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Causality in Maxwell's Equations and the Creation of Electromagnetic Fields

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Abstract

As well known, Maxwell's equations in a vacuum can be transformed into two wave equations, the solutions of which are the electric and magnetic fields as a functional of arbitrary charge and current densities and first derivatives of them. Accordingly, Jefimenko and others have argued that the true sources of the electromagnetic field are the charge and current densities, while the electric and magnetic fields are independent of each other. On the contrary, Maxwell, Weyl, and others interpret Maxwell's equations such that, in charge- and current-free regions, the electric and magnetic fields induce each other. As a matter of fact, Jefimenko's arguing, (i), discards the advanced solutions (describing incoming fields), (ii), implies action at a distance, and, (iii), is derived from non-fundamental equations. In contrast, the mutual creation of electric and magnetic fields emerges from fundamental equations and is free of that artifacts.

1. Introduction

"Causality. . . is a basic concept in physics – so basic, in fact, that it is hard to conceive of a useful model in which effects do not have causes. Indeed, the whole point of a physical model could be said to describe the process of cause and effect in some particular situation." (P. Kinsler 2018 [1]. p. 1)

In standard notation and SI units, the Maxwell-Heaviside equations read (SI units; cf. [2,3].)

$$\nabla \cdot \vec{D}(\vec{r},t) = \rho_{\text{free}}(\vec{r},t) \tag{1a}$$

$$\frac{\partial \vec{B}}{\partial t}(\vec{r},t) = -\nabla \times \vec{E}(\vec{r},t) \tag{1b}$$

$$\nabla \cdot \vec{B}(\vec{r},t) = 0 \tag{1c}$$

$$\nabla \times \vec{H}(\vec{r},t) = \vec{j}_{\text{free}}(\vec{r},t) + \frac{\partial \vec{D}}{\partial t}(\vec{r},t)\,. \tag{1d}$$

For charges and currents in a vacuum, the constitutive equations simplify to $\vec{D} = \varepsilon_0 \vec{E}$ and $\vec{H} = \vec{B}/\mu_0$. Then, the Maxwell-Heaviside equations (1) become (with $\varepsilon_0\mu_0 = 1/c_0^2$ and omitting the index 'free')

$$\nabla \cdot \vec{E}(\vec{r},t) = \rho(\vec{r},t)/\varepsilon_0 \tag{2a}$$

$$\frac{\partial \vec{B}}{\partial t}(\vec{r},t) = -\nabla \times \vec{E}(\vec{r},t) \tag{2b}$$

$$\nabla \cdot \vec{B}(\vec{r},t) = 0 \tag{2c}$$

$$\nabla \times \vec{B}(\vec{r},t) = \mu_0 \vec{j}(\vec{r},t) + \frac{1}{c_0^2}\frac{\partial \vec{E}}{\partial t}(\vec{r},t)\,. \tag{2d}$$

Furthermore, requiring $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$ to be sufficiently smooth functions of $\vec{r}$ and t, that four equations can be separated into two inhomogeneous wave equations (with the already separated constraints (2a) and (2c)) as

$$\frac{1}{c_0^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \Delta \vec{E} = -\mu_0 \frac{\partial \vec{j}}{\partial t} - \frac{1}{\varepsilon_0}\nabla\rho\,, \quad \nabla \cdot \vec{E} = \frac{1}{\varepsilon_0}\rho \tag{3a}$$

$$\frac{1}{c_0^2}\frac{\partial^2 \vec{B}}{\partial t^2} - \Delta \vec{B} = \nabla \times \vec{j}\,, \quad \nabla \cdot \vec{B} = 0\,. \tag{3b}$$

Thus, in Section II, we will sketch Jefimenko's and other's arguing that the wave equations (3) show that, (i), the charges and currents are the only sources of the fields $\vec{E}$ and $\vec{B}$ and, (ii), $\vec{E}$ and $\vec{B}$ are independent of each other. And we will present arguments against that interpretation. In Section III, we will bring forward further arguments in favor of the mutual creation of electric and magnetic fields as described by eqs. (2b) and (2d). Section IV will summarize and conclude this article.

