as follows. Section II sketches the most general representation of Huygens' principle in terms of propagators due

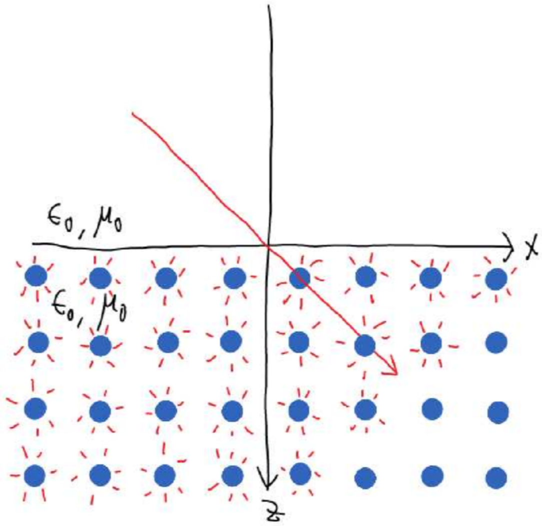

Figure 1: Ewald Oseen Extinction Theorem for a Dielectric

The incident pulse enters the medium without perturbation (red line), the atoms irradiate waves to Feynman. Section III presents a novel representation of the extinction theorem using that framework. For subsequent use, Section IV provides the Maxwell-Heaviside equations and sketches the macroscopic and microscopic approaches for describing the polarization as well as Hertz's potential and the electric Hertz vector, where a novel hypothesis is proposed. Basing on that, Section V treats the dielectric half-space, commenting on Born & Wolfs and Zangwill's treatments. Finally, Section VI summarizes and concludes this article.

It is a correction, clarification, and major extension of an earlier article by one of us [12-28].

2. Feynman's Representation of Huygens' Principle in Terms of Propagators

2.1 The Chapman-Kolmogorov Equation

According to Feynman, Huygens' principle can be expressed through propagators P_{ab} as

$$P_{ac} = \sum_b P_{ab} P_{bc} \,. \qquad (1)$$

That means, P_{ac}, representing the propagation from state (or space-time point) a to state c, can be constructed as a propagation, first, from the initial state a to all possible (accessible) intermediate states {b} and, then, from those to the final state c. The 'primary waves' $\{P_{ab}\}$ 'excite' a certain set of states {b}, which irradiates the 'secondary wavelets' $\{P_{bc}\}$ that sum up to the value of state c.

Formula (1) is a variant of the Chapman-Kolmogorov equation [29,30]. In the space-time domain, it reads

$$G(\vec{r}_c, t_c; \vec{r}_a, t_a) = \iiint_{V_b} G(\vec{r}_c, t_c; \vec{r}_b, t_b)\, G(\vec{r}_b, t_b; \vec{r}_a, t_a)\, \mathrm{d}V_b \,; \quad t_c \geq t_b \geq t_a \,, \qquad (2)$$

where *G* is an appropriate propagator *aka* Green's function [31,32]. It generalizes Huygens' construction from sharp to spreading wave fronts, where the domain of sources of secondary wavelets is not necessarily a surface but may be a *finite* volume V_b. Sharp wave fronts correspond to a δ-function in G whence the volume integral is reduced to a surface integral. In the usual representations of Huygens' construction, this surface is the location of the secondary sources.

Notice that the Chapman-Kolmogorov equation (2) is *not* fulfilled by the Green's function of d'Alembert's (Euler's) wave equation in 3+1d. However, that does *not* mean that Huygens' principle "is only approximately fulfilled in optics". One has to transform a wave equation into two partial differential equations of *first* order in time and to analyze the corresponding *matrix* Green's function.

2.2 Example

To illustrate that most general representation of Huygens' principle, let us consider a very simple example, *viz*., a two-dimensional network of (ideally) lossless transmission lines, see Figure 2 [33].

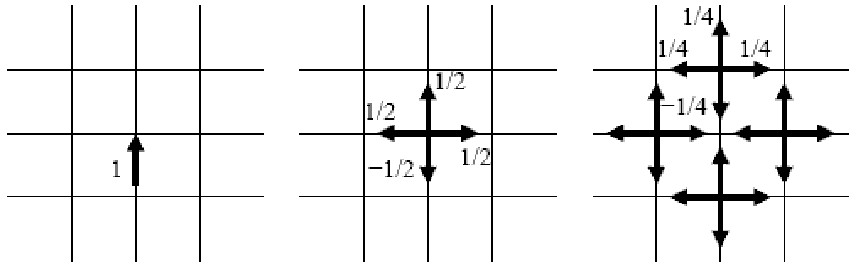

Figure 2: 2D TLM Mesh; Left: Incident Pulse, Middle: First Scattering, Right: Second Scattering

A node connects 4 lines, say, in the directions West, North, East, and South. Each line has the impedance *Z*. A very short pulse of voltage 1 (arbitrary units) incident from the South is scattered as follows.

