ISSN: 2997-6170
Space Science Journal
Research Article

# Brotherhood Relation

**Shrikrishna Jayraj Kalgaonkar***

*Retd Lecturer and Head of the Department Mathematics, Rajaram College, Kolhapur, India*

***Corresponding Author**
Shrikrishna Jayraj Kalgaonkar, Retd Lecturer and Head of the Department Mathematics, Rajaram College, Kolhapur, India

**Submitted:** 2025, Mar 14 **Accepted:** 2025, Apr 09 **Published:** 2025, Apr 18

**Citation:** Kalgaonkar, S. J. (2025). Brotherhood Relation. *Space Sci J, 2*(2), 01-11.

## 1. Introduction

In the discussion that follows it is expected that the reader is familiar with elementary concepts of Set Theory such as sets, elements of a set, the curly bracket notation, belonging relation, subsets of a set, equality of two sets, union and intersection operations and complement of a set.

With these elementary ideas, we now start the discussion of a topic that will take us to study 'Binary Relations'.

**Ordered pair:**

In a set the only important information is 'Which are the elements of it?' and nothing else. The order of elements in a set is not important.
e.g. The sets {3, 4} and {4, 3} are equal or in other words they are 'same' sets.
But in many instances, we require the elements taken in specific order.
e.g. In an army parade when soldiers are standing in rows, the third person in the fourth row is not same as the fourth person in the third row. In order to achieve this, we introduce the following definition
**Definition:** If a and b are two elements then the set {{a}, {a, b}} is called the ordered pair of a and b with the understanding that 'a' is the first element and 'b' is the second.
This will be denoted as (a, b).
Thus
(a, b) = {{a}, {a, b}}
It is easy to see and to prove that this concept certainly enjoys the following two properties
(i) $(a, b) \neq (b, a)$ unless $a = b$
and (ii) $(a, b) = (c, d)$ iff $a = c$ and $b = d$.
As (i) says that if $a \neq b$ then $(a, b) \neq (b, a)$; it justifies the name given to it as an 'ordered pair'.

## 2. Cartesian Product of Two Sets

Definition: If A and B are two non–empty sets then the Cartesin product of A with B is defined as
$\{(x, y) / x \in A, y \in B\}$ and is denoted as $A \times B$.
Thus
$A \times B = \{(x, y) / x \in A, y \in B\}$
It can be verified that $A \times B \neq B \times A$ unless of course $A = B$.

Also if A has ‘m’ number of elements and B has ‘n’ number of elements the set A×B will have ‘mn’ elements in it. But the nature of elements of A×B is quite different from those of A or B
e.g. If A = {x, y} and B = {1, 2, 3} then
A×B = {(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3)}.
and B×A = {(1, x), (2, x), (3, x), (1, y), (2, y), (3, y)}.

## 3. Binary Relations

In everyday life we use statements of the following type Arjun 'is the father of ' Abhimanyu

or Lakshman 'is a brother of ' Rama or Usha 'is taller than' Rahul etc.
In Mathematics we use similar statements 4 'is greater than' 3
or $L_1$ 'is parallel to' $L_2$
or ∆ABC 'is similar to' ∆PQR etc.

In all the above examples the phrase put in the inverted commas is called a relation which relates the element before the phrase with the element that follows the phrase.

e.g. In the statement "Arjun 'is the father of ' Abhimanyu" the relation considered is 'is the father of ' and it relates Arjun to Abhimanyu. Similarly, in the statement " 4 'is greater than' 3 " the relation considered is 'is greater than' and it relates 4 with 3.
Please see that when 4 is related to 3 under the relation 'is greater than'; 3 is not related to 4 although 3 may be related to 4 under different relation 'is less than' but certainly not under 'is greater than'.

Consider the following example
Let S = {2, 3, 4, 6, 8} and the relation being considered is 'is a factor of ' The set of true statements formed in this case are

2 is a factor of 2
2 is a factor of 4
2 is a factor of 6
2 is a factor of 8
3 is a factor of 3
3 is a factor of 6
4 is a factor of 4
4 is a factor of 8
6 is a factor of 6
8 is a factor of 8

To avoid repeatedly writing the phrase 'is a factor of ' we can say that let us denote this complete phrase by R (the first letter in the word relation).

