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Short Communication

A Unified Cross-Asset Framework with Value–Price Separation

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Abstract
This paper introduces a unified cross-asset framework that separates intrinsic "value" from market "price" and integrates both asset-specific and common risk factors in a single, coherent structure. By incorporating a state-space representation of key macroeconomic and market variables, the framework models the joint evolution of asset values and states. A unified pricing kernel links these values to observed prices, ensuring internal consistency across equities, bonds, and potentially other asset classes. We strengthen this framework by offering explicit conditions for no-arbitrage between equity and bond value formulations, refining the state-space dynamics to accommodate regime shifts or jumps, proposing a concrete empirical strategy grounded in a twostep Kalman filtering approach, and outlining practical implementation guidelines that bridge theory and practice. Through these enhancements, we provide a more robust theoretical foundation and clearer guidance for real-world application.

1. Introduction

Equities and bonds have traditionally been valued using distinct methodologies in both academic research and industry practice. Equity valuation techniques, such as discounted cash flow models, usually emphasize growth and earnings, while bond pricing relies more directly on coupon flows, default risks, and inflation considerations. Over time, researchers and practitioners have come to recognize that some risk factors, such as macroeconomic shocks and changes in investor risk appetite, affect both equities and bonds in a unified manner. These factors may influence both asset classes simultaneously, particularly during flight-to-quality episodes when correlations shift abruptly.

In this paper, we develop and refine a framework that applies a single discounting mechanism, often referred to as a pricing kernel, to both equities and bonds while preserving a distinction between the intrinsic or fundamental value of each asset and its observed market price. We build on earlier versions of this framework by addressing four key enhancements. First, we impose explicit no-arbitrage conditions that ensure that the equity and bond value specifications do not lead to exploitable price differentials. Second, we enrich the statespace model by allowing for jumps or regime-switches, which become particularly relevant when investors unexpectedly change their risk perception. Third, we propose a two-step Kalman filtering approach, which clarifies the empirical procedures needed to estimate the underlying state variables from macroeconomic data and asset market observations. Fourth, we outline guidelines to handle parameter instability, detect regime changes, and improve computational efficiency, thereby providing clearer directions for implementing the theory in real-world settings.

2. Literature Review

Several strands of literature converge in this unified framework. Consumption-based asset pricing, pioneered by Lucas (1978), Breeden (1979), and extended by Cochrane (2001), anchors the theoretical foundation [1-3]. This strand emphasizes the marginal rate of substitution in consumption as the basis for pricing all assets, suggesting a single pricing kernel can indeed be applied across multiple asset types. The term-structure literature, including classic contributions by Vasicek (1977) and Duffie and Kan (1996), has deepened our understanding of how risk-free rates, inflation, and other yield curve factors evolve and inform bond prices [4,5]. Macro-finance integration, explored by Ang and Piazzesi (2003) and studied further by Caballero and Krishnamurthy (2008) and Bekaert and Engstrom (2010), has shown that common factors such as growth and risk premia can affect multiple asset classes and can shift significantly under different market regimes [6-8].

Although many previous works have studied either equity or bond pricing under comprehensive economic conditions, few frameworks explicitly disentangle the fundamental value drivers of these assets from the market discounting process, while still imposing no-arbitrage across asset classes. Our present framework extends existing research by introducing such a no-arbitrage condition, which ensures that the separate value formulations for equities and bonds cannot be exploited to generate riskless profit. Furthermore, by enriching the statespace model with jumps or regime switches, we more closely capture the abrupt correlation changes often witnessed during crises.

3. Core Theoretical Innovation: The Value–Price Separation

3.1 Unified Value Function with No-Arbitrage Conditions

The key theoretical innovation of this paper lies in unifying the specification of an asset's intrinsic value for both equities and bonds while ensuring that no-arbitrage conditions hold. We introduce a value function, $V(t)$, defined separately for equities and bonds:

$$V(t) = \begin{cases} E_t \left[1 + g(S_t)\right] Q(S_t), & \text{(Equities)}, \\ C_t \left[1 - L(t)\right]\left[1 - D(\pi_t)\right], & \text{(Bonds)}, \end{cases}$$

where E_t denotes current equity earnings, $g(S_t)$ represents growth as a function of the state S_t, $Q(S_t)$ is a measure of firm-specific quality, C_t is a coupon-like payout for a bond, $L(t)$ is a loss or default probability, and $D(\pi_t)$ captures inflation-related effects.