• The pulse enters the node from a line with impedance *Z*. At the node, it meets three lines with impedance *Z* each. Because they are parallel, their common impedance at the node is $\frac{1}{3}Z$. For this, the reflection coefficient equals (a node works like a voltage divider)

$$\rho = \frac{\frac{1}{3}Z - Z}{\frac{1}{3}Z + Z} = -\frac{1}{2}\,. \tag{3}$$

A pulse of voltage $\rho = -\frac{1}{2}$ will travel back to the Southern neighbor.

• The sum of the voltages of the 3 pulses transmitted towards West, North, and East, respectively, amounts to the transmission coefficient (due to the continuity of voltage, $\rho + \tau = 1$)

$$\tau = 1 - \rho = \tfrac{3}{2}\,. \tag{4}$$

By virtue of symmetry, each transmitted pulse carries a voltage of $\frac{\tau}{3} = \frac{1}{2}$.

• At the moment of scattering, the voltage in the Southern line equals the sum of the voltages of the incident and reflected pulses,

$$1 + \rho = \tfrac{1}{2}\,. \tag{5}$$

Therefore, at the moment of scattering, *all* lines get *one and same* voltage $+\frac{1}{2}$ each. This reflects the symmetry of the node and Huygens' principle [34]. The summing (5) has been discarded in and $\rho^2 = (\tau/3)^2$ along all lines considered to represent Huygens' principle [35,36].

In *d* dimensions, the reflection coefficient equals

$$\frac{\frac{1}{2d-1}Z - Z}{\frac{1}{2d-1}Z + Z} = -\frac{d-1}{d} = \frac{1}{d} - 1\,. \tag{6}$$

The total voltage on the incoming line equals $(\frac{1}{d} - 1) + 1 = \frac{1}{d}$. The transmitted pulses exhibit a voltage of

$$\frac{1 - \left(\frac{1}{d} - 1\right)}{2d-1} = \frac{1}{d} \tag{7}$$

along each line. Again, the symmetry and the fulfillment of Huygens' principle are obvious.

The voltage impulses in that network are described by a set of difference equations of *first* order in the propagation (time) step. Its fundamental solution is a discrete *matrix* Green's function which obeys a Chapman-Kolmogorov equation [37].

BTW, TLM networks are (idealized) *physical* realizations of correlated random walks [38]. This makes their algorithms in numerical mathematics extremely stable.

3. The Ewald Oseen Extinction Theorem in Terms of Propaga Tors

Let a in formula (1) refer to a state (point) outside a medium (vacuum), c to one inside the medium, and b to one on its surface. Then, formula (1) reads

$$P_{ac}^{(\text{vac}\to\text{med})} = \sum_{b\,\in\,\text{surface}} P_{ab}^{(\text{vac})} P_{bc}^{(\text{med})}\,. \qquad (8)$$

Now, if there would be no medium, we would have

$$P_{ac}^{(\text{vac})} = \sum_{b\,\in\,\text{surface}} P_{ab}^{(\text{vac})} P_{bc}^{(\text{vac})}\,, \quad \text{or} \quad P_{ac}^{(\text{vac})} - \sum_{b\,\in\,\text{surface}} P_{ab}^{(\text{vac})} P_{bc}^{(\text{vac})} = 0\,. \qquad (9)$$

Adding the second formula (9) to the r.h.s. of formula (8), we obtain

$$P_{ac}^{(\text{vac}\to\text{med})} = P_{ac}^{(\text{vac})} + \sum_{b\,\in\,\text{surface}} P_{ab}^{(\text{vac})} \left(P_{bc}^{(\text{med})} - P_{bc}^{(\text{vac})}\right). \qquad (10)$$

This formula is a most general expression of the Ewald Oseen extinction theorem in terms of propagators. Each state on the surface b is supposed to create *two* secondary 'wavelets'.

1. $P_{bc}^{(\text{med})}$ describes the propagation of one 'wavelet' from the boundary point/state b to the medium point/state c according to the propagation conditions of the medium (e.g., speed of light $c = c_0/n$, n being the index of refraction of the medium);
2. $P_{bc}^{(\text{vac})}$ describes the propagation of the other 'wavelet' according to the propagation conditions in a vacuum (e.g., speed of light $c = c_0$); this 'wavelet' extinguishes the incident 'wave' $^{\text{P(vac)}}$ in the medium.