So now in this context R will always mean 'is a factor of ' The above set of true statements can now be reproduced as

2R2, 2R4, 2R6, 2R8, 3R3, 3R6, 4R4, 4R8, 6R6, 8R8.

Here is another way to look at the problem. Every statement of relation requires two elements. One is the element which is related and the other is the element to which the first is related. Hence such relations are called 'the binary relations'.
It is interesting to see that not only every statement incorporates a pair of elements but the order in which these elements occur is also important. For, in the above example 2R4 but $4 \not R 2$.

Hence the statement in a binary relation not only involves a pair but it involves 'an ordered pair'.
If it is stipulated that in the topic of binary relation (a, b) will mean that a is related to b i.e. in other words the related element will be written in the first place, then the above statements will now appear as (2, 2), (2, 4), (2, 6), (2, 8), (3, 3), (3, 6), (4, 4), (4, 8), (6, 6), (8, 8).
When the set of true statements is enclosed with the set bracket, the above relation will now look as {(2, 2), (2, 4), (2, 6), (2, 8), (3, 3), (3, 6), (4, 4), (4, 8), (6, 6), (8, 8)}.

This set will be called the relation set R. Thus in this example
R = {(2, 2), (2, 4), (2, 6), (2, 8), (3, 3), (3, 6), (4, 4), (4, 8), (6, 6), (8, 8)}
It can be easily seen that
$R \subset A \times A$ (although here $A \times A \not\subset R$)
Here the relation is first introduced with meaningful words like 'is a factor of ' and is developed to its abstract form as a subset of Cartesian product.

This development suggests us the definition of a binary relation as follows–
**Definition:** If A is a non–empty set then a non–empty subset R of A×A is called a binary relation in A.
Thus, $A \neq \phi$, $R \subset A \times A$ and $R \neq \phi$.
If $x, y \in A$ such that $(x, y) \in R$ we shall say that x is related to y in A by R. This is the same thing as writing xRy.
Thus $(x, y) \in R \equiv xRy$
Earlier we have seen that 2 is a factor of 4 but 4 is not a factor of 2.

i.e. 2R4 but $4 \not R 2$ or (2, 4) ∈ R but (4, 2) ∉ R
Thus the elements, here, cannot be interchanged without affecting the truth value of the statement.
We see that the elements 2, 4 are not symmetrically placed on both sides of R and indeed we are going to admit that this relation " is a factor of " is not symmetric.
Considering this and such other possibilities we define certain types of relations.

## 4. Types of Binary Relations

**1) Reflexive relation**

**Definition:** A binary relation R in a non–empty set A is said to be a reflexive relation if xRx for every x ∈ A i.e. if every element of A is related with itself by R.
The property is called reflexivity.

e.g.
(i) 'is a factor of' relation in any subset of integers is reflexive as every integer is a factor of itself.
(ii) If parallelism of lines is defined as 'two lines $L_1$ and $L_2$ are said to be parallel if they are co–planar and $L_1 \cap L_2 = \phi$' then parallelism cannot be reflexive. Because for any line L, although, L and L are co–planar $L \cap L = L$ and it cannot be empty.
(iii) But if parallelism is defined as 'two lines $L_1$ and $L_2$ in a plane are said to be parallel if their slopes are equal' then for any line L slope of L = slope of L and we shall have to accept that a line is always parallel with itself. Here the relation will be reflexive.
The above example will show that whether a relation is reflexive or not depends strictly on the definition of that relation i.e. it depends upon the meaning we attribute to the relation. The reader should keep this point in his mind to understand the further discussion.
(iv) In many books, on Set Theory, Modern Algebra the relation 'is a brother of' is considered in the set of humans. As the statement 'x is a brother of x' is found unacceptable to many authors the brotherhood relation is declared as non–reflexive in these books.

The reflexivity is discussed in detail because the main purpose of this writing is to establish the reflexivity of brotherhood relation.