To avoid arbitrage, the discounted payoff of an asset derived from these value formulas must not allow the creation of a riskless profit by combining or decomposing equity and bond claims on the same underlying cash flows. Formally, if a portfolio that synthetically replicates a firm's unlevered cash flows were to produce higher payoffs under one specification (equity) than under another (bond) while facing the same risk, an arbitrage condition would emerge. Therefore, we impose constraints on $g(S_t)$, $L(t)$, and $D(\pi_t)$ to ensure that neither the equity-related nor the bond-related part of the firm's cash flows produces a strictly higher payoff without corresponding risk exposure.

4. Mathematical Framework Refinement

4.1 Extended State-Space Dynamics with Jumps

We define a state vector,

$$S_t = \left[g_t,\, \pi_t,\, r_t,\, \lambda_t,\, \theta_t\right]^\top,$$

where g_t represents growth, π_t denotes inflation, r_t is the risk-free rate, λ_t is a measure of the market price of risk or risk aversion, and θ_t is a factor capturing cross-asset correlations. In earlier versions of the framework, the dynamics of the combined system, consisting of $V(t)$ and $S(t)$, were modeled in a linear fashion:

$$\left[V(t+1),\, S(t+1)\right] = F\left[V(t),\, S(t)\right] + \Sigma(t)\,\varepsilon(t+1).$$

This linear model captures incremental changes in both the value functions and the state variables.

In order to incorporate abrupt regime changes or jumps in risk perception, we enrich this specification as follows:

$$\left[V(t+1),\, S(t+1)\right] = F\left[V(t),\, S(t)\right] + \Sigma(t)\,\varepsilon(t+1) + \Gamma(t)\,J(t+1),$$

where $\Gamma(t)$ translates the size of the jump or regime shift into changes in $V(t)$ and $S(t)$, and $J(t+1)$ is a jump process that may be represented by a Poisson random variable or a Markov-switching regime indicator. This extended specification addresses phenomena such as flight-to-quality spikes, where the correlation between equities and bonds may suddenly shift, as well as crises where default probabilities or market risk aversion can jump sharply.

4.2 Unified Pricing Kernel

We employ a consumption-based or CRRA-type pricing kernel:

$$M(t,t+1) = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \exp\left\{-\lambda(S_t)^\top\,\Sigma(t)\,\varepsilon(t+1)\right\},$$

where β is the discount factor, $\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$ captures time-varying marginal utility of consumption for a CRRA investor, and $\lambda(S_t)$ interacts with $\Sigma(t)\,\varepsilon(t+1)$ to represent the statedependent pricing of risk. By design, this kernel is applied uniformly to the discounted value of both equity and bond payoffs, ensuring consistency across the two asset classes and eliminating opportunities for arbitrage that might arise if different discount rates were used.

4.3 Price Formation for Equities and Bonds

Under this unified approach, the price of an equity is given by

$$P_e(t) = \mathbb{E}_t\left[\sum_{s=t+1}^{\infty} M(t,s)\,\big(V_s + R(s)\big)\right] + \mathbb{E}_t\left[M(t,\infty)\,T(\infty)\right],$$

where $R(s)$ represents dividends or other near-term payouts, and $T(\infty)$ can be a terminal value in perpetuity or liquidation value. For bonds, the price becomes

$$P_b(t) = \mathbb{E}_t\left[\sum_{s=t+1}^{T} M(t,s)\,V_s\right] + \mathbb{E}_t\left[M(t,T)\,\text{Principal}\right],$$

where V_s here includes coupon-related value at each future date, and the principal is repaid (assuming no default) at maturity T. In both cases, jumps or regime changes affect expected future payouts through V_s and change the discount factor through $M(t,s)$, allowing the model to capture the abrupt price movements often witnessed in real markets.