Mathematically, both formulae (8) and (10) are equivalent. Physically, however, they are quite *different*. For Huygens' principle – formula (8) – states that the incoming 'wave' is absorbed at the boundary.

Remark 1 *What should be the reason for the secondary sources to irradiate two different secondary 'wavelets' into the medium according to formula (10)? (For double refraction, see.*

Remark 2 *Again, if the incident 'field' travels unperturbed through the medium, which 'field' is exciting the medium?*

4. General Electromagnetic Formulas for Later Use

4.1 The Maxwell-Heaviside Equations

In what follows, we will consider an electromagnetic wave traveling through a vacuum and a linear, electrically neutral, homogeneous, and isotropic dielectric. To avoid confusions in the notations, let us begin with the Maxwell-Heaviside equations (SI units; cf. [39,40]).

$$\nabla \times \vec{H} - \dot{\vec{D}} = \vec{j}_{\text{free}} \qquad (11a)$$

$$\nabla \times \vec{E} + \dot{\vec{B}} = 0 \qquad (11b)$$

$$\nabla \cdot \vec{D} = \rho_{\text{free}} \qquad (11c)$$

$$\nabla \cdot \vec{B} = 0 \qquad (11d)$$

Here, ρ_{free} and $\vec{j}_{free}$ are the free, *un*bound charges and (conduction, convection) current densities, respectively. They obey the equation of continuity

$$\nabla \cdot \vec{j}_{\text{free}} + \dot{\rho}_{\text{free}} = 0\,. \qquad (12)$$

In an electrically neutral dielectric, both vanish identically,

$$\vec{j}_{\text{free}}(\vec{r},t) \equiv \vec{0}, \quad \rho_{\text{free}}(\vec{r},t) \equiv 0 \tag{13}$$

4.2 Macroscopic Theory

Within a macroscopic theory, one sets, for the dielectric under consideration,

$$\vec{H} = \frac{1}{\mu_0}\vec{B}, \quad \vec{D} = \epsilon_r\epsilon_0\vec{E}. \tag{14}$$

The Maxwell-Heaviside equations (11) become

$$\frac{1}{\mu_0}\nabla\times\vec{B} - \epsilon_r\epsilon_0\dot{\vec{E}} = \vec{0} \tag{15a}$$

$$\nabla\times\vec{E} + \dot{\vec{B}} = \vec{0} \tag{15b}$$

$$\epsilon_r\epsilon_0\nabla\cdot\vec{E} = 0 \tag{15c}$$

$$\nabla\cdot\vec{B} = 0 \tag{15d}$$

As a consequence, one obtains wave equations for those field quantities with phase velocities c_0 in a vacuum and $c = c_0/n, n = \sqrt{\epsilon_r}$ in a dielectric.

The amplitudes of the reflected and *t*ransmitted fields are determined by the continuities and discontinuities of the electric and magnetic field quantities.

In the case of an incoming transverse plane wave moving perpendicularly to the surface $z = 0$ of a dielectric in the half-space $z \geq 0$, one has

$$\vec{E}^{(\mathrm{i})}(\vec{r},t) = \vec{E}_0\, e^{i(k_0 z-\omega t)}, \quad \vec{B}^{(\mathrm{i})}(\vec{r},t) = \vec{B}_0\, e^{i(k_0 z-\omega t)}, \quad \vec{B}_0 = \vec{k}_0\times\vec{E}_0\,. \tag{16}$$

One obtains for the reflected and transmitted electric waves

$$\vec{E}^{(\mathrm{r})}(\vec{r},t) = \frac{1-n}{1+n}\,\vec{E}_0\, e^{i(-k_0 z-\omega t)}, \quad \vec{E}^{(\mathrm{t})}(\vec{r},t) = \frac{2}{1+n}\,\vec{E}_0\, e^{i(kz-\omega t)}\,;$$
$$n = \sqrt{\epsilon_r}, \quad k = nk_0; \quad \vec{E}^{(\mathrm{r})}(\vec{0},t) + \vec{E}^{(\mathrm{i})}(\vec{0},t) = \vec{E}^{(\mathrm{t})}(\vec{0},t)\,, \tag{17}$$

and analogously for $\vec{B}^{(r)}$ and $\vec{B}^{(t)}$. $\vec{B}^{(r)}$ and $\vec{B}^{(t)}$ will play no role in what follows because the dielectric under consideration is not magnetizable.