**2) Symmetric Relation**

Definition: A binary relation R in A is said to be a symmetric relation
if xRy ⇒ yRx for all x, y ∈ A
The property is called symmetry.
As is seen the elements are symmetrically placed on both the sides of R and their interchange does not affect the truth value.
e.g.
(i) The relation 'is a factor of' is certainly not symmetric as we have seen that 2 is a factor of 4 but 4 is not a factor of 2 i.e. 2R4 but $4 \not R 2$ .
(ii) In the set of all triangles in a plane the congruency of triangles is symmetric.
Because $\Delta ABC \equiv \Delta PQR \Rightarrow \Delta PQR \equiv \Delta ABC$
(iii) In almost all books the brotherhood relation is declared to be non–symmetric in the set of all human beings. This is so because the statement that x 'is a brother of' y does not necessarily mean that y 'is a brother of ' x, as in some instances y may be a sister of x. But this immediately suggests that in the set of all <u>male</u> human beings the brotherhood relation will be symmetric. We will turn to this relation in detail later.

**3) Transitive relation**

Definition: A binary relation R in A is said to be a transitive relation if
xRy and yRz ⇒ xRz for all x, y, z ∈ A
The property is called transitivity.
It can be described in the words that if there are two statements of relation taken in specific order such that the second statement begins where the first ends then the relation is transited.
In xRy and yRz, the intermediate element y is sometimes looked upon as the media through which R gets transited from x to z.
(i) The relation 'is a factor of' is transitive.
If xRy and yRz then in this context it will mean that x is a factor of y
i.e. y = mx for some integer m and y is a factor of z i.e. z = ny for some integer n.
By substitution z = n(mx) = (nm)x and as nm is certainly an integer we get that x is a factor of z
i.e. xRz.
(ii) In a bookshop if we say that for two books x and y, xRy if the difference in their prices is less than Rs. 10; then this relation will be reflexive, symmetric but not transitive.
For,
a) |Price of x – Price of x| = 0 < Rs. 10
∴ xRx (Reflexivity

b) If xRy then
|Price of x – Price of y| < Rs. 10
$\therefore$ |Price of y – Price of x| < Rs. 10
[ since |a–b| = |b–a| ]
$\therefore$ yRx (Symmetry
c) Let x, y z be books such that Price of x = Rs. 100
Price of y = Rs. 108 Price of z = Rs. 116
Clearly here xRy and yRz but
| Price of x – Price of z | = 16 which is $\not<$ 10
$\therefore$ $x \not R z$

(iii) In almost all books of Set Theory , Modern Algebra, the brother relation is considered transitive probably because the following statement is ***felt*** to be obvious.
If x is a brother of y and y is a brother of z then x is a brother of z.

Now here is the main important analysis of this problem.
In the definition of transitivity, the condition xRy and yRz $\Rightarrow$ xRz for all x, y, z $\in$ A, it is not at all needed that x, y, z should be different from each other. The condition must be satisfied for *all* x, y, z whether equal or unequal. Hence a situation can arise of the following type.

Suppose that x and y are brothers of each other. Then we will have xRy and yRx.
The first statement ends on y where the second begins. Therefore y becomes the medium. Now if we are to agree that R is transitive then xRy and yRx must give us xRx i.e. we will have to accept that 'a person is brother of himself' i.e. brotherhood is reflexive at least among males. If we don't agree about the reflexivity then because of the above situation we have to say that the relation is not transitive. Thus brotherhood relation can be either reflexive and transitive or it can be non–reflexive and non– transitive. Truly speaking there is no reason why brotherhood should not be accepted as reflexive. The main trouble lies in the fact that the term "brother" is not defined here and it is taken as if it is a known concept. If at all we decide to define the term then our definition will obviously be of the following type.

**Definition:** x is said to be a brother of y if x is a male human being and both x and y have same parents.

Once this definition is given, whenever we consider a male human being x then the statement that 'x and x have same parents' becomes a true statement and we get xRx. i.e. ' x is a brother of x' and the brotherhood relation becomes reflexive at least in the set of all male humans (and therefore transitive also)
We now consider this problem from another angle in order to again establish the reflexivity of this brotherhood relation