2. On Jefimenko's and Similar Interpretations of EQS. (3)

The retarded solutions of the wave equations (3) are [4].

$$\vec{E}(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\iiint \left[\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\,\rho(\vec{r}',t_r) + \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^2}\frac{1}{c_0}\frac{\partial\rho(\vec{r}',t_r)}{\partial t} - \frac{1}{|\vec{r}-\vec{r}'|}\frac{1}{c_0^2}\frac{\partial\vec{j}(\vec{r}',t_r)}{\partial t}\right]\mathrm{d}^3r' \tag{4a}$$

$$\vec{B}(\vec{r},t) = -\frac{\mu_0}{4\pi}\iiint \left[\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\times\vec{j}(\vec{r}',t_r) + \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^2}\times\frac{1}{c_0}\frac{\partial\vec{j}(\vec{r}',t_r)}{\partial t}\right]\mathrm{d}^3r'\,; \quad t_r := t - \frac{|\vec{r}-\vec{r}'|}{c_0}\,. \tag{4b}$$

In view of that solutions, Jefimenko argues as follows.

“. . . neither Maxwell’s equations nor their solutions indicate an existence of causal links between electric and magnetic fields. Therefore, we must conclude that an electromagnetic field is a dual entity always having an electric and a magnetic component simultaneously created by their common sources: time- variable electric charges and currents. . . ” [5].

As a matter of fact, in the retarded solutions (4) of the wave equations (3), the values of the fields depend on *earlier* values of solely the charges and currents. This leads to the conclusion that the latter ones are the (only) causes of the former ones ([6]. p. 382). In particular, “. . . since each of these [Maxwell-Heaviside] equations connects quantities si- multaneous in time, none of these equations can represent a causal relation.” [5]. However, the following arguments speak against such interpretations.

1. They do not apply to the advanced solutions. The latter ones are not nonphysical, do not violate the causality principle as they describe incoming fields [7].
2. They are drawn from non-fundamental equations. As a matter of fact, in contrast to the Maxwell equations (2), the wave equations (3) are not fundamental.

• They are *derived* from Maxwell’s equations (2).
• The Green’s function of them does not obey a Chapman-Kolmogorov equation which expresses Huygens’ principle, cf. [9,8].

3. If the field interaction is excluded, then the creation of electromagnetic fields in regions free of charges and currents is action at a distance. Despite Newton has stressed more than once the absence of action at a distance, that view discards the energy, momentum, and angular momentum carried by the field itself.

3.On the Mutual Creation of Electric and Magnetic Fields

According to Weyl, the principle of causality requires the basic equations to be in the form of partial differential equations of first order in time. Referring to Mie, he deals with the following subset of Maxwell’s [11] original equations (in nowadays notation) [10].

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\vec{j} = 0 \tag{5a}$$

$$\frac{\partial\vec{B}}{\partial t} + \nabla\times\vec{E} = 0 \tag{5b}$$

$$\frac{\partial\vec{A}}{\partial t} + \nabla\Phi = -\vec{E} \tag{5c}$$

$$\frac{\partial\vec{D}}{\partial t} - \nabla\times\vec{H} = -\vec{j} \tag{5d}$$

They comprise the constraints $\nabla\cdot\vec{D}\equiv\rho$ and $\nabla\cdot\vec{B}\equiv 0$ ($\vec{B}=\nabla\times\vec{A}$).

In a region without charges and currents, Weyl’s eqs. (5) and the Maxwell-Heaviside equations (2) can be rewritten as the two evolution equations

$$\frac{\partial\vec{E}}{\partial t}(\vec{r},t) = c_0^2\,\nabla\times\vec{B}(\vec{r},t) \tag{6a}$$

$$\frac{\partial\vec{B}}{\partial t}(\vec{r},t) = -\nabla\times\vec{E}(\vec{r},t)\,. \tag{6b}$$

They contain the constraints $\nabla\cdot\vec{E}(\vec{r},t)\equiv 0$ and $\nabla\cdot\vec{B}(\vec{r},t)\equiv 0$ which express the transver- sality of the electromagnetic field in a vacuum.

For that equations, one can define a 2 × 2 *matrix* Green’s function $\hat{G}$ as follows.

$$\begin{pmatrix}\frac{\partial}{\partial t} & -c_0^2\,\nabla\times\\ \nabla\times & \frac{\partial}{\partial t}\end{pmatrix}\hat{G} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\delta(\vec{r}-\vec{r}')\delta(t-t')\,. \tag{7}$$

This Green’s function obeys a Chapman-Kolmogorov equation representing Huygens’ prin- ciple (see above).