4.3 Polarization

The relative dielectric constant εr in eqs. (14) can also been represented by the polarization $\vec{P}$ and the polarizability χ as

$$\vec{D} = \epsilon_0\vec{E} + \vec{P}, \quad \vec{P} = \epsilon_0\chi\vec{E}\,. \tag{18}$$

Here, $\vec{E}$ is the (total) electric field strength inside the dielectric. Since the dielectric under consideration is optically homogeneous, isotropic, and linear, χ and the index of refraction $n = \sqrt{1+\chi}$ are constant, and the vectors $\vec{P}$ and $\vec{E}$ are parallel to the vector $\vec{E}^{(i)}$.

The microscopic theory to be considered next should be compatible with that macroscopic picture. As a matter of fact, this has been more or less implicitly used in the representations we are aware of, notably, in [25,26], as will be indicated below.

4.4 Microscopic Theory

The polarization $\vec{P}$ (18) is related to the *bound* charge and current densities as

$$\nabla \vec{P} = -\rho_{\text{bound}}, \quad \dot{\vec{P}} = \vec{j}_{\text{bound}}; \quad \nabla \cdot \vec{j}_{\text{bound}} + \dot{\rho}_{\text{bound}} = 0\,. \tag{19}$$

The Maxwell-Heaviside equations (11) become

$$\frac{1}{\mu_0}\nabla \times \vec{B} - \epsilon_0 \dot{\vec{E}} = \vec{j}_{\text{free}} + \vec{j}_{\text{bound}} =: \vec{j}_{\text{tot}} \tag{20a}$$

$$\nabla \times \vec{E} + \dot{\vec{B}} = 0 \tag{20b}$$

$$\epsilon_0 \nabla \cdot \vec{E} = \rho_{\text{free}} + \rho_{\text{bound}} =: \rho_{\text{tot}} \tag{20c}$$

$$\nabla \cdot \vec{B} = 0 \tag{20d}$$

In our case, by virtue of the identities (13), we have $\rho_{tot} = \rho_{bound}$ and $\vec{j}_{tot} = \vec{j}_{bound}$. Notice that some authors call $\vec{j}_{free} + \partial\vec{D}/\partial t$ the total current (density) [41].

4.5 Hertz's Potential

To calculate the radiation field of a point-like linear dipole in a vacuum oscillating along the z-axis, Heinrich Hertz has invented the following ingenious calculation [42]. Outside the dipole, the Maxwell-Heaviside equations (20) read (Hertz's equation numbers)

$$(1) \quad \frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}, \qquad (2) \quad \frac{1}{c_0^2}\frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B}.$$

$$(3) \quad \nabla \cdot \vec{B} = 0, \qquad \nabla \cdot \vec{E} = 0.$$

Due to the cylindrical symmetry of the dipole and the linearity of the Maxwell equations, the fields $\vec{E}$, $\vec{B}$ are cylindrical symmetric, too. (For the general case, see, e.g. [43]). By virtue of its rotational nature, $\vec{B}$ circulates parallel to the x, y-plane around the z-axis, while $\vec{E}$ is perpendicular to it. In cylindrical coordinates ($\rho = \sqrt{x^2+y^2}$), this means (our E_ρ is Hertz's R, our B_ϕ – his P)

$$\vec{E} = \vec{E}(\rho, z, t) = (E_\rho, E_\phi \equiv 0, E_z), \quad E_\rho = \frac{x}{\rho}E_x + \frac{y}{\rho}E_y; \tag{21a}$$

$$\vec{B} = \vec{B}(\rho, z, t) = (B_\rho \equiv 0, B_\phi, B_z \equiv 0), \quad B_\phi = \frac{y}{\rho}B_x - \frac{x}{\rho}B_y. \tag{21b}$$

There are only three non-vanishing field components. They are subject to the two constraints (3). For this, there is only *one* independent field variable, e.g. the Hertz potential Π. As a consequence, "a possible solution" ([Hertz 2001] p. 150) of eqs. (1). . . (3) is

$$E_x = -\frac{\partial^2 \Pi}{\partial x\,\partial z} = -\frac{\partial^2 \Pi}{\partial \rho\,\partial z}\,\frac{x}{\rho}, \quad E_y = -\frac{\partial^2 \Pi}{\partial y\,\partial z} = -\frac{\partial^2 \Pi}{\partial \rho\,\partial z}\,\frac{y}{\rho}, \quad E_\rho = -\frac{\partial^2 \Pi}{\partial \rho\,\partial z},$$

$$E_z = \frac{\partial^2 \Pi}{\partial x^2} + \frac{\partial^2 \Pi}{\partial y^2} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \Pi}{\partial \rho}\right) = \frac{1}{c_0^2}\frac{\partial^2 \Pi}{\partial t^2} - \frac{\partial^2 \Pi}{\partial z^2}, \tag{22a}$$