**Equivalance Relations**
A binary relation R in A is said to be an equivalence relation if, R is
i) reflexive
ii) symmetric
and iii) transitive
Before establishing various properties of equivalence relation we will first show that in the above definition, all the three conditions are essential and no two of them can imply the third. This becomes clear from the following examples
i) In a bookshop if we say that for two books x and y, xRy if the difference in their prices is less than Rs. 10; then this relation will be reflexive, symmetric but not transitive.
Thus
Reflexivity and symmetry $\not\Rightarrow$ Transitivity
ii) If we consider the relation and 'is a factor of' then it becomes reflexive, transitive but not symmetric.
Thus
Reflexivity and Transitivity $\not\Rightarrow$ symmetry
iii) The above two cases are readily accepted but many students (and some teachers/authors) feel that the condition of reflexivity is a redundancy, it is really not necessary; it follows from symmetry and transitivity together; that it is only mentioned in the definition of equivalence relation as a formality just as the closure property is mentioned in the definition of a group. This thinking is based on the following logic.
Let R be a symmetric and transitive relation in a set A.
Now
xRy $\Rightarrow$ yRx by symmetry
We then get xRy and yRx

As the relation is transitive we get xRx and reflexivity.
But this argument is wrong. It only shows that in a symmetric and transitive relation if an element is related to some element of the set then it is related to itself. But it does not show that if an element is not related to any other element, then will it be related to itself or not. The answer is 'not necessarily'.

This suggests the construction of the following example Let A= {x, y, z}
Let R = {(x, x), (x, y), (y, x) (y, y)}
The reader is asked to verify that this relation is symmetric and transitive. But is it reflexive? No. Because $z \in A$ but $(z, z) \notin R$.
Thus
symmetry and Transitivity $\not\Rightarrow$ Reflexivity

These examples show that all the three conditions i.e. reflexivity, symmetry and transitivity are required in the definition of an equivalence relation.

The concept of equivalence relation has its roots in the concept of equality. Irrespective of the nature of objects we certainly have in our mind that

i) $x = x$ for any x
ii) $x = y \Rightarrow y = x$ for all x, y
and iii) $x = y$ and $y = z \Rightarrow x = z$ for all x, y, z

These properties are generalised and are named as reflexive, symmetric and transitive properties respectively. This clearly indicates that equality is indeed an example of equivalence relation.
<u>Some other examples</u>
i) Let I denote the set of all integers i.e. I = { . . ., −3, −2, −1, 0, 1, 2, 3,…}
We fix some positive integer, say 5. Once fixed this will not be changed now.
In I we define a relation as follows–
For $x,y \in I$ we say that xRy if 5/y–x. This is read as "5 divides y – x" meaning that y – x is divisible by 5 i.e. y – x has a factor 5 or y – x = 5.m for some $m \in I$
We will first show that this is an equivalence relation in I

a) Let $x \in I$

Then $x - x = 0 = 5 \cdot 0$, $0 \in I$.

5/x–x and xRx which mean R is reflexive.

b) Let $x, y \in I$ with xRy

$\therefore$ 5/y–x

$\therefore$ $y - x = 5 \cdot m$ for some $m \in I$

$\therefore$ $x - y = 5 \cdot (-m)$ where $-m \in I$

$\therefore$ 5/x–y

$\therefore$ yRx

$\therefore$ R is symmetric.

c) Let $x, y, z \in I$ with xRy and yRz

$\therefore$ 5/y–x and 5/z–y

$\therefore$ $y - x = 5 \cdot m$ and $z - y = 5 \cdot n$ for some $m, n \in I$

Adding these two equations

$(y - x) + (z - y) = 5m + 5n$

$\therefore$ $z - x = 5 \cdot (m + n)$ As m, n are in I, $(m+n) \in I$

$\therefore$ 5/z–x

$\therefore$ xRz

$\therefore$ R is transitive.

Thus R is an equivalence relation.
ii) We are very much (and mainly) interested in the "brother" relation. We investigate this in detail. For the time being we restrict ourselves to the set of all male human beings.
Let M = {all male humans} Let R mean "is a brother of"
Since we are analysing this relation now strictly Mathematically, it is necessary for us to define the term 'brother' formally. As mentioned earlier, we define
**Definition:** x is said to be a brother of y if x is male and both x and y have same parents.
Here

a) Let $x \in M$

$\therefore$ (x is male) and (x and x have same parents)

$\therefore$ xRx and R is reflexive.

b) Let $x, y \in M$ such that xRy

$\therefore$ (x and y are males) and (x and y have same parents)

$\therefore$ (y is male) and (y and x have same parents)

$\therefore$ yRx and R is symmetric.

c) Let $x, y, z \in M$ such that xRy and yRz

$\therefore$ x, y and z are males such that

x and y have same parents also y and z have same parents

$\therefore$ x and z have same parents

Thus (x is male) and (x and z have same parents)

$\therefore$ xRz and R is transitive.