5. Portfolio Optimization

Consider a portfolio consisting of equities and bonds, with values $V_e(t)$ and $V_b(t)$ respectively, and weights we and wb. The total portfolio value at time t is

$$V_p(t) = w_e^\top V_e(t) + w_b^\top V_b(t).$$

The goal is to maximize expected utility of the terminal portfolio value $V_p(T)$:

$$\max_{w_e, w_b} \mathbb{E}\Big[U\big(V_p(T)\big)\Big] \quad \text{subject to} \quad w_e + w_b = 1 \quad \text{and} \quad w_e, w_b \geq 0.$$

In dynamic settings, where flight-to-quality behavior or regime shifts can abruptly alter the correlation structure captured by $\Sigma(t)$ and $\Gamma(t)\, J\,(t+1)$, the portfolio weights may be recalibrated to adapt to the new market environment. This dynamic perspective acknowledges that both the payoffs and the risk premia for equities and bonds may move in concert, particularly in stress regimes.

6. Empirical Strategy

6.1 Two-Step Kalman Filtering for State Estimation

To implement and estimate the proposed model, we suggest a two-step Kalman filtering procedure, as outlined in the pseudocode below:

```
def estimate_states(data, params):
"""
Two-step Kalman Filter for Macro + Asset States
" " "
# 1) Estimate Macro States
kf_macro = KalmanFilter(
state_transition=derive_transition_matrix(params,for_macro=True),
observation_matrix=derive_observation_matrix(params, macro_obs=True)
)
states_macro = kf_macro.filter(data.macro_data)
# 2) Estimate Full States (Incorporating Asset Prices)
kf_full = KalmanFilter(
state_transition=derive_transition_matrix(params,for_full=True),
observation_matrix=derive_observation_matrix(params, asset_obs=True)
)
states_full = kf_full.filter(data.asset_data, initial_state=states_macro[-1])
return states_full
```

In the first step, macroeconomic variables such as GDP growth, inflation, and interest rates are used to filter out the underlying growth component g_t, inflation π_t, and risk aversion λ_t. In the second step, the filtered macro states are used as an initial foundation when incorporating equity and bond price data. This hierarchical approach allows the model to disentangle the macroeconomic environment from asset-specific signals, providing a more stable identification of the cross-asset factor θ_t.

6.2 Handling Regime or Jump Dynamics

When including the jump or regime-switching component $\Gamma(t)\, J\,(t+1)$, it may be necessary to use extensions of the Kalman filter designed for non-linear or switching processes. One can adopt a Markov-switching Kalman filter or a Hamilton filter if jumps follow a Markov process, or turn to particle filters for more general jump specifications. These methods can better capture episodes where large shocks alter default probabilities or market perceptions in a discrete manner.

6.3 Calibration and Model Validation

Researchers and practitioners can calibrate the functions $g(S_t)$, $\lambda(S_t)$, $L(t)$, and $D(\pi_t)$ by maximizing the likelihood of the observed macro and asset prices, or by employing Bayesian methods with informative priors if so desired. Validation of the model involves comparing the implied bond yields, equity returns, and correlation structures to their real-world counterparts over both in-sample and out-of-sample periods. In particular, one can test how effectively the model anticipates shifts in flight-to-quality behavior, by examining changes in correlations and price dynamics during known stress episodes.

7. From Mean–Variance to the New Approach

Traditional mean–variance portfolio construction proceeds by specifying an expected return vector and a covariance matrix, then solving a quadratic optimization problem. Formally, weights w are selected to maximize

$$\mathbf{w}^\top \boldsymbol{\mu} - \frac{\kappa}{2}\mathbf{w}^\top \Sigma\, \mathbf{w},$$

subject to the sum of weights being one. Although this approach has proven useful over decades, it assumes the expected return and covariance estimates remain stable or can be estimated from historical data.

We outline a method to bridge mean–variance with our state-based framework by approximating the state-dependent returns through a linearization. If $\tilde{r}_t(S_t)$ is the return function and S_t is the state vector, we can expand $\tilde{r}_t(S_t)$ around a baseline S_0:

$$\tilde{r}_t(\mathbf{S}_t) \approx \tilde{r}_t(\mathbf{S}_0) + \nabla_{\mathbf{S}} \tilde{r}_t(\mathbf{S}_0)\,\big(\mathbf{S}_t - \mathbf{S}_0\big).$$

Using the Kalman filter estimates of S_t, practitioners can derive implied means and covariances of returns, μ^* and Σ^*, which can be fed into a conventional mean–variance optimizer. Re-optimizing these parameters periodically (or whenever a regime shift is detected) ensures the portfolio construction remains responsive to the time-varying risk environment modeled in our framework.