$$B_x = \frac{1}{c_0^2}\frac{\partial^2 \Pi}{\partial y \partial t} = \frac{1}{c_0^2}\frac{\partial^2 \Pi}{\partial \rho\,\partial t}\frac{y}{\rho}, \quad B_y = -\frac{1}{c_0^2}\frac{\partial^2 \Pi}{\partial x \partial t} = -\frac{1}{c_0^2}\frac{\partial^2 \Pi}{\partial \rho\,\partial t}\frac{x}{\rho}, \quad B_\phi = -\frac{1}{c_0^2}\frac{\partial^2 \Pi}{\partial \rho \partial t}, \tag{22b}$$

where the Hertz potential Π obeys the homogeneous wave equation,

$$\frac{1}{c_0^2}\,\frac{\partial^2 \Pi}{\partial t^2} = \Delta\,\Pi. \tag{23}$$

The symmetry of the electromagnetic field is obvious; in particular, Π is independent of ϕ. The vector and scalar potentials equal $\vec{A} = (0, 0, -\partial\Pi/\partial t)$ and $\Phi = \partial\Pi/\partial z$, respectively. For this, the Hertz potential is also called a 'super-potential'.

4.6 The Electric Hertz Vector

The boundary conditions used in eqs. (17) are not needed when looking for a solution in whole space. Such a solution can be obtained using the electric Hertz vector.

The homogeneous Maxwell-Heaviside equations (20b), (20d) are identically fulfilled through introducing the vector $\vec{A}$ and scalar potentials Φ, respectively, as

$$\vec{B} = \nabla \times \vec{A}, \quad \vec{E} = -\dot{\vec{A}} - \nabla\Phi\,. \tag{24}$$

For exploiting the electric Hertz vector, it is necessary to impose the Lorenz gauge,

$$\nabla \cdot \vec{A} + \frac{1}{c_0^2}\dot{\Phi} = 0\,. \tag{25}$$

It leads to the wave equations

$$\frac{1}{c_0^2}\ddot{\vec{A}} - \nabla^2\vec{A} = \mu_0\,\vec{j}_{\mathrm{bound}} = \mu_0\,\dot{\vec{P}}\,, \tag{26a}$$

$$\frac{1}{c_0^2}\ddot{\Phi} - \nabla^2\Phi = \frac{1}{\epsilon_0}\rho_{\mathrm{bound}} = -\frac{1}{\epsilon_0}\nabla\cdot\vec{P}. \tag{26b}$$

Analogously to the relations (19) between the polarization and the bound charge and current densities, the Lorenz gauge (25) can be automatically fulfilled by means of the electric Hertz vector $\vec{\pi}_e$,

$$\vec{A} = \frac{1}{c_0^2}\dot{\vec{\pi}}_{\mathrm{e}}\,, \quad \Phi = -\nabla\cdot\vec{\pi}_{\mathrm{e}} \tag{27}$$

Inserting these expressions into the wave equations (26) yields

$$\frac{1}{c_0^2}\frac{\partial}{\partial t}\left(\frac{1}{c_0^2}\ddot{\vec{\pi}}_{\mathrm{e}} - \nabla^2\vec{\pi}_{\mathrm{e}}\right) = \mu_0\frac{\partial}{\partial t}\vec{P}\,, \quad \frac{1}{c_0^2}\ddot{\vec{\pi}}_{\mathrm{e}} - \nabla^2\vec{\pi}_{\mathrm{e}} = \frac{1}{\epsilon_0}\vec{P} + \vec{f}(\vec{r})\,; \tag{28a}$$

$$-\nabla\cdot\left(\frac{1}{c_0^2}\ddot{\Phi} - \nabla^2\Phi\right) = -\frac{1}{\epsilon_0}\nabla\cdot\vec{P}\,, \quad \frac{1}{c_0^2}\ddot{\vec{\pi}}_{\mathrm{e}} - \nabla^2\vec{\pi}_{\mathrm{e}} = \frac{1}{\epsilon_0}\vec{P} + \vec{g}(t). \tag{28b}$$

Both equations are compatible, iff $\vec{f}(\vec{r}) = \vec{g}(t) = \text{const.}$. This constant must be equal zero for the advanced and retarded solutions to them to converge. Therefore, the electric Hertz vector obeys the wave equation

$$\frac{1}{c_0^2}\ddot{\vec{\pi}}_{\mathrm{e}} - \nabla^2\vec{\pi}_{\mathrm{e}} = \frac{1}{\epsilon_0}\vec{P}\,. \tag{29}$$

Remark 3 *That wave equation means that*
1. The phase speed of the electric Hertz vector $\vec{\pi}_e$ equals c_0 both in the vacuum and the dielectric;
2. $\vec{\pi}_e$ is calculated using a seemingly vacuum equation; due to the polarization, however, it contains both vacuum and medium terms. see below.