Thus R is an equivalence relation.
Of course, for the sake of argument, one may say that if in the definition of 'brother' we introduce a condition that $x \neq y$ then reflexivity will be impossible. i.e. If we say that x is a brother of y if x is a male, $x \neq y$ and both x and y have same parents then brotherhood will not be reflexive (and of course it will not be transitive also)
But I think it will be more natural to accept the reflexivity of "brother" relation rather than trying to remove it by putting such additional condition. The reason for my thinking on these lines lies in the effect of equivalence relation in a set and the effect of a partition in a set. We proceed to study this in detail.

**Equivalence class of a set**
**Definition:** Let R be an equivalance relation in a set S. Let $a \in S$. Then the set of all elements of S which are related to 'a' is called as the equivalence class of a in S under R.

It is denoted as [a] (read as class a).
Thus

$[a] = \{x \in S / xRa\}$ or $[a] = \{x \in S / aRx\}$

(Because of the symmetry of R i.e. $xRa \equiv aRx$)

Clearly $[a] \subset S$ Consider the example,
Let I denote the set of all integers i.e. $I = \{..-3, -2, -1, 0, 1, 2, 3,\dots\}$
We fix some positive integer, say 5. Once fixed this will not be changed now. In I we define a relation as follows–
For $x, y \in I$ we say that xRy if 5/y–x. This is read as "5 divides y – x" meaning that y – x is divisible by 5 i.e. y – x has a factor 5 or y – x = 5.m for some $m \in I$

This is an equivalence relation in I.
Now if we take $0 \in I$ then

$[0] = \{ x \in I / xR0 \}$

$= \{ x \in I / 5|(0-x) \}$

$= \{ x \in I / 5 \text{ is a factor of } -x \}$

$= \{ x \in I / -x \text{ (hence x) is a multiple of } 5 \}$

$= \{ \ldots -15, -10, -5, 0, 5, 10, 15, 20, \ldots \}$

Similarly
$[1] = \{ \ldots -9, -4, 1, 6, 11, 16, \ldots \}$
The reader may try to find other equivalence classes and see how many he gets. Here are some important properties of equivalence classes.

**Property 1: An equivalence class is never empty.**
This follows almost immediately. As it is an equivalence relation, it is reflexive. Hence for any $a \in S$ we must have aRa.

$\therefore a \in [a]$

$\therefore [a] \neq \phi$

Thus an element always belongs to its equivalence class. [a] may or may not have any other element but it will certainly have 'a'.

**Property 2: Any two equivalence classes are either disjoint or equal.**
Proof: Let R be an equivalence relation in S. Let $a, b \in S$ and consider [a] and [b].

Obviously either $[a] \cap [b] = \phi$ or $[a] \cap [b] \neq \phi$
Case i) If $[a] \cap [b] = \phi$ then the two classes are disjoint and there is nothing to prove.
Case ii) If $[a] \cap [b] \neq \phi$ then there must exist at least one $x \in S$ such that $x \in [a] \cap [b]$.

$\therefore x \in [a]$ and $x \in [b]$

$\therefore$ xRa and xRb (by definition of equivalence classes

$\therefore$ aRx and xRb (by symmetry

$\therefore$ aRb (by transitivity

Now for any $y \in [a]$
we have yRa
But since aRb is already established, we get yRa and aRb

$\therefore$ yRb (by transitivity

$\therefore y \in [b]$

Thus, $y \in [a] \Rightarrow y \in [b]$

$\therefore [a] \subset [b]$ …..(i)

This result is derived from the statement aRb.

But by symmetry, aRb also gives us bRa.

Hence we can show that $[b] \subset [a]$ …..(ii)

From (i) and (ii) we get
$[a] = [b]$
Thus for any two equivalence classes [a] and [b] they are either disjoint or equal.
This can be described also as, two equivalence classes either have 'nothing' in common or have 'everything' in common.

For an equivalence relation R in S we may construct the set of all distinct equivalence classes of S under R and denote such set as $\wp$.
Thus, $\wp = \{[a] / a \in S$ and $[a]$ occurs only once$\}$
Then no two members of $\wp$ are equal hence as seen in property 2 they must be disjoint.