8. Practical Implementation Guidelines

8.1 Handling Parameter Instability

It is important to note that parameter estimates may change over time, particularly if market conditions evolve or if there are multiple structural breaks. In practice, users may employ rolling or expanding windows to recalibrate the state transition and observation matrices. They may also incorporate outlier detection techniques to distinguish genuine regime shifts from transient noise or anomalous data points.

8.2 Dealing with Regime Changes

Because regimes can shift rapidly, practitioners can apply Markov-switching Kalman filters or Hamilton filters when there is reason to believe that different macroeconomic regimes appear according to a Markov process. Alternatively, if data suggests more random or discrete jumps in key state variables, particle filters can be deployed to capture the multimodal distribution of jumps. Early warning indicators, such as credit spreads or volatility indices, can serve as triggers to intensify the jump processes through $\Gamma(t)$ during periods of heightened market stress.

8.3 Computational Efficiency

In large-scale or high-frequency scenarios, computational efficiency becomes a critical concern. Techniques such as vectorization in NumPy or JAX can speed up the linear algebra operations required by the Kalman filter, and parallelization can be pursued for particle filtering in more advanced jump models. These optimizations are essential to ensure that real-time or near-real-time risk assessment remains feasible in practice.

8.4 Linking Theory to Practice

In practical settings, macro data might be available at monthly or quarterly intervals, while market data for equities and bonds may be updated daily or weekly. Researchers should therefore pay special attention to aligning the Kalman filter state updates with the actual data frequencies, possibly by using bridging or interpolation techniques. In communicating results, it can be helpful to summarize the model output as approximate means and covariances, allowing practitioners more accustomed to mean–variance methods to interpret and act on the results without having to fully rewrite existing processes.

9. Discussion of Academic Contribution

The theoretical and empirical contributions of this framework can be summarized by highlighting four main advances. First, the explicit no-arbitrage constraints across equity and bond valuations mark a step forward in ensuring that a single discounting mechanism cannot be exploited through trivial manipulations of either asset class. Second, the introduction of a jump or regime-switching component $\Gamma(t)\, J\,(t+1)$ systematically accounts for flight-toquality phenomena and abrupt correlation changes that have been well-documented in crisis periods. Third, the proposed two-step Kalman filter procedure clarifies a practical route for combining macroeconomic and asset-level data in a robust manner, thereby strengthening the operational viability of the model. Fourth, the discussion of bridging the new framework with traditional mean–variance provides a valuable method for practitioners to gradually incorporate state-space insights into existing portfolio management strategies.

10. Conclusion

This paper refines and strengthens a unified cross-asset framework that distinguishes intrinsic asset value from its market price and applies a coherent pricing kernel to both equity and bond valuation. We have demonstrated how no-arbitrage can be enforced by ensuring that separate formulas for equity and bond values cannot be combined to generate riskless profit. The core framework has been extended to allow for jumps or regime shifts, capturing the type of abrupt price and correlation changes frequently observed during market stress. A twostep Kalman filtering approach was proposed to handle state estimation, and we explained how rolling, expanding, or switching methods can be applied to accommodate parameter instability over time.

We also outlined how practitioners, accustomed to conventional mean–variance portfolio construction, can integrate the richer state-based model into their existing workflows through linearization of state-dependent returns and periodic re-optimization. Future studies could expand the framework to include additional asset classes such as credit, foreign exchange, or commodities. Moreover, exploring robust optimization under parametric uncertainty might help further mitigate the risk of miscalibration, especially when historical data offers limited insight into the frequency or magnitude of tail events. Overall, these refinements and practical guidelines underscore the feasibility of a comprehensive macro-finance model that unifies multiple asset classes under a single economic umbrella, thereby offering coherent valuation, dynamic portfolio management, and deeper insights into cross-asset risk.

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