Moreover,

$$\vec{E} = -\frac{1}{c_0^2}\ddot{\vec{\pi}}_{\mathrm{e}} + \nabla\left(\nabla\cdot\vec{\pi}_{\mathrm{e}}\right) = \nabla\times\nabla\times\vec{\pi}_{\mathrm{e}} - \frac{\vec{P}}{\epsilon_0}\,. \tag{30}$$

Hypothesis 1 *The Hertz potential Π and its generalizations to the electric Hertz vector $\vec{\pi}_e$ in polarizable media and the magnetic Hertz vector $\vec{\pi}_m$ in magnetizable media represent a complete set of independent dynamical variables.*

If that hypothesis holds true, it provides a reason for the fact that they have proven useful for solving many radiation problems (for a throughout analysis, see [44-46], for references of pedagogical purpose [47]). In contrast to the cumbersome traditional calculations (e.g., Section 2.4), the electric Hertz vector allows for a relatively direct treatment of the extinction theorem in non-magnetically, electrically neutral dielectrics (e.g. as will be discussed in what follows.

5. The Ewald-Oseen Extinction Theorem for a Dielectric Half- Space

5.1 Experimental Setup

Zangwill (Section 20.9) considers a dielectric as above which occupies the half-space $z \geq 0$ (see his Figure 20.23 on p. 763). There is an incident plane wave of electric field strength $\vec{E}^{(i)}$ moving perpendicularly to the surface $z = 0$ of the dielectric,

$$\vec{E}^{(\mathrm{i})}(\vec{r}, t) = \vec{E}_0\, e^{i(k_0 z - \omega t)}\,. \tag{31}$$

The vector $\vec{E}_0$ lies parallel to the x, y-plane.

Remark 4 *As noticed before, this ansatz supposes the incident wave to penetrate without perturbation into the dielectric. Then, which field is exciting the polarization in it?*

As mentioned at the beginning of Subsection IV F, matching conditions play no role, if one seeks a solution that is valid for the whole space $\mathbb{R}^3$ p. 763, after formula (20.244). This is surely a benefit of such an approach. For the setup under consideration, they play no role anyway, because, (i), the electric field is transverse and, (ii), the medium is not magnetizable.

5.2 Electric Hertz Vector

Due to the linearity of all equations, inside the dielectric, the total electric field can be split into the incident field $\vec{E}^{(i)}$ (31) and the field $\vec{E}^{(p)}$ due to the polarization of the dielectric,

$$\vec{E} = \vec{E}^{(\mathrm{i})} + \vec{E}^{(\mathrm{p})}. \tag{32}$$

Accordingly, the electric Hertz vector can be split as

$$\vec{\pi}_{\mathrm{e}} = \vec{\pi}_0 + \vec{\pi}_{\mathrm{p}} \tag{33}$$

with, using formulas (30) and (31),

$$\frac{1}{c_0^2}\frac{\partial^2\vec{\pi}_0}{\partial t^2} - \nabla^2\vec{\pi}_0 = 0\,, \quad \vec{E}^{(\mathrm{i})} = -\frac{1}{c_0^2}\ddot{\vec{\pi}}_0 + \nabla\left(\nabla\cdot\vec{\pi}_0\right) \tag{34a}$$

$$\frac{1}{c_0^2}\frac{\partial^2\vec{\pi}_{\mathrm{p}}}{\partial t^2} - \nabla^2\vec{\pi}_{\mathrm{p}} = \frac{1}{\epsilon_0}\vec{P}\,, \quad \vec{E}_p = -\frac{1}{c_0^2}\ddot{\vec{\pi}}_{\mathrm{p}} + \nabla\left(\nabla\cdot\vec{\pi}_{\mathrm{p}}\right). \tag{34b}$$

$\vec{\pi}_0$ is not of interest here except that it represents the solution to the homogeneous part of eq. (29) which is given by the incident electric field (31). Due to that, it is sufficient to seek just one particular solution $\vec{\pi}_p$ to eq. (34b). Zangwill chooses the retarded integral

$$\vec{\pi}_{\mathrm{p}}(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\vec{P}(\vec{r}',t-|\vec{r}-\vec{r}'|/c_0)}{|\vec{r}-\vec{r}'|}\,\mathrm{d}x'\mathrm{d}y'\mathrm{d}z'. \tag{35}$$

"The physical solution of interest" (after formula (20.247)) contains a polarization of the form ($k = nk_0$)

$$\vec{P}(\vec{r}',t) = \vec{P}_0\, e^{i(kz-\omega t)}\,. \tag{36}$$

As a matter of fact, this expression is imposed without further reasoning; actually, it follows from the macroscopic theory, see formulas (16) f. Thus, Born & Wolf's and Zangwill's treatments are not purely microscopically [48].