Consider $\bigcup_{[a]\in\wp} [a]$

$$x \in \bigcup_{[a]\in\wp} [a] \quad \Rightarrow \quad x \in [a] \text{ for some } [a] \in \wp$$

$$\Rightarrow \quad x \in S \quad \text{as } [a] \subset S \qquad \ldots\ldots\ldots\text{(iii)}$$

Equally

$$x \in S \quad \Rightarrow \quad x \in [x] \in \wp$$

$$\Rightarrow \quad x \in \bigcup_{[a]\in\wp} [a] \qquad \ldots\ldots\ldots\text{(iv)}$$

By (iii) and (iv)

$$\boxed{S = \bigcup_{[a]\in\wp} [a]}$$

Thus S is split into different subsets (as classes) such that any two are disjoint and all together give us S. To use a favourite language among Mathematicians; these classes are mutually disjoint and collectively exhaustive. But such a splitting of a set in different mutually disjoint subsets is called a partition of S. Here is the definition.

**Definition:** Let S be a non–empty set. A set $\wp$ of subsets of S is said to be a partition of S if

(i) $A \in \wp$ and $B \in \wp \Rightarrow A \cap B = \phi$

and (ii) $\bigcup_{A\in\wp} A = S$

The elements of $\wp$ are called as the members of partition.

With this definition and the earlier discussion we conclude that an equivalence relation in S induces a partition in S wherein the equivalence classes are the members of the partition.
Consider the example,
Let I denote the set of all integers i.e. I = { ..., −3, −2, −1, 0, 1, 2, 3,…}
We fix some positive integer, say 5. Once fixed this will not be changed now. In I we define a relation as follows–
For x, y ∈ I we say that xRy if 5/y–x. This is read as "5 divides y – x" meaning that y – x is divisible by 5 i.e. y – x has a factor 5 or y – x = 5·m for some m∈I.
The reader must have found that here we get only five distinct classes namely
[0] = {. . . –15, –10, –5, 0, 5,10, 15, 20, . . .}
[1] = {. . . –9, –4, 1, 6, 11, 16, . . .}
[2] = {. . . –13, –8, –3, 2, 7, 12, 17, . . .}
[3] = {. . . –12, –7, –2, 3, 8, 13, 18, . . .}
[4] = {. . . –11, –6, –1, 4, 9, 14, 19, . . .}
If we continue further we will get same classes again and again as [5] = [0]
[6] = [1]
and so on.
Hence the partition induced in I is
$\wp = \{[0], [1], [2], [3], [4]\}$

Just as an equivalence relation induces a partition it is equally true that a partition induces an equivalence relation in such a manner that the members of the partition become the equivalence classes. Here is the proof of this.

Let $\wp$ be a partition of a non–empty set S.

Hence S is split into different mutually disjoint subsets which are members of $\wp$. Define a relation R in S as follows
For x, y $\in$ S, we say that xRy iff both x and y belong to same member of the partition.
We then get

a) For any $x \in S$

since $S = \bigcup_{A \in \wp} A$,

$x \in A$ for some $A \in \wp$.

But then $x \in A$ and $x \in A$ [$p \Rightarrow p \wedge p$

$\therefore$ x and x belong to the same member A of $\wp$.

$\therefore$ xRx

$\therefore$ R is reflexive

b) Let $x, y \in S$ such that xRy.

By the definition of R, it means there is some $A \in \wp$ such that both x and $y \in A$.

i.e. $x \in A$ and $y \in A$

$\therefore$ $y \in A$ and $x \in A$ [ $p \wedge q \Leftrightarrow q \wedge p$

$\therefore$ y and x belong to same member, namely, A of $\wp$.

$\therefore$ yRx

$\therefore$ R is symmetric.

c) Let $x, y, z \in S$ such that xRy and yRz.

$xRy \Rightarrow \exists A \in \wp$ such that $x \in A$ and $y \in A$.

$yRz \Rightarrow \exists B \in \wp$ such that $y \in B$ and $z \in B$.

$\because$ A and $B \in \wp$ and $\wp$ is a partition, $A = B$ or $A \cap B = \phi$.