Thus, the two evolution equations (6) are *fundamental* equations, since they are – in contrast to the wave equations (3) – differential equations of *first* order in time, cf. also [9]. Hence, correct conclusions drawn from them are expected to be fundamental, too.

Eq. (6b) states that an electric field with rotational vortex ($\nabla\times\vec{E}\neq\vec{0}$) creates a (change of the) magnetic field. In turn, eq. (6a) states that a magnetic field with rotational vortex ($\nabla\times\vec{B}\neq\vec{0}$) creates a (change of the) electric field. That mutual creation of electric and magnetic fields is depicted in Figure 1 [12]. The corresponding propagation of electromagnetic waves is shown in Figure. 2 [13].

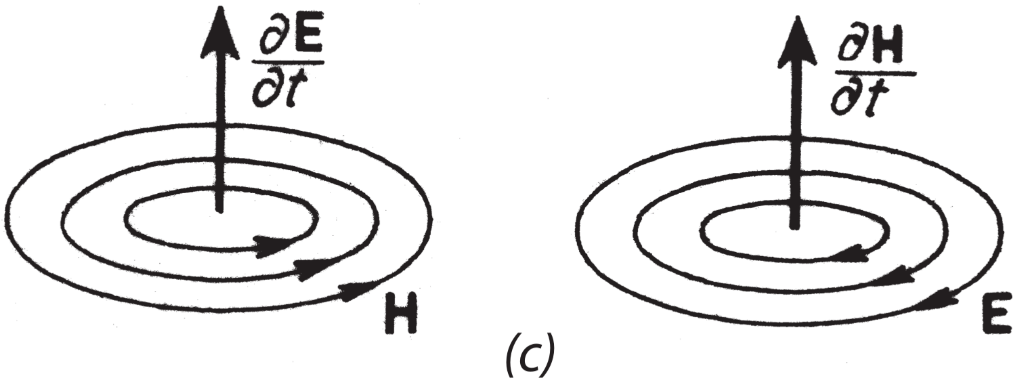

Figure 1: Mutual Creation of Electric and Magnetic Fields in a Vacuum Without Charges and Currents; $E \equiv \vec{E}$, $H = \vec{B}/\mu0$

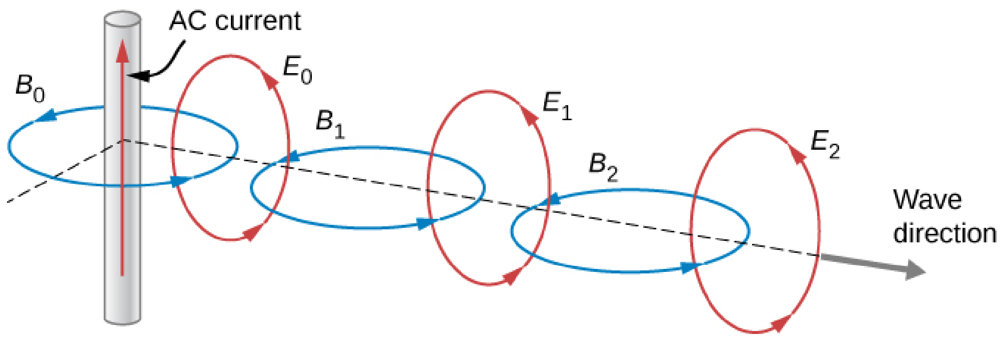

Figure 2: Propagation of Electric and Magnetic Fields in a Vacuum Without Charges and Currents by Mutual Creation

4. Summary and Conclusions

There are two different, actually excluding each other views on the causality in the prop- agation of electromagnetic waves at least in a vacuum.

The [*non*-fundamental] wave equations for the electric and magnetic fields (3) show that the fields are created solely by the charges and currents and thus independent of each other [5,6].

The *fundamental* eqs. (6) show the electric and magnetic fields mutually creating each other as illustrated in Figures. 1 and 2.

Kinsler has formulated the following causality criterion. The highest order of time derivative on the l.h.s [1]. should be higher than that of the r.h.s. Unfortunately, it applies to both the wave equations (3) and the evolution equations (6).

However, by virtue of the fact that the Maxwell-Heaviside equations (1) are fundamental (at least more fundamental than the wave equations, cf. also, the second view should be accepted and taught .

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