Inserting the r.h.s. of the ansatz (36) into the integral (35) yields ($\omega/c_0 = k_0$)

$$\vec{\pi}_{\mathrm{p}}(\vec{r},t) = \frac{\vec{P}\,e^{-i\omega t}}{4\pi\epsilon_0}\,e^{ikz}\int\limits_0^\infty e^{ik(z'-z)}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\frac{e^{ik_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\,\mathrm{d}x'\mathrm{d}y'\mathrm{d}z'. \tag{37}$$

The advantage of putting the factor e^{ikz} before and the factor e^{-ikz} inside the integral will become visible in formulas (39) ff. below.

The integrals over x', y' equal (for a detailed calculation, see (17), [49].

$$\frac{2\pi i}{k_0}\,e^{ik_0|z'-z|}\,. \tag{38}$$

Therefore,

$$\vec{\pi}_{\mathrm{p}}(\vec{r},t) = \frac{\vec{P}\,e^{-i\omega t}}{2\epsilon_0 k_0}\,I, \quad I := i\,e^{ikz}\int\limits_0^\infty e^{ik(z'-z)}\,e^{ik_0|z'-z|}\,\mathrm{d}z'. \tag{39}$$

As to be expected for symmetry reasons, $\vec{\pi}_p$ is independent of x, y.

The integral I must be separately calculated for $z < 0$ (Case 1 below) and $z > 0$ (Case 2 below) due to the absolute value $|z' - z|$ in the integrand. Indeed,

$$e^{ik(z'-z)}\, e^{ik_0|z'-z|} = \begin{cases} e^{ik(z'-z)}\, e^{-ik_0(z'-z)} = e^{i(k-k_0)(z'-z)} & z' \leq z\,, \\ e^{ik(z'-z)}\, e^{ik_0(z'-z)} = e^{i(k+k_0)(z'-z)} & z' \geq z\,. \end{cases} \tag{40}$$

Case 1. $z < 0$ (vacuum): For all $\{z' \,|\, 0 \leq z' < \infty\}$, we have $z' > z$. The integral I in formula (39) thus equals

$$I = I_{z<0} := i\, e^{ikz} \int_0^\infty e^{i(k+k_0)(z'-z)}\, \mathrm{d}z' = -\frac{e^{-ik_0 z}}{k + k_0}\,. \tag{41}$$

Case 2. $z \geq 0$ (dielectric): The integration interval $0 \leq z' < \infty$ in formula (39) must be split into the intervals $0 \leq z' \leq z$ and $z \leq z' < \infty$,

$$I = I_{z\geq 0} := i\, e^{ikz} \int_0^z e^{i(k-k_0)(z'-z)}\, \mathrm{d}z' + e^{ikz} \int_z^\infty e^{i(k+k_0)(z'-z)}\, \mathrm{d}z'$$
$$= \frac{e^{ikz} - e^{ik_0 z}}{k - k_0} - \frac{e^{ikz}}{k + k_0} = -\frac{e^{ik_0 z}}{k - k_0} + \frac{2k_0 e^{ikz}}{k^2 - k_0^2}\,. \tag{42}$$

Inserting those results into formula (39) gives

$$\vec{\pi}_\mathrm{p}(\vec{r}, t) = -\frac{\vec{P}_0\, e^{-\omega t}}{2\epsilon_0 k_0} \begin{cases} \frac{1}{k+k_0}\, e^{-ik_0 z} & z < 0\,, \\ \frac{1}{k-k_0}\, e^{ik_0 z} + \frac{2k_0}{k_0^2 - k^2}\, e^{ikz} & z \geq 0\,. \end{cases} \tag{43}$$

5.3 Ewald Oseen Extinction Theorem

We are now in the position to calculate $\vec{E}_\mathrm{p}$ according to formula (34b). Since $\vec{\pi}_\mathrm{p} \parallel \vec{P}$ and $\vec{\pi}_\mathrm{p}$ – as $\vec{P}$ does – depends only on z, we have $\nabla \cdot \vec{\pi}_\mathrm{p} = \nabla \cdot \vec{P} = 0$. Due to that and $\omega^2/c_0^2 = k_0^2$,

$$\vec{E}_\mathrm{p} = -\frac{1}{c_0^2} \ddot{\vec{\pi}}_\mathrm{p} = k_0^2 \vec{\pi}_\mathrm{p}\,, \quad \vec{E} = \vec{E}_0\, e^{i(k_0 z - \omega t)} + k_0^2 \vec{\pi}_\mathrm{p}\,. \tag{44}$$

Again, we have to discriminate between the two cases $z < 0$ and $z \geq 0$.