But as y is common to both A and B, $A \cap B \neq \phi$

$\therefore A = B$

Calling B as A now, we can write

($x \in A$ and $y \in A$) and ($y \in A$ and $z \in A$)

i.e. $x, y, z \in A$

In particular, x and z belong to the same member A of $\wp$.

$\therefore$ xRz

$\therefore$ R is transitive.

R is thus an equivalence relation in S induced by the partition $\wp$.

If we consider any element a of S then as $S = \bigcup_{A \in \wp} A$. We will get some A in $\wp$ such that $a \in A$.

Now [a] being the equivalence class of a in S under R,

$[a] = \{ x \in S / xRa \}$

As $a \in A$, xRa will mean $x \in A$ ($\because xRa \Rightarrow$ x and a both are in same member of the partition.)

$\therefore \quad [a] = \{ x \in S / x \in A \}$

i.e. $[a] = A$

Thus this equivalence relation induced by the partition $\wp$ is such that it's equivalence classes are exactly the members of the partition. We have seen now that with an equivalence relation there corresponds a partition and with a partition there corresponds an equivalence relation such that equivalence classes are same as the

members of the partition. Hence we can say that an equivalence relation and a partition are but two sides of the same coin.
We turn our attention once again to the set of all male human beings.
i.e. M = {all male humans}
Suppose we introduce a split in this set by saying "all brothers should form their groups"

The set M will be broken into different subsets such that each subset will comprise of men who are brothers of each other. If a certain male human being is the only son of his parents then he will form a singleton subset in this split. These different subsets of M would be mutually disjoint and if the union of all of them is taken then we will get our set M back. But this means that such a split forms a partition of M. We have seen earlier a partition induces an equivalence relation. So this partition of M will give us an equivalence relation in M such that the equivalence classes will be the members of the partition. In this equivalence relation an equivalence class will be comprised of men who are brothers of each other.

Since the members of an equivalence class are all related to each other, this relation that we get is nothing but 'the brother relation'. The very fact that we are getting this as an equivalence relation indicates that it is reflexive. If a person x is the only son of his parents (thus forming a singleton set in this partition) then also we have to accept xRx.

**In view of all that is said so far about equivalence relation and partition it is obvious that the brother relation should be accepted as reflexive.**

**Extension to all human beings**
The difference between the words 'brother' and 'sister' is only of gender. The remaining part of the condition of 'having same parents' is common to both of them. Hence it is suggested that this gender difference should be abandoned. In many instances we do this. For example, when we talk about 'international brotherhood' or when we say that 'the peaceful progress of mankind is possible if we behave like brothers irrespective of our race, religion or nationality' do we exclude all female human beings from our consideration? Certainly not.

Neglecting this gender difference we can identify the words 'brother' and 'sister' to mean same thing.
When this is done it is no longer necessary for us to consider only male human beings. We can extend the entire argument made so far to all human beings.

**Thus if x is any human being and R means 'is a brother of ' then we conclude xRx**
**or x is a brother of x or R is reflexive**

*Now, there is no intention, whatsoever, of connecting Mathematics and Meta–Physics. Nor I am trying to analyse anything said in the 'Srimad Bhagwadgeeta', as such an attempt will be beyond my capacity. Yet, when in the concluding part of the above epilogue I mentioned my humble opinion that 'brother relation' should be accepted as reflexive, I was reminded of similar statement in Srimad Bhagwadgeeta somewhere.*

*I searched and found that in chapter 6 stanza 5 Lord Krishna has said to Arjuna that*

आत्मैव ह्यात्मनो बंधूः ।

*meaning that 'we (and only we) are brothers of ourselves'.*

*Because of the word* एव *coming in* आत्मैव *(this is a combination of* आत्म एव *) the statement can be symbolically written as follows*

*If xRy means 'x is a brother of y' then for men x and y*

*xRx and* $(x \neq y \Rightarrow x \not R y)$

*Since* $p \wedge q \Rightarrow p$ *always, the above statement certainly gives us xRx.*

*Let me make this clear that the context of the above statement is quite different. It is cited here, only because, the similarity was felt interesting.*

।। श्रीकृष्णार्पणमस्तू ।।

(Srikrishnarpanmastu)

*Meaning that whatever is here written and done be offered to Lord Krishna.*