Case 1. $z < 0$ (vacuum): Substituting formula (43) for $z < 0$ into formula (44) gives

$$\vec{E}(z < 0, t) = \vec{E}_0\, e^{i(k_0 z - \omega t)} - \frac{\vec{P}}{2\epsilon_0} \frac{k_0}{k_0 + k}\, e^{i(-k_0 z - \omega t)}\,. \tag{45}$$

The first term describes the incident wave, the second one the reflected wave. According to formulas (17), (44), and $k = nk_0$, we have

$$\vec{E}^{(\mathrm{r})} = \frac{1-n}{1+n}\, \vec{E}_0 = -\frac{\vec{P}_0}{2\epsilon_0} \frac{k_0}{k_0 + k} = -\frac{\vec{P}_0}{2\epsilon_0} \frac{1}{1+n}\,, \quad \vec{P}_0 = 2\epsilon_0 (n-1) \vec{E}_0. \tag{46}$$

Case 2. $z \geq 0$ (dielectric): Doing the same for $z \geq 0$ gives, using the second formula (46),

$$\vec{E}(z \geq 0, t) = \vec{E}_0\, e^{i(k_0 z - \omega t)} - \frac{\vec{P}}{2\epsilon_0} \frac{k_0}{k - k_0}\, e^{i(k_0 z - \omega t)} - \frac{\vec{P}}{\epsilon_0} \frac{k_0^2}{k_0^2 - k^2}\, e^{i(kz - \omega t)}$$
$$= \vec{E}_0\, e^{i(k_0 z - \omega t)} - \vec{E}_0\, e^{i(k_0 z - \omega t)} - 2(n-1) \vec{E}_0 \frac{1}{1 - n^2}\, e^{i(kz - \omega t)}$$
$$= \frac{2}{1+n} \vec{E}_0\, e^{i(kz - \omega t)}\,. \tag{47}$$

The second line displays the extinction of the incident wave, while the last line describes the observed transmitted wave $\vec{E}(t)$ (17) in the dielectric.

Our early recursion to the macroscopic theory in formulas (46) saves the calculations in (20.255) ff.

Obviously, that exposition is formally, mathematically correct. However, it contradicts Huygens' principle, see Remark 4.

6. Summary and Conclusions

In our understanding, Huygens' construction involves that the incident wave is *completely extinguished* by having excited the sources of the secondary wavelets; each of these sources irradiates *one* secondary wavelet according to its local propagation conditions (in case of double refraction, two secondary wavelets are irradiated).

On the contrary, the Ewald Oseen extinction theorem describes refraction this way: The incident wave moves through the refracting medium without any alteration;

There is a polarization and/or a magnetization of the medium creating *three or four* waves:

(a) the reflected wave,
(b) one wave which extinguishes the incoming wave,
(c) one wave which corresponds to the observed one (in case of double refraction two waves).

Here, the question arises, how the medium is excited when the incident wave moves through it *without* alteration, i.e. *without* interacting with the medium? Hence, as the excitation of a medium has not any cause, the theorem is both physically and philosophically doubtful. Therefore, it resembles Vaihinger's *philosophy of 'as if'* [50]. It can be useful to act "as if" it were physically (and philosophically) correct.

As an example, we have shown in Section III that an abstract propagator description is mathematically compatible with both points of view. This corroborates Feynman's interpretation of Huygens' principle in terms of the Chapman-Kolmogorov equation (2). However, both descriptions are mathematically but *not* physically equivalent.

Notice that all those treatments discard the interaction principle in that the incident wave acts upon the secondary sources, while there is no back-reaction from the secondary sources upon the incident wave (cf. end-note 9).

For the sake of completeness, let us remark this. The Ewald Oseen extinction theorem imagines that each secondary source in Huygens' principle re-irradiates more than one secondary wavelet. This resembles the standard interpretation of the surface integral terms in Kirchhoff's integral theorem in that it "involves two types of sources of varying strength." [51,52]. For a criticism of that interpretation as well as considering Kirchhoff's theorem as a representation of Huygens' principle, see.

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